PARAKRAM

JEE MAIN

CURATED BY EXPERT FACULTY OF PW

MATHEMATICS

1500+

Selected MCQs
to Boost your Confidence

EDITION: First

Published By: Physicswallah Limited

Physics Wallah Publication

ISBN:	978-93-48446-91-6
MRP:	549/-
Mobile App:	Physics Wallah (Available on Play Store)
Website:	www.pw.live
Youtube Channel:	Physics Wallah - Alakh Pandey JEE Wallah Competition Wallah NCERT Wallah
Email:	publication@pw.live
SKU Code:	3cb1fd75-f284-46cd-889e-f0d4bbd630a7

PREFACE

A highly skilled professional team of Physics Wallah (PW) works arduously to ensure that the students receive the best content for the **JEE** exam.

From the beginning, the whole content team comprising faculties, DTP operators, Proofreaders and others are involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

Faculties have adopted a new style of presenting the content in easy-to-understand language and have provided the team with their guidance and supervision throughout the creation of this Study Material.

Physics Wallah (PW) strongly believes in conceptual and fun-based learning. PW provides highly exam-oriented content to bring quality and clarity to the students.

A plethora of **JEE Study Material** is available in the market but PW professionals are continuously working to provide the supreme Study Material for our **JEE** students.

This Study Material adopts a multi-faceted approach to master and understanding the concepts by having a rich diversity of questions asked in the examination and equip the students with the knowledge for the competitive exam.

The main objective of the study material is to provide a large number of quality problems with varying cognitive levels to facilitate the teaching-learning of concepts that are presented through the book.

It has become popular among aspirants because of its easy-to-understand language.

Students can benefit themselves by attempting the exercise given in this problem booklet.

The questions are strictly designed in accordance with the exam relevant topics that help to develop examination temperament within the students.

Mastering the Physics Wallah (PW) study material curated by the PW team, the students can easily qualify for the exam with a top Rank in the **JEE**.

In each chapter, for better understanding, questions have been classified according to the latest syllabus of **JEE Mains**.

- The nature and diversity of the equations help students to ace the examination.
- Quality questions to strengthen the concept of the topic at the zenith level.

BOOK FEATURES

- Topic wise **MCQs** and Integer type questions
- Strictly as per the latest **NTA** syllabus
- Assertion Reason, Matrix match & Statement based questions also included in exercises.

CONTENTS

CHAPTER 01

SETS

Single Option Correct Type Questions (01 to 57)

1. The set of intelligent students in a class is-
(1) a null set
(2) a singleton set
(3) a finite set
(4) not a well defined collection

2. Which of the following is the empty set
(1) $\{x : x$ is a real number and $x^2 - 1 = 0\}$
(2) $\{x : x$ is a real number and $x^2 + 1 = 0\}$
(3) $\{x : x$ is a real number and $x^2 - 9 = 0\}$
(4) $\{x : x$ is a real number and $x^2 = x + 2\}$

3. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is
(1) Null set
(2) Singleton set
(3) Infinite set
(4) not a well defined collection

4. Which of the following is true?
(1) $[3, 7] \subseteq (2, 10)$
(2) $(0, \infty) \subseteq (4, \infty)$
(3) $(5, 7] \subseteq [5, 7)$
(4) $[2, 7] \subseteq (2.9, 8)$

5. The number of subsets of the power set of set $A = \{7, 10, 11\}$ is:
(1) 32 (2) 16
(3) 64 (4) 256

6. Which of the following collections is not a set?
(1) The collection of natural numbers between 2 and 20
(2) The collection of numbers which satisfy the equation $x^2 - 5x + 6 = 0$
(3) The collection of prime numbers between 1 and 100.
(4) The collection of all intelligent women in Jalandhar.

7. The set $A = \{x : x$ is a positive prime $< 10\}$ in the tabular form is
(1) $\{1,2,3,5,7\}$ (2) $\{1,3,5,7,9\}$
(3) $\{2,3,5,7\}$ (4) $\{1 ,3,5,7\}$

8. Which of the following sets is an infinite set?
(1) Set of divisors of 24
(2) Set of all real number which lie between 1 and 2
(3) Set of all human beings living in India.
(4) Set of all three digits natural numbers

9. Power set of the set $A = \{\phi, \{\phi\}\}$ is :
(1) $\{\phi, \{\phi\}, \{\{\phi\}\}\}$
(2) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$
(3) $\{\phi, \{\phi\}, A\}$
(4) $\{\{\phi\}, \{\{\phi\}\}\}$

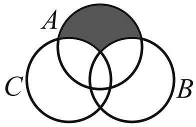
10. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is
(1) $\{3\}$ (2) $\{1, 2, 3, 4\}$
(3) $\{1, 2, 4, 5\}$ (4) $\{1, 2, 3, 4, 5, 6\}$

11. Let $A = \{x : x \in R, -1 < x < 1\}$, $B = \{x : x \in R, x \le 0 \text{ or } x \ge 2\}$ and $A \cup B = R - D$, then the set D is
(1) $\{x : 1 < x \le 2\}$ (2) $\{x : 1 \le x < 2\}$
(3) $\{x : 1 \le x \le 2\}$ (4) $\{x : 1 < x < 2\}$

12. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is:
(1) $\{2, 3, 5\}$ (2) $\{3, 5, 9\}$
(3) $\{1, 2, 5, 9\}$ (4) $\{1, 2, 3, 5, 9\}$

13. If $A = \{2, 3, 4, 8, 10\}$, $B = \{3, 4, 5, 10, 12\}$, $C = \{4, 5, 6, 12, 14\}$ then $(A \cap B) \cup (A \cap C)$ is equal to
(1) $\{3, 4, 10\}$ (2) $\{2, 8, 10\}$
(3) $\{4, 5, 6\}$ (4) $\{3, 5, 14\}$

14. The shaded region in the given figure is



(1) $A \cap (B \cup C)$ (2) $A \cup (B \cap C)$
(3) $A \cap (B - C)$ (4) $A - (B \cup C)$

15. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$A = \{1, 2, 5\}$, B $= \{6, 7\}$, then $A \cap B'$ is
(1) B' (2) A
(3) A' (4) B

16. If $A = \{x : x = 4n + 1, n \leq 5, n \in N\}$ and $B = \{3n : n \leq 8, n \in N\}$, then $A - (A - B)$ is :
(1) $\{9, 21\}$ (2) $\{9, 12\}$
(3) $\{6, 12\}$ (4) $\{6, 21\}$

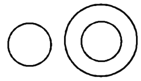
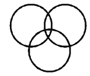
17. $A \cup B = A \cap$ B if:
(1) $A \subset B$ (2) $A = B$
(3) $A \supset B$ (4) $A \subseteq B$

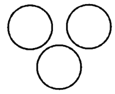
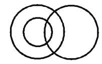
18. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N, b \geq 2, c \geq 2$ are relatively prime, then which one of the following is correct ?
(1) $b = cd$
(2) $c = bd$
(3) $d = bc$
(4) $d^2 = bc$

19. Consider the following statements:
1. $N \cup (B \cap Z) = (N \cup B) \cap Z$ for any subset B of R, where N is the set of positive integers, Z is the set of integers, R is the set of real numbers.
2. Let $A = \{n \in N : 1 \leq n \leq 24, n$ is a multiple of 3$\}$. There exists no subset B of N such that the number of elements in A is equal to the number of elements in B.

Which of the above statement(s) is/are correct?
(1) 1 only
(2) 2 only
(3) Both 1 and 2
(4) Neither 1 nor 2

20. Which of the following venn-diagrams best represents the sets of females, mothers and doctors?

(1)  (2) 

(3)  (4) 

21. Let A and B be two sets. Then:
(1) $n(A \cup B) \leq n(A \cap B)$
(2) $n(A \cap B) \leq n(A \cup B)$
(3) $n(A \cap B) = n(A \cup B)$
(4) can't be say

22. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers are-
(1) at least 30 (2) at most 20
(3) exactly 25 (4) exactly 30

23. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons traveling by car or bus is
(1) 80 percent (2) 40 percent
(3) 60 percent (4) 70 percent

24. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy newspaper A only is
(1) 3100 (2) 3300
(3) 2900 (4) 1400

25. In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali. Number of people who can speak Hindi only is
(1) 300 (2) 400
(3) 500 (4) 600

26 In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali. Number of people who can speak Bengali only is

(1) 150 (2) 250
(3) 50 (4) 100

27. In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali. . Number of people who can speak both Hindi and Bengali is

(1) 50 (2) 100
(3) 150 (4) 200

28. Let A_1, A_2 and A_3 be subsets of a set X. Which one of the following is correct ?

(1) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing elements of each of A_1, A_2 and A_3
(2) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing either A_1 or $A_2 \cup A_3$ but not both
(3) The smallest subset of X containing $A_1 \cup A_2$ and A_3 equals the smallest subset of X containing both A_1 and $A_2 \cup A_3$ only if $A_2 = A_3$
(4) None of these

29. Let A, B, C be distinct subsets of a universal set U. For a subset X of U, let X' denote the complement of X in U.

Consider the following sets :

1. $((A \cap B) \cup C)' \cap B')' = B \cap C$
2. $(A' \cap B') \cap (A \cup B \cup C') = (A \cup (B \cup C))'$

Which of the above statement(s) is/are correct?

(1) 1 only
(2) 2 only
(3) Both 1 and 2
(4) Neither 1 nor 2

30. Let U be set with number of elements in U is 2009.

Consider the following statements:

I. If A, B are subsets of U with $n(A \cup B) = 280$, then $n(A' \cap B') = x_1^3 + x_2^3 = y_1^3 + y_2^3$ for some positive integers $x_1, x_2 y_1, y_2$

II. If A is a subset of U with $n(A) = 1681$ and out of these 1681 elements, exactly 1075 elements belong to a subset B of U, then $n(A - B) = m^2 + p_1 p_2 p_3$ for some positive integer m and distinct primes p_1, p_2, p_3

Which of the statements given above is / are correct ?

(1) I only (2) II only
(3) Both I and II (4) Neither I nor II.

31. Consider the following statements:

1. If $A = \{(x, y) \in [R \times R : x^3 + y^3 = 1]$ and $B = \{(x, y) \in [R : x - y = 1]\}$, then $A \cap B$ contains exactly one element.
2. If $A = \{(x, y) \in [R \times R : x^3 + y^3 = 1]$ and $B = \{(x, y) \in [R : x + y = 1]\}$, then $A \cap B$ contains exactly two elements.

Which of the above statement(s) is/are correct?

(1) 1 only
(2) 2 only
(3) Both 1 and 2
(4) Neither 1 and 2

32. In a class of 42 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is:

(1) 15 (2) 30
(3) 22 (4) 27

33. For real numbers x and y, we write $x\ R\ y \Rightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is
(1) Reflexive
(2) Symmetric
(3) Transitive
(4) Equivalence relation

34. Let X and Y be two sets.
Statement-1 $X \cap (Y \cup X)' = \phi$
Statement-2 If $X \cup Y$ has m elements and $X \cap Y$ has n elements then symmetric difference $X \Delta Y$ has $m - n$ elements.
(1) Statement-1 is True, Statement-2 is True
(2) Statement-1 is False, Statement-2 is False
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

35. Match the set P in column one with its superset Q in column II

Column-I (Set P)		Column-II (Set Q)	
A.	$[3^{2n} - 8n - 1 : n \in N\}$	**P.**	$\{49\ (n-1) : n \in N$
B.	$\{2^{3n} - 1 : n \in N\}$	**Q.**	$\{64\ (n-1) : n \in N\}$
C.	$\{3^{2n} - 1 : n \in N\}$	**R.**	$\{7n : n \in N\}$
D.	$\{2^{3n} - 7n - 1 : n \in N\}$	**S.**	$\{8n : n \in N\}$

	A	B	C	D
(1)	Q	R	S	P
(2)	R	S	P	Q
(3)	S	P	Q	R
(4)	P	Q	R	S

36. A and B are two sets such that $n(A) = 3$ and $n(B) = 6$, then
(1) minimum value of $n(A \cup B) = 6$
(2) minimum value of $n(A \cup B) = 9$
(3) maximum value of $n(A \cup B) = 6$
(4) maximum value of $n\,(A \cap B) = 6$

37. In a survey, it was found that 21 persons liked product A, 26 liked product B and 29 liked product C. If 14 persons liked products A and B, 12 liked products C and A, 13 persons liked products B and C and 8 liked all the three products then which of the following is true?
(1) The number of persons who liked the product C only = 14
(2) The number of persons who like the products A and B but not C = 6
(3) The number of persons who liked the product C only = 6
(4) The number of persons who like the products A and B but not C = 12

38. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then:
(1) $A = C$ (2) $B = C$
(3) $A \cap B = \phi$ (4) $A = B$

39. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is:
(1) 5^2 (2) 3^5
(3) 2^5 (4) 5^3

40. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to
(1) X (2) Y
(3) N (4) $Y - X$

41. If a set contains m element and another set contains n element. If 56 is the difference between the number of subsets of both sets then find (m, n)
(1) 3, 6 (2) 6, 3
(3) 8, 3 (4) 3, 8

42. Which of the following sets is empty set?
(1) $A = \{x : x \in N, 3 < x \leq 4\}$
(2) $B = \{x : x$ is prime , $90 < x < 96\}$
(3) $C = \{\, x : x$ is an even prime$\}$
(4) $D = \{x : x \in$ Rational numbers & $1 < x < 2\}$

43. Which of the following are pairs of equivalent sets?
(1) $A = \{1,2,3\}$ $B = \{3, 6, 9\}$
(2) $A = \{0\}$ $B = \phi$
(3) $A = \{-2, -1\}$ $B = \{1,2,3\}$
(4) $A = (x ; x \in N, x < 3\}$
$B = (x : x \in W, x < 3\}$

44. Which of the following is false?
(1) Set of all triangles in a plane is infinite set
(2) Set of all lines parallel to the y-axis is infinite set
(3) Set of all points on the circumference of a circle is finite set
(4) Set of all positive integers greater than 100 is infinite set.

45. If $A = \{1,\{2,3\},4\}$. Which of the following statement is false?
(1) $\{2,3\} \in A$
(2) $\{\{2,3\}\} \subseteq A$
(3) $\{1,4\} \subseteq A$
(4) $\phi \in A$

46. Which of the following sets is a pair of disjoint sets?
(1) $A = \{1, 2, 3, 4\}$ $B = \{x : x$ is prime number, $x \leq 11\}$
(2) $A = \{x : x \in N\}$, B $\{x : x$ is prime number, $x \geq 3\}$
(3) $A = \{x : x \in N$, x is even$\}$, $B = \{x : x$ is prime, $x \geq 4\}$
(4) $A = \{x : x \in N$, x is odd$\}$ $C = \{x : x$ is composite number$\}$

47. If A and B are two sets containing 5 and 7 elements respectively, then maximum and minimum number of elements in $A \cup B$ respectively are
(1) 7,5
(2) 12,5
(3) 7,0
(4) 12, 7

48. If $A = \{x : x$ is a prime number $< 25\}$ and $B = \{x : x$ is composite number $< 20\}$ then
(1) $n(A \cup B) = 20$
(2) $n(A \cap B) = 1$
(3) $n(A \cup B) = 18$
(4) $n(A \cap B') = 9$

49. If $P(A)$ denotes power set of A, then which of the following is correct?
(1) $n(P(P(P(\phi)))) = 2$
(2) $n(P(P(P(P(\phi))))) = 8$
(3) $n(P(P(\phi)) + n(P(P(P(P(\phi)))) = 18$
(4) $n(P(\phi)) + n(P(P(\phi))) = 4$

50. $A' \cup \{(A \cup B) \cap B'\} =$
(1) $A' \cap B'$
(2) $(A \cap B)'$
(3) $A' \cap B$
(4) $A \cap B'$

51. If X and Y are two sets, then $X \cap (X \cap Y)'$ equals
(1) X
(2) Y
(3) ϕ
(4) $X \cup Y$

52. If $N_a = \{an : n \in N\}$, then $N_3 \cap N_5 =$
(1) N_3
(2) N_6
(3) N_{15}
(4) N_5

53. The number of elements in the set $\{(x, y) : x^2 + 4y^2 = 45, x, y \in Z$, where Z is the set of all integers) is:
(1) 0
(2) 4
(3) 8
(4) 12

54. If a set contains m element and another set contains n elements. If 144 is the sum of number of subsets of both sets then (m,n) can be**:**
(1) 4, 6
(2) 7,2
(3) 7,4
(4) 8,1

55. Let $X = \{1,2,3,4)$. The number of different ordered pairs (Y,Z) that can be formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty is:
(1) 81
(2) 16
(3) 243
(4) 64

56. An investigator interviewed 100 students to determine their preferences for the three drink: milk (M), coffee (C) and tea (T). He reported the following: 10 students had all the three drinks M, C and T; 20 had M and C; 30 had C and T; 25 had M and T; 12 had M only; 5 had C only; and 8 had T only. How many did not take any of the three drinks?

(1) 20 (2) 16
(3) 25 (4) 80

57. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. then total number of subsets of X having atleast one and atmost eight elements is

(1) 510 (2) 511
(3) 255 (4) 254

Integer Type Questions (58 to 67)

58. If $A = \{x : -3 < x < 3, x \in Z\}$ then the number of subsets of A is –

59. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?

60. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A' \cap B') =$

61. A class has 175 students. The following data shows the number of students choosing one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have chosen Mathematics alone?

62. 31 candidates appeared for an examination, 15 candidates passed in English, 15 candidates passed in Hindi, 20 candidates passed in Sanskrit. 3 candidates passed only in English. 4. candidates passed only in Hindi, 7 candidates passed only in Sanskrit. 2 candidates passed in all the three subjects How many candidates passed only in two subjects?

63. In an examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects? (in percentage)

64. In a survey of 100 students, the number of students studying the various languages is found as, English only 18, English but not Hindi 23, English and Sanskrit 8, Sanskrit and Hindi 8, English 26, Sanskrit 48 and no language 24 then number of students studying Hindi is

65. The number of positive integers from 1 to 1000, which are not divisible by 2, 3 or 5 is

66. There are three clubs A, B, C in a town with 40,50,60 members respectively. 10 people are members of all the three clubs, 80 members belong to only one club . Then the number of members which belongs to atleast two clubs is

67. If $n(A) = 12$, $n(B) = 15$. Isf x and y are minimum and maximum value of $n(A' \cap B)$ then $x + y =$

CHAPTER 02

RELATIONS & FUNCTIONS

Single Option Correct Type Questions (01 to 63)

1. Let R be a relation over the set $N \times N$ and it is defined by $(a, b)\ R\ (c, d) \Rightarrow a + d = b + c$. Then R is

(1) Symmetric only (2) Transitive only

(3) Reflexive only (4) Equivalence only

2. Let S be a set of all square matrices of order 2 . If a relation R defined on set S such that

$AR\,B \Rightarrow AB = O$, where O is zero square matrix of order 2,then relation R is

$(A, B \in S)$

(1) Reflexive (2) Transitive

(3) Symmetric (4) Not equivalence

3. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{2x^2 - x + 5}{7x^2 + 2x + 10}$, then f is

(1) one-one but not onto

(2) onto but not one-one

(3) onto as well as one-one

(4) neither onto nor one-one

4. If $f(x) = \sin\left(\sqrt{[a]}x\right)$ (where [.] denotes the greatest integer function) has π as its fundamental period, then

(1) $a = 1$ (2) $a = 9$

(3) $a \in [1, 2)$ (4) $a \in [4, 5)$

5. For real numbers x and y we write $xRy \Rightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is

(1) Reflexive

(2) Symmetric

(3) Transitive

(4) Equivalence relation

6. Let $A = N \times N$ be the Certesian product of N and N. Let $S = \{((m, n), (p, q)) \in A \times A : m + q = n + p\}$

Consider the following statements:

I. $((m, n), (p, q) \in S$, and $((p, q), (r, s)) \in S$ then $((r, s)\ (m, n)) \in S$

II. There exists at least one element $((m, n), (p, q)) \in S$ such that $((p, q), (m, n)) \in S$

Which of the statements given above is/are correct?

(1) I only (2) II only

(3) Both I and II (4) Neither I nor II

7. Let $A = Z$, the set of integers. Let $R_1 = \{(m, n) \in Z \times Z$: $(m + 4n)$ is divisible by 5 in $Z\}$.

Let $R_2 = \{(m, n) \in Z \times Z$: $(m + 9n)$ is divisible by 5 in $Z\}$

Which one of the following is correct?

(1) R_1 is a proper subset of R_2

(2) R_2 is a proper subset of R_1

(3) $R_1 = R_2$

(4) R_1 is not a symmetric relation on Z

8. Let X be the set of all persons living in a state. Elements x, y in X are said to be related if '$x < y$', whenever y is 5 years older than x. Which one of the following is correct?

(1) The relation is an equivalence relation

(2) The relation is transitive only

(3) The relation is transitive and symmetric, but not reflexive

(4) The relation is neither reflexive, nor symmetric, nor transitive

9. If the solution set of $[x]+\left[x+\frac{1}{2}\right]+\left[x-\frac{1}{3}\right]=8$ is $[a, b)$, then $(a + b)$ equals to (where [ ] denotes greatest integer function)

(1) $\frac{19}{3}$ (2) $\frac{20}{3}$

(3) 6 (4) 7

10. If

$$f(x)=\{x\}+\left\{x+\left[\frac{x}{1+x^2}\right]\right\}+\left\{x+\left[\frac{x}{1+2x^2}\right]\right\}$$

$$+\left\{x+\left[\frac{x}{1+3x^2}\right]\right\}......+\left\{x+\left[\frac{x}{1+99x^2}\right]\right\}$$, then

values of $[f(\sqrt{3})]$ is (where [.] denotes greatest integer function and {.} represent fractional part function)

(1) 5050 (2) 4950

(3) 17 (4) 73

11. Number of integral solutions of the inequation $x^2-10x+25\,\text{sgn}(x^2+4x-32)\le 0$

(1) infinite (2) 6

(3) 7 (4) 8

12. If $[x+[2x]]<3$, where [.] denotes the greatest integer function, then x is

(1) $[0,1)$ (2) $\left(-\infty,\frac{3}{2}\right]$

(3) $(1,\infty)$ (4) $(-\infty,1)$

13. The set of all values of x for which $\frac{\sqrt{-x^2+5x-6}}{\sqrt{1-2\{x\}}}\ge 0$ is (where {.} denotes the fractional part function)

(1) $\left[2,\frac{5}{2}\right)\cup\{3\}$ (2) $(2,3)$

(3) $\left(\frac{5}{2},3\right]$ (4) $\left[2,\frac{5}{2}\right)\cup\left(\frac{5}{2},3\right]$

14. The domain of the function

$$f(x)=\log_{1/2}\left(-\log_2\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$$ is

(1) $0 < x < 1$ (2) $0 < x \le 1$

(3) $x \ge 1$ (4) null set

15. If $q^2 - 4pr = 0$, $p > 0$, then the domain of the function $f(x) = \log(px^3 + (p + q)x^2 + (q + r)x + r)$ is

(1) $R-\left\{-\frac{q}{2p}\right\}$

(2) $R-\left[(-\infty,-1]\cup\left\{-\frac{q}{2p}\right\}\right]$

(3) $R-\left[(-\infty,-1)\cap\left\{-\frac{q}{2p}\right\}\right]$

(4) R

16. Let $f(x)=\frac{x-[x]}{1+x-[x]}$, $f:R\to A$ is onto then set A is (where {.} and [.] represent fractional part and greatest integer part functions respectively)

(1) $\left(0,\frac{1}{2}\right]$ (2) $\left[0,\frac{1}{2}\right]$

(3) $\left[0,\frac{1}{2}\right)$ (4) $\left(0,\frac{1}{2}\right)$

17. Let f be a real valued function defined by $f(x)=\frac{e^x-e^{-|x|}}{e^x+e^{|x|}}$, then the range of $f(x)$ is

(1) R (2) [0, 1]

(3) [0, 1) (4) $\left[0,\frac{1}{2}\right)$

18. The range of the function $f(x)=\log_{\sqrt{2}}(2-\log_2(16\sin^2 x+1))$ is

(1) $(-\infty,1)$ (2) $(-\infty,2)$

(3) $(-\infty,1]$ (4) $(-\infty,2]$

19. If domain of $f(x)$ is $(-\infty, 0]$, then domain of $f(6\{x\}^2 - 5\{x\} + 1)$ is (where $\{.\}$represents fractional part function)

(1) $\bigcup_{n\in I}\left[n+\frac{1}{3}, n+\frac{1}{2}\right]$

(2) $(-\infty, 0)$

(3) $\bigcup_{n\in I}\left[n+\frac{1}{6}, n+1\right]$

(4) $\bigcup_{n\in I}\left[n-\frac{1}{2}, n-\frac{1}{3}\right]$

20. Let $f:(e,\infty)\to R$ be defined by $f(x) = \ell n(\ell n(\ell n x))$, then

(1) f is one-one but not onto
(2) f is onto but not one-one
(3) f is one-one and onto
(4) f is neither one-one nor onto

21. If $f(x) = 2[x] + \cos x$, then $f: R\to R$ is: (where [ ] denotes greater integer function)

(1) one-one and onto
(2) one-one and into
(3) many-one and into
(4) many-one and onto

22. If $f: R\to R$ be a function such that

$f(x) = \begin{cases} x|x| - 4; x\in Q \\ x|x| - \sqrt{3}; x\notin Q \end{cases}$, then $f(x)$ is

(1) one-one, onto (2) many one, onto
(3) one-one, into (4) many one, into

23. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x) = 2^{x(x-1)}$ then f^{-1} is

(1) $(1/2)^{x(x-1)}$

(2) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$

(3) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$

(4) Not defined

24. Let $f: N\to N$, where $f(x) = x + (-1)^{x-1}$, then the inverse of f is

(1) $f^{-1}(x) = x + (-1)^{x-1}, x\in N$

(2) $f^{-1}(x) = 3x + (-1)^{x-1}, x\in N$

(3) $f^{-1}(x) = x, x\in N$

(4) $f^{-1}(x) = (-1)^{x-1}, x\in N$

25. A function $g(x)$ satisfies the following conditions

(i) Domain of g is $(-\infty, \infty)$

(ii) Range of g is [–1, 7]

(iii) g has a period π and

(iv) $g(2) = 3$

Then which of the following may be possible

(1) $g(x) = 3 + 4\sin(n\pi + 2x - 4), n\in I$

(2) $g(x) = \begin{cases} 3 & ; x = n\pi \\ 2 + 4\sin x; & x\neq n\pi \end{cases}$

(3) $g(x) = 3 + 4\cos(n\pi + 2x - 4), n\in I$

(4) $g(x) = 3 - 8(n\pi + 2x - 4), n\in I$

26. Let f: (–1,1) $\to R$ be such that $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)

(1) $1-\sqrt{\frac{3}{2}}$ (2) $2+\sqrt{\frac{3}{2}}$

(3) $1-\sqrt{\frac{2}{3}}$ (4) $1+\sqrt{\frac{2}{3}}$

27. Consider the following relations:

R: {(x, y)| x, y are real numbers and $x = wy$ for some rational number w}

$S = \{\left(\frac{m}{n}, \frac{p}{q}\right)$| m,n, p and q are integers such that n, $q \neq 0$ and $qm = pn\}$

Then

(1) neither R nor S is an equivalence relation
(2) S is an equivalence relation but R is not an equivalence relation
(3) R and S both are equivalence relations
(4) R is an equivalence relation but S is not equivalence relation

28. Consider the following relation R on the set of real square matrices of order 3.
$R = \{(A, B) | A = P^{-1} BP$ for some invertible matrix $P\}$
Statement-1: R is equivalence relation
Statement-2: For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$
(1) Statement-1 is true, statement-2 is a correct explanation for statement-1
(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1
(3) Statement-1 is true, statement-2 is false
(4) Statement-1 is false, statement-2 is true.

29. The number of functions f from $\{1, 2, 3,\ldots\ldots,20\}$, onto $\{1, 2, 3,\ldots.,20\}$ such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4, is
(1) $5! \times 6!$ (2) $(15)! \times 6!$
(3) $6^5 \times (15)!$ (4) $5^6 \times 15!$

30. If x and y satisfy the equation $y = 2[x] + 3$ and $y = 3\,[x–2]$ simultaneously, where [.] denotes the greatest integer function, the $[x + y]$ is equal to
(1) 21 (2) 9
(3) 30 (4) 12

31. Which of the following pair of functions are identical?
(1) $\sqrt{1+\sin x}, \sin\frac{x}{2}+\cos\frac{x}{2}$
(2) $\sin^{-1}\frac{2x}{1+x^2}, 2\tan^{-1}x$
(3) $\sqrt{x^2}, (\sqrt{x})^2$
(4) $\ell nx^3+\ell nx^2, 5\ell nx$

32. If the graph of the function $f(x)=\frac{a^x-1}{x^n(a^x+1)}$ is symmetric about y-axis, then n is equal to
(1) 2 (2) 2/3
(3) 1/4 (4) –1/3

33. The function $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1$ is
(1) an odd function
(2) an even function
(3) neither an odd nor an even function
(4) a periodic function

34. If $f: R \to [-1,1]$, where $f(x)=\sin\frac{\pi}{2}[x]$, (where [ ] denotes the greatest integer function), then which of the following is false
(1) $f(x)$ is onto (2) $f(x)$ is onto
(3) $f(x)$ is periodic (4) $f(x)$ is many one

35. The mapping $f: R \to R$ given by $f(x)=x^3+ax^2+bx+c$ is a bijection if
(1) $b^2 \le 3a$ (2) $a^2 \le 3b$
(3) $a^2 \ge 3b$ (4) $b^2 \ge 3a$

36. If 'f' and 'g' are bijective functions and gof is defined, then, gof must be:
(1) injective (2) surjective
(3) bijective (4) into only

37. The fundamental period of function $f(x)=[x]+\left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right]-3x+15$ where [.] denotes greatest integer function, is
(1) $\frac{1}{3}$ (2) $\frac{2}{3}$
(3) 1 (4) non-periodic

38. The fundamental period of $f(x) = [\sin 3x] + |\cos 6x|$, (where [.] denotes greatest integer function) is
(1) 2π (2) π
(3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{2}$

39. $f(x) = |x–1|$, $f: R^+ \to R$, $g(x) = e^x$, $g: [–1, \infty) \to R$. If the function fog (x) is defined, then its domain and range respectively are
(1) $(0,\infty)$ and $(0,\infty)$
(2) $[-1,\infty)$ and $[0,\infty)$
(3) $(-1,\infty)$ and $\left[1-\frac{1}{e},\infty\right)$
(4) $(-1,\infty)$ and $\left[\frac{1}{e}-1,\infty\right)$

40. If $f:(2,4)\to(1,3)$ be a function defined by $f(x)=x-\left[\frac{x}{2}\right]$ (where [.] denotes the greatest integer function), then $f^{-1}(x)$ is equal to:

(1) $2x$ (2) $x+\left[\frac{x}{2}\right]$

(3) $x+1$ (4) $x-1$

41. Statement-1: If $f(x)$ and $g(x)$ both are one one and $f(g(x))$ exists, then $f(g(x))$ is also one one.

Statement-2: If $f(x_1)=f(x_2)\Leftrightarrow x_1=x_2$, then $f(x)$ is one-one

(1) Statement-1 is true; statement-2 is a correct explanation for statement-1

(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1

(3) Statement-1 is true, statement-2 is false

(4) Statement-1 is false, statement-2 is true

42. **Statement - 1** If $y=f(x)$ is increasing in $[\alpha,\beta]$, then its range is $[f(\alpha),f(\beta)]$

Statement - 2 Every increasing function need not to be continuous

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

43. **Statement-1**: All points of intersection of $y=f(x)$ and $y=f^{-1}(x)$ lies on $y=x$ only

Statement – 2: If point $P(\alpha,\beta)$ lies on $y=f(x)$, then $Q(\beta,\alpha)$ lies on $y=f^{-1}(x)$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

44. Which of the following is not a periodic function

(1) $\sin 2x+\cos x$ (2) $\cos\sqrt{x}$

(3) $\tan 4x$ (4) $\log\cos 2x$

45. A function 'f' from the set of natural numbers to integers defined by

$$f(n)=\begin{cases}\frac{n-1}{2}, & \text{when } n \text{ is odd}\\ -\frac{n}{2}, & \text{when } n \text{ is even}\end{cases}$$, then f is:

(1) one-one (2) many-one

(3) one-one and onto (4) into

46. If $f:R\to R$ satisfies $f(x+y)=f(x)+f(y)$, for all $x,y\in R$ and $f(1)=7$, then $\sum_{r=1}^{n} f(r)$ is

(1) $\frac{7n}{2}$ (2) $\frac{7(n+1)}{2}$

(3) $7n(n+1)$ (4) $\frac{7n(n+1)}{2}$

47. Domain of definition of the function $f(x)=\frac{3}{4-x^2}+\log_{10}(x^3-x)$ is

(1) $(1,2)$

(2) $(-1,0)\cup(1,2)$

(3) $(1,2)\cup(2,\infty)$

(4) $(-1,0)\cup(1,2)\cup(2,\infty)$

48. The function $f(x)=\log\left(x+\sqrt{x^2+1}\right)$, is

(1) an even function

(2) an odd function

(3) a periodic function

(4) neither an even nor an odd function

49. If $f:R\to S$, defined by $f(x)=\sin x-\sqrt{3}\cos x+1$, is onto, then the interval of S is:

(1) [0, 3] (2) [–1,1]
(3) [0,1] (4) [–1, 3]

50. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then :
(1) $f(x + 2) = f(x-2)$
(2) $f(2+x) = f(2-x)$
(3) $f(x) = f(-x)$
(4) $f(x) = -f(x)$

51. Let f be a function defined by $f(x) = (x-1)^2 + 1, (x \geq 1)$

Statement-1: The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$

Statement-2: f is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

52. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$, and $S = \{x \in R : f(x) = f(-x)\}$; then S
(1) contains exactly one element
(2) contains exactly two elements
(3) contains more than two elements
(4) is an empty set

53. For $x \in R, f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then
(1) $g'(0) = \cos(\log 2)$
(2) $g'(0) = -\cos(\log 2)$
(3) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$
(4) g is not differentiable at $x = 0$

54. The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is
(1) Invertible
(2) injective but not surjective
(3) surjective but not injective
(4) neither injective nor surjective

55. Suppose $f(x) = (x + 1)^2$ for $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equals
(1) $-\sqrt{x} - 1, x \geq 0$
(2) $\frac{1}{(x+1)^2}, x > -1$
(3) $\sqrt{x+1}, x > -1$
(4) $\sqrt{x} - 1, x \geq 0$

56. Let function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then f is
(1) one to one and onto
(2) one to one but not onto
(3) onto but not one to one
(4) neither one to one nor onto

57. Range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}; x \in R$ is:
(1) $(1, \infty)$ (2) $\left(1, \frac{11}{7}\right]$
(3) $\left(1, \frac{7}{3}\right]$ (4) $\left(1, \frac{7}{5}\right]$

58. If $f(x) = \sin x + \cos x$ and $g(x) = x^2 - 1$.if g [$f(x)$] is invertible, then value 'x' is
(1) $\left[0, \frac{\pi}{2}\right]$ (2) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$
(3) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (4) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

59. If the functions $f(x)$ and $g(x)$ are defined on $R \to R$ such that $f(x)=\begin{cases}0, & x\in \text{rational}\\ x, & x\in \text{irrational}\end{cases}$, $g(x)=\begin{cases}0, & x\in \text{irrational}\\ x, & x\in \text{rational}\end{cases}$, then $(f-g)(x)$ is

(1) one-one and onto
(2) neither one-one nor onto
(3) one-one but not onto
(4) onto but not one-one

60. Let $f(x)=x^2$ and $g(x)=\sin x$ for all $x\in R$. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) $(x)=f(g(x))$, is

(1) $\pm\sqrt{n\pi}, n\in\{0,1,2,.....\}$
(2) $\pm\sqrt{n\pi}, n\in\{1,2,.....\}$
(3) $\frac{\pi}{2}+2n\pi, n\in\{...-2,-1,0,1,2.,.....\}$
(4) $2n\pi, n\in\{.....,-2,-1,0,1,2,......\}$

61. The function $[0, 3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(1) one-one and onto
(2) onto but not one-one
(3) one-one but not onto
(4) neither one-one nor onto

62. Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ be given by $f(x)=(\log(\sec x+\tan x))^3$. Then

(1) $f(x)$ is an odd function
(2) $f(x)$ is a many one function
(3) $f(x)$ is an into function
(4) $f(x)$ is an even function

63. If $f(x)=x^2+1$, then $f^{-1}(17)$ and $f^{-1}(-3)$ will be

(1) 4, 1
(2) 4, 0
(3) 3, 2
(4) None of these

Integer Type Questions (64 to 73)

64. If f and g are two distinct linear function defined on R such that they map $[-1,1]$ onto $[0,2]$ and $h : R - \{-1, 0, 1\} \to R$ defined by $h(x)=\frac{f(x)}{g(x)}$, then $|h(h(x))+h(h(1/x))|>n$. Then maximum integral values of n is

65. If $f(x)=ax^7+bx^3+cx-5; a,b,c$ are real number constants and $f(-7)=7$ then maximum value of $f(7)+17\cos x$ is

66. Let f be a one-one function with domain {21, 22, 23} and range $\{x,y,z\}$. It is given that exactly one of the following statements is true and the remaining two are false. $f(21) = x$; $f(22) \neq x$; $f(23) \neq y$. then value of $f^{-1}(x)$ is :

67. If $f(y)=\log y$, then $f(y)+f\left(\frac{1}{y}\right)$ is equal to

68. The number of surjection from A to B, where $A=\{1,2,3,4\}$, $B=\{a,b\}$, is

69. The reciprocal of the value of 'x' satisfying equation $|2x-1| = 3[x] + 2\{x\}$. (where [.] and { . } denote greatest integer and fractional part function respectively)

70. The number of solution(s) of the equation $x^2 - 4x + [x] + 3 = 0$ (where $[x]$ denotes integral part of x)

71. The function

$$f(x)=\sin\frac{\pi x}{2}+2\cos\frac{\pi x}{3}-\tan\frac{\pi x}{4}$$

is periodic with period

72. The number of positive integral values of x satisfying $\left[\frac{x}{9}\right]=\left[\frac{x}{11}\right]$ is

(where [.] = G. I. F)

73. The number of values of $f(x)=\left[\frac{x}{15}\right]\left[\frac{-15}{x}\right]$ can take where $x\in(0,90)$ (where [.]= G.I.F)

CHAPTER

03 TRIGONOMETRY (TRI)

Single Option Correct Type Questions (01 to 65)

1. If cosec A + cot $A = \frac{11}{2}$, then tan A is equal to

(1) $\frac{21}{22}$ (2) $\frac{15}{16}$ (3) $\frac{44}{117}$ (4) $\frac{117}{43}$

2. If tan α + cot α = a then the value of $\tan^4 \alpha + \cot^4 \alpha$ is equal to

(1) a^4+4a^2+2 (2) a^4-4a^2+2

(3) a^4-4a^2-2 (4) a^4+4a^2-2

3. If $\tan\theta = -\frac{5}{12}$, θ is not in the second quadrant, then $\frac{\sin(360^0-\theta)+\tan(90^0+\theta)}{-\sec(270^0+\theta)+\text{cosec}(-\theta)} =$

(1) $\frac{131}{338}$ (2) $\frac{181}{338}$

(3) $-\frac{181}{338}$ (4) $-\frac{131}{338}$

4. $\frac{\tan\left(x-\frac{\pi}{2}\right).\cos\left(\frac{3\pi}{2}+x\right)-\sin^3\left(\frac{7\pi}{2}-x\right)}{\cos\left(x-\frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2}+x\right)}$

when simplified reduces to

(1) $\sin x\cos x$ (2) $-\sin^2 x$

(3) $-\sin x\cos x$ (4) $\sin^2 x$

5. The expression 3 $\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi+\alpha)\right]$ is equal to

(1) 0 (2) 1

(3) 3 (4) sin 4α + sin 6α

6. $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + ...\sin^2 85^\circ + \sin^2 90^\circ =$

(1) 7 (2) 8

(3) $9\frac{1}{2}$ (4) 10

7. The value of $\frac{\sin 24^\circ\cos 6^\circ-\sin 6^\circ\sin 66^\circ}{\sin 21^\circ\cos 39^\circ-\cos 51^\circ\sin 69^\circ}$ is

(1) −1 (2) 1

(3) 2 (4) 0

8. If tan A – tan B = x and cot B – cot A = y, then cot (A – B) is equal to

(1) $\frac{1}{y}-\frac{1}{x}$ (2) $\frac{1}{x}-\frac{1}{y}$

(3) $\frac{1}{x}+\frac{1}{y}$ (4) $x + y$

9. If 3 sinα =5 sinβ, then $\frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$ is equal to

(1) 1 (2) 2

(3) 3 (4) 4

10. The expression $\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{\cos 5x+5\cos 3x+10\cos x}$ is equal to

(1) cos 2x (2) 2 cos x

(3) $\cos^2 x$ (4) 1 + cos x

11. If tan 25° = x, then $\dfrac{\tan 155° - \tan 115°}{1 + \tan 155° \ \tan 115°}$ is equal to

(1) $\dfrac{1-x^2}{2x}$ (2) $\dfrac{1+x^2}{2x}$

(3) $\dfrac{1+x^2}{1-x^2}$ (4) $\dfrac{1-x^2}{1+x^2}$

12. $2 \sin^2 \beta + 4 \cos (\alpha + \beta) \sin \alpha \sin \beta + \cos 2 (\alpha + \beta) =$

(1) $\sin 2\alpha$ (2) $\cos 2\beta$

(3) $\cos 2\alpha$ (4) $\sin 2\beta$

13. If $\cos A = \dfrac{3}{4}$, then the value of $16\cos^2\left(\dfrac{A}{2}\right) - 32 \sin\left(\dfrac{A}{2}\right) \sin\left(\dfrac{5A}{2}\right)$ is

(1) – 4 (2) – 3

(3) 3 (4) 4

14. If $\cos\theta = \dfrac{1}{2}\left(a + \dfrac{1}{a}\right)$, then $\cos 3\theta$ in terms of 'a' is

(1) $\dfrac{1}{4}\left(a^3 + \dfrac{1}{a^3}\right)$ (2) $\dfrac{1}{2}\left(a^3 + \dfrac{1}{a^3}\right)$

(3) $4\left(a^3 + \dfrac{1}{a^3}\right)$ (4) $\left(a^3 + \dfrac{1}{a^3}\right)$

15. If $\sin t + \cos t = \dfrac{1}{5}$ then $\tan \dfrac{t}{2}$ is equal to:

(1) $-1, 2$ (2) $-\dfrac{1}{3}, 2$

(3) $-2, \dfrac{1}{3}$ (4) $-\dfrac{1}{6}$

16. The value of the expression $\left(1+\cos\dfrac{\pi}{10}\right)\left(1+\cos\dfrac{3\pi}{10}\right)\left(1+\cos\dfrac{7\pi}{10}\right)\left(1+\cos\dfrac{9\pi}{10}\right)$ is

(1) $\dfrac{1}{8}$ (2) $\dfrac{1}{16}$

(3) $\dfrac{1}{4}$ (4) 0

17. $\dfrac{\cos 20° + 8\sin 70° \sin 50° \sin 10°}{\sin^2 80°}$ is equal to:

(1) 1 (2) 2

(3) $\dfrac{3}{4}$ (4) $\dfrac{1}{2}$

18. If $A = \tan 6° \tan 42°$ and $B = \cot 66° \cot 78°$, then value of $\dfrac{A}{B}$ is

(1) 1 (2) $\dfrac{1}{2}$

(3) $\dfrac{1}{3}$ (4) $\dfrac{1}{4}$

19. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha =$

(1) $\tan\alpha$ (2) $\cot\alpha$

(3) $\cot 16\alpha$ (4) $16 \cot\alpha$

20. If $A + B + C = \dfrac{3\pi}{2}$, then $\cos 2A + \cos 2B + \cos 2C$ is equal to

(1) $1 - 4\cos A \cos B \cos C$

(2) $4 \sin A \sin B \sin C$

(3) $1 + 2 \cos A \cos B \cos C$

(4) $1 - 4 \sin A \sin B \sin C$

21. The value of $\cos 0 + \cos\dfrac{\pi}{9} + \cos\dfrac{2\pi}{9} + \cos\dfrac{3\pi}{9} + \cos\dfrac{4\pi}{9} + \cos\dfrac{5\pi}{9} + \cos\dfrac{6\pi}{9} + \cos\dfrac{7\pi}{9} + \cos\dfrac{8\pi}{9}$ is

(1) $\dfrac{1}{2}$ (2) $-\dfrac{1}{2}$

(3) 0 (4) 1

22. If ϕ is the exterior angle of a regular polygon of n sides and θ is any constant, then $\sin\theta + \sin(\theta+\phi) + \sin(\theta+2\phi) + \ldots\ldots\ldots\ldots$ up to n terms =

(1) $\sin n\theta$ (2) $\sin n\phi$

(3) $2n\pi$ (4) 0

23. $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11} =$

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$

(3) $\frac{1}{16}$ (4) $\frac{1}{32}$

24. The greatest and least value of $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$ are respectively

(1) 11, 1 (2) 10, 2

(3) 12, – 4 (4) 11, – 1

25. The difference between maximum and minimum value of the expression $y = 1 + 2\sin x + 3\cos^2 x$ is

(1) $\frac{16}{3}$ (2) $\frac{13}{3}$

(3) 7 (4) 8

26. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are

(1) $\frac{\pi}{3}$ & $\frac{\pi}{6}$ (2) $\frac{\pi}{8}$ & $\frac{3\pi}{8}$

(3) $\frac{\pi}{4}$ & $\frac{\pi}{4}$ (4) $\frac{\pi}{5}$ & $\frac{3\pi}{10}$

27. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$ then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

(1) 3 (2) 2

(3) 1 (4) 0

28. In a triangle ABC if $\tan A < 0$ then:

(1) $\tan B.\tan C > 1$ (2) $\tan B.\tan C < 1$

(3) $\tan B.\tan C = 1$ (4) $\tan B.\tan C \geq 1$

29. If $\alpha\cos^2 3\theta + \beta\cos^4\theta = 16\cos^6\theta + 9\cos^2\theta$ is an identity then

(1) $\alpha = 1, \beta = 18$ (2) $\alpha = 1, \beta = 24$

(3) $\alpha = 3, \beta = 24$ (4) $\alpha = 4, \beta = 2$

30. If in a triangle ABC, $\angle C = \frac{2\pi}{3}$, then the value of $\cos^2 A + \cos^2 B - \cos A.\cos B$ is equal to-

(1) $\frac{3}{4}$ (2) $\frac{3}{2}$

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

31. The value of $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ}$ is

(1) $\frac{2\sqrt{3}}{3}$ (2) $\frac{4\sqrt{3}}{3}$

(3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

32. If $a\sec\theta = 1 - b\tan\theta$ and $a^2\sec^2\theta = 5 + b^2\tan^2\theta$, then

(1) $a^2b^2 - 4a^2 = 9b^2$

(2) $a^2b^2 + 2a^2 = 9b^2$

(3) $a^2b^2 + 4a^2 = 9b^2$

(4) $a^2b^2 + 4a^2 = 3b^2$

33. If $\sin x + \sin y = a$ & $\cos x + \cos y = b$, then $\tan\frac{x-y}{2} =$

(1) $\pm\sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$

(2) $\pm\sqrt{\frac{4 + a^2 - b^2}{a^2 + b^2}}$

(3) $\pm\sqrt{\frac{2 - a^2 - b^2}{a^2 + b^2}}$

(4) $\pm\sqrt{\frac{4 - a^2 - b^2}{a^2 - b^2}}$

34. The greatest and least value of $y = 3\cos\left(\theta + \frac{\pi}{3}\right) + 5\cos\theta + 3$ are respectively

(1) 11, –5 (2) 3, –3

(3) 3, 0 (4) 10, – 4

35. **STATEMENT-1:** $\sin 2 > \sin 3$

STATEMENT-2: If $x, y \in \left(\frac{\pi}{2}, \pi\right)$, $x < y$, then $\sin x > \sin y$

(1) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(2) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(3) STATEMENT-1 is true, STATEMENT-2 is false
(4) STATEMENT-1 is false, STATEMENT-2 is true

36. **STATEMENT-1:** There is no value of θ for which $\frac{\tan\theta}{\tan 3\theta} = 2$

STATEMENT-2: If $y = \frac{\tan\theta}{\tan 3\theta}$, then $y \in \left(-\infty, \frac{1}{3}\right) \cup (3, \infty) - \{0\}$, where $\theta \neq \frac{n\pi}{3} + \frac{\pi}{6}$, $\theta \neq \frac{m\pi}{3}$, for $n, m \in I$

(1) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(2) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(3) STATEMENT-1 is true, STATEMENT-2 is false
(4) STATEMENT-1 is false, STATEMENT-2 is true

37. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to

(1) $\frac{24}{25}$ (2) $-\frac{24}{25}$
(3) $\frac{13}{18}$ (4) $-\frac{13}{18}$

38. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$ then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$ is equal to

(1) 1 (2) –1
(3) 0 (4) None of these

39. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if

(1) $x - y \neq 0$ (2) $x = y, x \neq 0, y \neq 0$
(3) $x = y$ (4) $x \neq 0, y \neq 0$

40. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by

(1) $2\left(a^2 + b^2\right)$ (2) $2\sqrt{a^2 + b^2}$
(3) $(a + b)^2$ (4) $(a - b)^2$

41. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\left(\frac{\alpha - \beta}{2}\right)$ is

(1) $\frac{-3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$
(3) $\frac{6}{65}$ (4) $\frac{-6}{65}$

42. *ABCD* is a trapezium such that *AB* and *CD* are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then *AB* is equal to:

(1) $\frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

(2) $\frac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(3) $\frac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$

(4) $\frac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

43. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as

(1) $\sin A \cos A + 1$
(2) $\sec A \operatorname{cosec} A + 1$
(3) $\tan A + \cot A$
(4) $\sec A + \operatorname{cosec} A$

44. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$
(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

45. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is

(1) $\frac{-3}{5}$ (2) $\frac{1}{3}$
(3) $\frac{2}{9}$ (4) $-\frac{7}{9}$

46. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then

(1) $t_1 > t_2 > t_3 > t_4$ (2) $t_2 < t_1 < t_3 < t_4$
(3) $t_3 > t_1 > t_2 > t_4$ (4) $t_2 > t_3 > t_1 > t_4$

47. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(1) $P \subset Q$ and $Q - P \neq \varnothing$
(2) $Q \not\subset P$
(3) $P \not\subset Q$
(4) $P = Q$

48. The value of
$$\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$ is equal to

(1) $3-\sqrt{3}$ (2) $2(3-\sqrt{3})$
(3) $2(\sqrt{3}-1)$ (4) $2(2+\sqrt{3})$

49. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then

(1) $2n = m(n^2 - 1)$ (2) $2m = n(m^2 - 1)$
(3) $2n = m(m^2 - 1)$ (4) $2m = n^2(n - 1)$

50. The ratio of the greatest value of $2 - \cos x + \sin^2 x$ to its least value is

(1) $\frac{1}{4}$ (2) $\frac{9}{4}$
(3) $\frac{13}{4}$ (4) $\frac{7}{4}$

51. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$, then the value of $A + B$ is

(1) 0 (2) $\frac{\pi}{2}$
(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

52. If $\sin(A + B + C) = 1$, $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\sec(A + C) = 2$, then

(1) $A = 90°, B = 60°, C = 30°$
(2) $A = 120°, B = 60°, C = 0°$
(3) $A = 60°, B = 30°, C = 0°$
(4) $A = 45°, B = 30°, C = 60°$

53. $\frac{3\cos\theta + \cos 3\theta}{3\sin\theta - \sin 3\theta}$ is equal to

(1) $1 + \cot^2\theta$ (2) $\cot^4\theta$
(3) $\cot^3\theta$ (4) $2\cot\theta$

54. If $A = \sin^2 x + \cos^4 x$, then for all real x

(1) $\frac{3}{4} \leq A \leq 1$ (2) $\frac{13}{16} \leq A \leq 1$
(3) $1 \leq A \leq 2$ (4) $\frac{3}{4} \leq A \leq \frac{13}{16}$

55. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(1) infinite number of real roots
(2) no real roots
(3) exactly one real root
(4) exactly four real roots

56. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is

(1) $\frac{4-\sqrt{7}}{3}$ (2) $-\left(\frac{4+\sqrt{7}}{3}\right)$

(3) $\frac{1+\sqrt{7}}{4}$ (4) $\frac{1-\sqrt{7}}{4}$

57. Let A and B denote the statements

A: $\cos\alpha + \cos\beta + \cos\gamma = 0$

B: $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then

(1) A is false and B is true

(2) both A and B are true

(3) both A and B are false

(4) A is true and B is false

58. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{56}{33}$ (2) $\frac{19}{12}$

(3) $\frac{20}{7}$ (4) $\frac{25}{16}$

59. If $\sin\theta = \frac{1}{3}$, then the quadratic equation whose roots are $\tan\frac{\theta}{2}$ and $\cot\frac{\theta}{2}$ is

(1) $x^2 - 6x + 1 = 0$ (2) $x^2 + 6x + 1 = 0$

(3) $x^2 - 6x - 1 = 0$ (4) $x^2 + 6x - 1 = 0$

60. Value of $\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$

(1) $\frac{1}{16}$ (2) $\frac{1}{32}$

(3) $\frac{1}{64}$ (4) $-\frac{1}{16}$

61. Range of value of $13 + 12\sin\frac{11\theta}{2} + 5\cos\frac{11\theta}{2}$ is

(1) [0, 26] (2) (0, 26)

(3) [–13, 13] (4) [13, 26]

62. If $\alpha + \beta + \gamma = 2\pi$, then

(1) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(2) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

(3) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(4) none of these

63. The equation $\sin^6 x + \cos^6 x = a^2$ has real solution if

(1) $a \in (-1, 1)$

(2) $a \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

(3) $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

(4) $a \in (-2, 2)$

64. Match the column-

Column- I		**Column- II**	
I	$\tan 9° - \tan 27° - \tan 63° + \tan 81°$	P	2
II	$2(\text{cosec}10° - \sqrt{3}\sec 10°)$	Q	8
III	$\sqrt{2}\sin 10° \left[\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right]$	R	3
IV	$\sqrt{3}\,(\cot 70° + 4\cos 70°)$	S	4

(1) I-Q; II-S; III-R; IV-P

(2) I-S; II-Q; III-P; IV-R

(3) I-P; II-R; III-S; IV-Q

(4) I-R; II-P; III-Q; IV-S

65. Match the column-

Column- I		Column- II	
I	Number of solutions of $\sin^2\theta + 3\cos\theta = 3$ in $[-\pi, \pi]$	P	2
II	If $\sin\theta + \text{cosec}\,\theta = 2$, then $\sin^{2008}\theta + \text{cosec}^{2008}\theta$ is equal to	Q	1
III	Maximum value of $\sin^4\theta + \cos^4\theta - 1$ is	R	0
IV	Least value of $2\sin^2\theta + 3\cos^2\theta - 3$ is	S	-1

(1) I-S; II-R; III-P; IV-Q
(2) I-R; II-S; III-Q; IV-P
(3) I-Q; II-P; III-R; IV-S
(4) I-P; II-Q; III-S; IV-R

Integer Type Questions (66 to 75)

66. If $a\cos\theta + b\sin\theta = 3$ & $a\sin\theta - b\cos\theta = 4$ then value of $a^2 + b^2$ is

67. The value of $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$ is

68. The value of $\dfrac{(1+\tan 8°)(1+\tan 37°)}{(1+\tan 22°)(1+\tan 23°)}$ is

69. The maximum value of $12\sin\theta - 9\sin^2\theta$ is

70. If $\tan^2\theta = 2\tan^2\phi + 1$, then the value of $\cos 2\theta + \sin^2\phi$ is

71. If $f(\theta) = (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$, then minimum value of $f(\theta)$ is

72. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$ respectively, then the value of $2 + q - p$ is

73. The value of $\dfrac{\tan 70° - \tan 20°}{\tan 50°}$ is equal to

74. If $\sin\alpha \sin\beta - \cos\alpha \cos\beta + 1 = 0$, then the value of $1 + \cot\alpha \tan\beta$ is

75. The positive integral value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$ is

CHAPTER

04

QUADRATIC EQUATIONS

Single Option Correct Type Questions (01 to 65)

1. If one of the factors of $ax^2 + bx + c$ and $bx^2 + cx + a$ is common, then :

(1) $a = 0$

(2) $a^3 + b^3 + c^3 = 3abc$

(3) $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

(4) $b = 0$

2. The value of m for which one root of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is

(1) 0, – 2 (2) 0, 2

(3) 2, 4 (4) 2, –2

3. If the quadratic equations $ax^2 + bx + c = 0$ ($a, b, c \in R, a \neq 0$) and $x^2 + 4x + 5 = 0$ have a common root, then a, b, c must satisfy the relations

(1) $a > b > c$

(2) $a < b < c$

(3) $a = k; b = 4k; c = 5k$ ($k \in R, k \neq 0$)

(4) $a = 5b = 6c$

4. The value of m, for which the equation $(1 + m) x^2 - 2(1 + 3m) x + (1 + 8m) = 0$ has equal roots, is

(1) 0, 3 (2) 1

(3) 2 (4) 3

5. If a,b are roots of the equation $x^2 + qx + 1 = 0$ and c,d are roots of $x^2 + px + 1 = 0$, then the value of $(a - c)(b - c)(a + d)(b + d)$ will be

(1) $q^2 - p^2$ (2) $p^2 - q^2$

(3) $-p^2 - q^2$ (4) $p^2 + q^2$

6. In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as – 10 in place of – 11 and the roots were found to be 4 and 6. The roots of the correct equation are

(1) 8, 3 (2) 4, 3

(3) 6, 3 (4) 5, 6

7. If α,β are roots of the equation $(3x + 2)^2 + p(3x + 2) + q = 0$, then roots of $x^2 + px + q = 0$ are

(1) α,β (2) $3\alpha + 2, 3\beta + 2$

(3) $\frac{1}{3}(\alpha-2), \frac{1}{3}(\beta-2)$ (4) $\alpha - 2, \beta - 2$

8. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$, then $h =$

(1) $\left(\frac{b}{a} - \frac{q}{p}\right)$ (2) $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

(3) $-\frac{1}{2}\left(\frac{a}{b} - \frac{p}{q}\right)$ (4) $-\frac{1}{2}\left(\frac{a}{b} + \frac{p}{q}\right)$

9. If α, β be the roots of the equation $(x - a)(x - b) + c = 0$ ($c \neq 0$), then the roots of the equation $(x - c - \alpha)(x - c - \beta) = c$ are

(1) a and $b + c$ (2) $a + b$ and b

(3) $a + c$ and $b + c$ (4) $a - c$ and $b - c$

10. Let α, β, γ be the roots of $(x - a)(x - b)(x - c) = d$, $d \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ are :

(1) $a + 1, b + 1, c + 1$ (2) a, b, c

(3) $a - 1, b - 1, c - 1$ (4) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

11. Let α, β be the roots of $x^2 + (3 - \lambda) x - \lambda = 0$. The value of λ for which $\alpha^2 + \beta^2$ is minimum, is

(1) 0 (2) 1

(3) 2 (4) 3

12. If a, b are non-zero real numbers and α, β are the roots of $x^2 + ax + b = 0$, then

(1) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$

(2) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax - 1 = 0$

(3) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b + a^2)x + b = 0$

(4) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

13. The values of k for which the expression $kx^2 + (k + 1)x + 2$ will be a perfect square of linear factor are

(1) $3 \pm 2\sqrt{2}$ (2) $4 \pm 2\sqrt{2}$

(3) 6 (4) 5

14. If $x^2 + (a - b)x + (1 - a - b) = 0$, $a, b \in R$ then the value of 'a' for which both roots of the equation are real and unequal $\forall\, b \in R$ is

(1) $(2, \infty)$ (2) $(3, \infty)$

(3) $(1, \infty)$ (4) $(-\infty, 1)$

15. If α, β are the real and distinct roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always

(1) Imaginary roots

(2) Two negative roots

(3) Two positive roots

(4) One positive root and one negative root

16. If $a < b < c < d$, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are

(1) Real and distinct

(2) Imaginary

(3) Real and equal

(4) Can't say anything

17. The values of k, for which the equation $x^2 + 2(k - 1)x + k + 5 = 0$ possess atleast one positive root, are:

(1) $[4, \infty)$

(2) $(-\infty, -1] \cup [4, \infty)$

(3) $[-1, 4]$

(4) $(-\infty, -1]$

18. If the two equations $x^2 - cx + d = 0$ and $x^2 - ax + b = 0$ have one common root and the second equation has equal roots, then $2(b + d) =$

(1) 0 (2) $a + c$

(3) ac (4) $-ac$

19. Let α, β be the roots of $f(x) = 3x^2 - 4x + 5 = 0$.

STATEMENT-1 : The equation whose roots are $2\alpha, 2\beta$ is given by $3x^2 + 8x - 20 = 0$.

STATEMENT-2: To obtain, from the equation $f(x) = 0$, having roots α and β, the equation having roots $2\alpha, 2\beta$ one needs to change x to $\frac{x}{2}$ in $f(x) = 0$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is False, Statemetn-2 is True

(4) Statement-1 is True, Statement-2 is False

20. **STATEMENT - 1** : Maximum value of $\log_{1/3}(x^2 - 4x + 5)$ is '0'.

STATEMENT - 2 : $\log_a x \le 0$ for $x \ge 1$ and $0 < a < 1$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statemetn-2 is True

21. If the quadratic equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then the equation containing their other roots is

(1) $x^2 + a(b + c)x - a^2bc = 0$

(2) $x^2 - a(b + c)x + a^2bc = 0$

(3) $a(b + c)x^2 + (b + c)x - bc = 0$

(4) $a(b + c)x^2 - (b + c)x + abc = 0$

22. If α, β are roots of $x^2 + 3x + 1 = 0$, then

(1) $(7 - \alpha)(7 - \beta) = 0$

(2) $(2 - \alpha)(2 - \beta) = 11$

(3) $\frac{\alpha^2}{3\alpha+1} + \frac{\beta}{3\beta-1} = -2$

(4) $\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{\alpha+1}\right)^2 = 15$

23. All the values of 'm' for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 lie in the interval

(1) $m > 3$ (2) $-1 < m < 3$

(3) $1 < m < 4$ (4) $-2 < m < 0$

24. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are :

(1) 6, 1 (2) 4, 3

(3) –6 , –1 (4) –4 , –3

25. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a,b,c \in R$, have a common root, then $a : b : c$ is

(1) 1 : 2 : 3 (2) 3 : 2 : 1

(3) 1 : 3 : 2 (4) 3 : 1 : 2

26. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval :

(1) $(-2, -1)$

(2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-1, 0) \cup (0, 1)$

(4) $(1, 2)$

27. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q ,r are in the A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$

(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

28. For the equation $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to

(1) 1/3 (2) 1

(3) 3 (4) 2/3

29. If α and β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then

(1) $0 < \alpha < \beta$

(2) $\alpha < 0 < \beta < |\alpha|$

(3) $\alpha < \beta < 0$

(4) $\alpha < 0 < |\alpha| < \beta$

30. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$, has:

(1) Both roots in $[a, b]$

(2) Both roots in $(-\infty, a)$

(3) Both roots in $[b, \infty)$

(4) One root in $(-\infty, a)$ and other in (b, ∞)

31. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min $f(x) >$ max $g(x)$, then the relation between b and c, is

(1) No relation (2) $0 < c < b/2$

(3) $|c| < \sqrt{2}|b|$ (4) $|c| > \sqrt{2}|b|$

32. If the quadratic expression $x^2 + 2ax - 3a + 10 > 0 \ \forall \ x \in R$, then

(1) $a > 5$ (2) $|a| < 5$

(3) $-5 < a < 2$ (4) $2 < a < 3$

33. If one root of the equation $x^2 + px + q = 0$ is square of other, then the relation between p, q is

(1) $p^3 - q(3p - 1) + q^2 = 0$

(2) $p^3 + q(3p + 1) + q^2 = 0$

(3) $p^3 + q(3p - 1) + q^2 = 0$

(4) $p^3 - q(3p + 1) + q^2 = 0$

34. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

(1) $\frac{2}{9}(p-q)(2q-p)$

(2) $\frac{2}{9}(q-p)(2p-q)$

(3) $\frac{2}{9}(q-2p)(2q-p)$

(4) $\frac{2}{9}(2p-q)(2q-p)$

35. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(1) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(2) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(3) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(4) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

36. A value of b for which the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have one root in common is

(1) $-\sqrt{2}$ (2) $-i\sqrt{3}$
(3) $i\sqrt{5}$ (4) $\sqrt{2}$

37. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has

(1) Only purely imaginary roots
(2) All real roots
(3) Two real and two purely imaginary roots
(4) Neither real nor purely imaginary roots

38. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(1) $2(\sec\theta - \tan\theta)$ (2) $2\sec\theta$
(3) $-2\tan\theta$ (4) 0

39. The number of values of 'a' for which $a(a^2 - 3a + 2)x^2 + (a^3 - 5a^2 + 6a)x + a^2 - 2a = 0$ is an identity in x, is

(1) 0 (2) 1
(3) 2 (4) 3

40. The number of triplet (a,b,c) for which $a(2\cos^2 x - 1) + b\sin^2 x + c = 0$ is satisfied by all real (where $a, b, c \in N$)

(1) 0 (2) 1
(3) 2 (4) Infinite

41. If $a, b \in R$ and $a \neq b$ then the roots of the quadratic equation $(a-b)x^2 - 5(a+b)x - 2(a-b) = 0$ are

(1) Real and equal
(2) Real and unequal
(3) Complex
(4) Rational and equal

42. The condition for which equation $\frac{1}{x} + \frac{1}{x+b} = \frac{1}{m} + \frac{1}{m+b}$ has real roots with equal in magnitude but opposite in sign, is

(1) $b^2 = m^2$ (2) $b^2 = 2m^2$
(3) $2b^2 = m^2$ (4) $4b^2 = m^2$

43. If ratio of the roots of the equation $ax^2 + bx + c = 0$ is $p : q$ then

(1) $c(p+q)^2 = -pqb$
(2) $ac(p+q)^2 = b^2pq$
(3) $ac(p+q)^2 + b^2pq = 0$
(4) $a(p+q)^2 + cpq = 0$

44. If roots of $a_1x^2 + b_1x + c_1 = 0$ are α_1, β_1 and roots of $a_2x^2 + b_2x + c_2 = 0$ are α_2, β_2 such that $\alpha_1\alpha_2 = \beta_1\beta_2 = 1$ then

(1) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (2) $\frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$

(3) $a_1a_2 = b_1b_2 = c_1c_2$ (4) $\frac{a_1}{b_2} = \frac{b_1}{c_2} = \frac{c_1}{a_2}$

45. Subset of the values of a for which the quadratic equation $3x^2 + 2(a^2 + 1)x + a^2 - 3a + 2 = 0$ possess roots of opposite sign is

(1) $(-\infty, 1)$ (2) $(-\infty, 0)$

(3) $(-1, 3)$ (4) $\left(\frac{3}{2}, 2\right)$

46. If $c > 0$ and $4a + c < 2b$ then $ax^2 - bx + c = 0$ has a roots in the interval

(1) $(0,2)$ (2) $(2,4)$

(3) $(0,1)$ (4) $(-2,0)$

47. If x is real, then the least value of the expression $\frac{x^2 - 6x + 5}{x^2 + 2x + 1}$ is

(1) -1 (2) $-\frac{1}{2}$

(3) $-\frac{1}{3}$ (4) 4

48. If S be the set of real values of p for which the equation $x^2 = p(x + p)$ has its roots greater than p, then p is equal to

(1) $(-2, -1/2)$ (2) $\left(\frac{-1}{2}, \frac{1}{4}\right)$

(3) ϕ (4) $(-\infty, 0)$

49. If α, β are roots of the equation $ax^2 + 2x + 5 = 0$ then the value of $\frac{(\alpha - 1)(\beta - 1)}{\left[(\alpha + 1)(\beta + 1) + \left(\frac{4}{a}\right)\right]}$ is

(1) $\frac{1}{a+2}$ (2) $a + 2$

(3) 1 (4) 2

50. If α, β are roots of the equation $3x^2 + 6x + c = 0$ then equation having roots $\alpha^2 + 2\alpha$, $\beta^2 + 2\beta$ is

(1) $3x^2 + 6cx + c^2 = 0$

(2) $9x^2 + 6cx + c^2 = 0$

(3) $3x^2 - 6cx + c^2 = 0$

(4) $9x^2 - 6cx + c^2 = 0$

51. If $(2x^2 + bx + c)(x^2 + bx - c) = 0$, $b, c \in R$ then equation has

(1) four real roots

(2) two real and two imaginary roots

(3) at least two real roots

(4) four imaginary roots

52. If α, β, γ are roots of equation $x^3 - 3x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is

(1) 0 (2) -3

(3) 6 (4) 9

53. If $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$, then the quadratic equation whose roots are α and β is

(1) $x^2 + 2x - 16 = 0$ (2) $x^2 + 2x + 15 = 0$

(3) $x^2 + 2x - 12 = 0$ (4) $x^2 + 2x - 8 = 0$

54. If the equation $x^2 + y^2 - 10x + 21 = 0$ has real roots $x = \alpha$ and $y = \beta$, then

(1) $3 \le x \le 7$ (2) $3 \le y \le 7$

(3) $-5 \le y \le 1$ (4) $-2 \le x \le 2$

55. If $2 + i$ and $\sqrt{5} - 2i$ are the roots of the equation $(x^2 + ax + b)(x^2 + cx + d) = 0$, where a, b, c, d are real constants, then product of all roots of the equation is

(1) 40 (2) $9\sqrt{5}$

(3) 45 (4) 35

56. If α and α^2 are the roots of the equation $x^2 - 6x + c = 0$, then the positive value of c is

(1) 2 (2) 8

(3) 4 (4) 9

57. If $a(p+q)^2 + 2apq + c = 0$ and $a(p+r)^2 + 2apr + c = 0$, then qr equals

(1) $p^2 + \frac{c}{a}$ (2) $p^2 + \frac{a}{c}$

(3) $p^2 + \frac{a}{b}$ (4) $p^2 + \frac{b}{a}$

58. Roots of the equation $(a + b - c)x^2 - 2ax + (a - b + c) = 0$, $(a, b, c \in Q)$ are

(1) Rational

(2) Irrational

(3) Complex

(4) Can't be determine

59. If a, b, c, d are real numbers, then the number of real roots of the equation $(x^2 + ax - 3b)(x^2 - cx + b)(x^2 - dx + 2b) = 0$ are

(1) 3 (2) 4
(3) 6 (4) At least 2

60. If roots of the equation $x^2 - 10ax - 11b = 0$ are c and d and those of $x^2 - 10cx - 11d = 0$ are a and b then the value of $a + b + c + d$ is (where a, b, c, d are all distinct numbers)

(1) 1210 (2) 110
(3) 1100 (4) 1200

61. If 'x' is real, then $\dfrac{x^2 - x + c}{x^2 + x + 2c}$ can take all real values if :

(1) $c \in [0, 6]$
(2) $c \in [-6, 0]$
(3) $c \in (-\infty, -6) \cup (0, \infty)$
(4) $c \in (-6, 0)$

62. If $p, q, r, s \in R$, then equation $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx - 2q) = 0$ has

(1) 6 real roots
(2) At least two real roots
(3) 2 real and 4 imaginary roots
(4) 4 real and 2 imaginary roots

63. If two roots of the equation $(a - 1)(x^2 + x + 1)^2 - (a + 1)(x^4 + x^2 + 1) = 0$ are real and distinct, then 'a' lies in the interval

(1) $(-2, 2)$
(2) $(-\infty, -2) \cup (2, \infty)$
(3) $(2, \infty)$
(4) $(-3, 3)$

64. Number of solutions of equation $(a+x)^{2/3} + (x - a)^{2/3} = 4(a^2 - x^2)^{1/3}$ are :

(1) 3 (2) 4
(3) 1 (4) 2

65. The roots of the equation $(3-x)^4 + (2-x)^4 = (5-2x)^4$ are

(1) Two real and two imaginary
(2) All imaginary
(3) All real
(4) One real and three imaginary

Integer Type Questions (66 to 75)

66. The number of roots of the equation $\sqrt{x^2 - 4} - (x - 2) = \sqrt{x^2 - 5x + 6}$ is

67. The value of 'a' for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is -

68. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$ respectively, then the value of $2 + q - p$ is

69. If 'x' is real, then maximum value of $\dfrac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is -

70. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less $\sqrt{5}$, then the positive integral possible values of 'a' is

71. Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is :

72. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is equal to :

73. The number of real roots of the equation $(x^2 - 6x + 1)(x^2 + 3x + 6) = 0$ is

74. Let a, b, c be three distinct positive real number then the number of positive roots of $ax^2 + 2b|x| + c = 0$ is

75. The number of real solutions of the equation $27^{1/x} + 12^{1/x} = 2(8^{1/x})$ is

CHAPTER

05

COMPLEX NUMBERS

Single Option Correct Type Questions (01 to 62)

1. The real part of $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is

(1) $\frac{1}{3+5\cos\theta}$ (2) $\frac{1}{5-3\cos\theta}$

(3) $\frac{1}{3-5\cos\theta}$ (4) $\frac{1}{5+3\cos\theta}$

2. If $z = \frac{1+i\sqrt{3}}{\sqrt{3}+i}$, then $(\bar{z})^{100}$ lies in

(1) I quadrant (2) II quadrant

(3) III quadrant (4) IV quadrant

3. The principal value of the $arg(z)$ and $|z|$ of the complex number $z = 1 + \cos\left(\frac{11\pi}{9}\right) + i\sin\left(\frac{11\pi}{9}\right)$ are respectively:

(1) $\frac{11\pi}{18}, 2\cos\frac{\pi}{18}$

(2) $-\frac{7\pi}{18}, 2\cos\frac{7\pi}{18}$

(3) $\frac{2\pi}{9}, 2\cos\frac{7\pi}{18}$

(4) $-\frac{\pi}{9}, -2\cos\frac{\pi}{18}$

4. If $(2 + i)(2 + 2i)(2 + 3i) \ldots\ldots (2 + 9i) = x + iy$, then 5.8.13. ....85 =

(1) $x^2 + y^2$ (2) $x^2 - y^2$

(3) $(x^2 + y^2)^2$ (4) $(x^2 - y^2)^2$

5. If $|z_1 + z_2| = |z_1 - z_2|$ then the value of $|\text{amp } z_1 - \text{amp } z_2|$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$

(3) π (4) $\frac{\pi}{3}$

6. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is

(1) equal to 1 (2) less than 1

(3) greater than 3 (4) equal to 3

7. If $|z - 2 + i| = 2$, then the greatest and least value of $|z|$ are respectively

(1) $\sqrt{5}+2, \sqrt{5}-2$ (2) $\sqrt{5}+2, 2-\sqrt{5}$

(3) $\sqrt{5}+2, 0$ (4) $\sqrt{5}-2, 0$

8. The vector $z = -4 + 5i$ is turned counter clockwise through an angle of 180º & stretched 1.5 times. The complex number corresponding to the newly obtained vector is

(1) $6 - \frac{15}{2}i$ (2) $-6 + \frac{15}{2}i$

(3) $6 + \frac{15}{2}i$ (4) $6 + 15i$

9. If $z = x + iy$ and $|z - 2 + i| = |z - 3 - i|$, then locus of z is

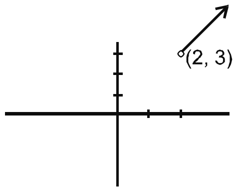
(1) $2x + 4y - 5 = 0$

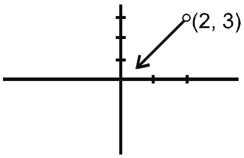
(2) $2x - 4y - 5 = 0$

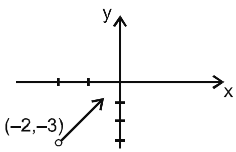
(3) $x + 2y = 0$

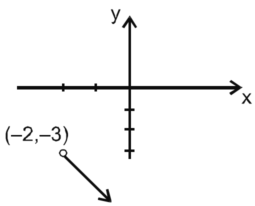
(4) $x - 2y + 5 = 0$

10. If $Arg\,(z-2-3i)=\frac{\pi}{4}$, then the locus of z is

(1) 

(2) 

(3) 

(4) 

11. Let A, B, C represent the complex numbers z_1, z_2, z_3 respectively on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number

(1) $z_1+z_2-z_3$ (2) $z_2+z_3-z_1$
(3) $z_3+z_1-z_2$ (4) $z_1+z_2+z_3$

12. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is

(1) Of area zero
(2) Right angled isosceles
(3) Equilateral
(4) Obtuse angled isosceles

13. The value of

$$\frac{(\cos 2\theta-i\sin 2\theta)^4(\cos 4\theta+i\sin 4\theta)^{-5}}{(\cos 3\theta+i\sin 3\theta)^{-2}(\cos 3\theta-i\sin 3\theta)^{-9}}$$ is

(1) $\cos 49\theta-i\sin 49\theta$
(2) $\cos 23\theta-i\sin 23\theta$
(3) $\cos 49\theta+i\sin 49\theta$
(4) $\cos 21\theta+i\sin 21\theta$

14. $\left[\frac{1+\cos(\pi/8)+i\sin(\pi/8)}{1+\cos(\pi/8)-i\sin(\pi/8)}\right]^8$ is equal to

(1) – 1 (2) 0
(3) 1 (4) 2

15. If $(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)(\cos 3\theta+i\sin 3\theta)\ldots(\cos n\theta+i\sin n\theta)=1$, then the value of θ is

(1) $4m\pi,\ m\in Z$
(2) $\frac{2m\pi}{n\,(n+1)},\ m\in Z$
(3) $\frac{4m\pi}{n\,(n+1)},\ m\in Z$
(4) $\frac{m\pi}{n\,(n+1)},\ m\in Z$

16. If $i=\sqrt{-1}$, then $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(1) $1-i\sqrt{3}$ (2) $-1+i\sqrt{3}$
(3) $i\sqrt{3}$ (4) $-i\sqrt{3}$

17. If $z_r=\cos\frac{2r\pi}{5}+i\sin\frac{2r\pi}{5}$, $r=0, 1, 2, 3, 4, \ldots$ then the value of $z_1.\,z_2.\,z_3.\,z_4.\,z_5$ is

(1) 3 (2) 5
(3) 1 (4) –1

18. The modulus and the principal argument of the complex number $z=-2\,(\cos 30^\circ+i\sin 30^\circ)$ are respectively

(1) $2,\ -\frac{\pi}{6}$ (2) $2,\ -\frac{5\pi}{6}$
(3) $-2,\ \frac{\pi}{6}$ (4) $2,\ \frac{7\pi}{6}$

19. The modulus and the principal argument of the complex number $z = 1 + \cos\frac{18\pi}{25} + i\sin\frac{18\pi}{25}$ are respectively

(1) $2\cos\frac{9\pi}{25}, \frac{9\pi}{25}$

(2) $2\sin\frac{9\pi}{25}, \frac{9\pi}{25}$

(3) $\cos\frac{9\pi}{25}, \frac{9\pi}{25}$

(4) $2\cos\frac{9\pi}{25}, \frac{16\pi}{25}$

20. If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to

(1) $\beta + i\alpha$ (2) $\alpha - i\beta$

(3) $\beta - i\alpha$ (4) $-\alpha - i\beta$

21. If $|z_1 + z_2|^2 = |z_1|^2 + |z_z|^2$ then

(1) $amp\ \frac{z_1}{z_2}$ may be equal to $\frac{\pi}{2}$

(2) $\frac{z_1}{z_2}$ is purely imaginary

(3) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

(4) All of these

22. If z satisfies the inequality $|z - 1 - 2i| \le 1$, then

(1) min $(arg(z)) = \tan^{-1}\left(\frac{3}{4}\right)$

(2) max $(arg(z)) = \frac{\pi}{6}$

(3) min $(|z|) = \sqrt{5}$

(4) max $(|z|) = \sqrt{5}$

23. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, ...9$.

Column- I		**Column- II**	
I	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	P	True
II	There exists $k \in \{1, 2,....,9\}$ such that $z_1 . z = z_k$ has no solution z in the set of complex numbers.	Q	False
III	$\frac{\lvert 1-z_1\rvert \lvert 1-z_2\rvert .....\lvert 1-z_9\rvert}{10}$ equals	R	1
IV	$1 - \sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals	S	2

	I	II	III	IV
(1)	P	Q	S	R
(2)	Q	P	R	S
(3)	P	Q	R	S
(4)	Q	P	S	R

24. If a and b are real numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, then

(1) $a = b = 2 + \sqrt{3}$

(2) $a = b = 2 - \sqrt{3}$

(3) $a = 2 - \sqrt{3}$ and $b = 2 + \sqrt{3}$

(4) $a = 2 + \sqrt{3}$ and $b = 2 - \sqrt{3}$

25. If $|z_1| = |z_2| = |z_3| = 1$ and z_1, z_2, z_3 are represented by the vertices of an equilateral triangle then

(1) $z_1 + z_2 + z_3 = 0$ (2) $z_1z_2z_3 = 1$

(3) $z_1 + z_2 + z_3 = 1$ (4) $z_1z_2z_3 = 0$

26. If three complex numbers are in *A.P.*, then they lie on

(1) A circle in the complex plane

(2) A straight line in the complex plane

(3) A parabola in the complex plane

(4) Can not say

27. The radius of the circle $z\bar{z} + (4 - 3i)z + (4 + 3i)\bar{z} + 5 = 0$ is

(1) $2\sqrt{5}$ (2) $\sqrt{5}$

(3) $3\sqrt{5}$ (4) $4\sqrt{5}$

28. The value of expression $\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right)\left(\cos\frac{\pi}{2^3} + i\sin\frac{\pi}{2^3}\right)$ .... to ∞ is

(1) -1 (2) 1

(3) 0 (4) 2

29. The expression $\left[\frac{1+i\tan\alpha}{1-i\tan\alpha}\right]^n - \frac{1+i\tan n\alpha}{1-i\tan n\alpha}$ when simplified reduces to

(1) zero (2) $2\sin n\alpha$
(3) $2\cos n\alpha$ (4) $-2\cos n\alpha$

30. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then

(1) $x^n + \frac{1}{x^n} = 2\cos(n\theta)$
(2) $x^n + \frac{1}{x^n} = 2\sin(n\theta)$
(3) $x^n - \frac{1}{x^n} = 2\cos(n\theta)$
(4) $y^n + \frac{1}{y^n} = 2\sin(n\phi)$

31. Let ω be the non real cube root of unity which satisfy the equation $h(x) = 0$ where $h(x) = xf(x^3) + x^2g(x^3)$.
If $h(x)$ is polynomial with real coefficient then which statement is incorrect?

(1) $f(1) = 0$ (2) $g(1) = 0$
(3) $h(1) = 0$ (4) $g(1) \neq f(1)$

32. Let ω be an imaginary root of $x^n = 1$. Then $(5 - \omega)(5 - \omega^2) \ldots. (5 - \omega^{n-1})$ is

(1) 1 (2) $\frac{5^n+1}{4}$
(3) 4^{n-1} (4) $\frac{5^n-1}{4}$

33. If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of $x^5 - 1 = 0$, then the value of $\frac{\omega-\alpha_1}{\omega^2-\alpha_1}\,\frac{\omega-\alpha_2}{\omega^2-\alpha_2}\,\frac{\omega-\alpha_3}{\omega^2-\alpha_3}\,\frac{\omega-\alpha_4}{\omega^2-\alpha_4}$ ... is . (where ω is imaginary cube root of unity.)

(1) ω (2) ω^2 (3) 2ω (4) $2\omega^2$

34. If $\alpha = e^{i2\pi/11}$ then Real $(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5)$ equals to:

(1) $\frac{1}{2}$ (2) 1 (3) $-\frac{1}{2}$ (4) -1

35. If $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$ is

(1) $x^2 + x - 2 = 0$ (2) $x^2 - x + 2 = 0$
(3) $x^2 - x - 2 = 0$ (4) $x^2 + x + 2 = 0$

36. **STATEMENT-1 :** $Arg\,(2 + 3i) + Arg\,(2 - 3i) = 0$ ($Arg\,z$ stands for principal argument of z)

STATEMENT-2 : $Arg\,z + Arg\,\bar{z} = 0$, $z = x + iy$, $\forall\ x, y \in R$ ($Arg\,z$ stands for principal argument of z)

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true
(3) Statement-1 is false, statement-2 is false
(4) Statement-1 is true, statement-2 is false.

37. **Statement-1:** Roots of the equation $(1 + z)^6 + z^6 = 0$ are collinear.

Statement-2: If z_1, z_2, z_3 are in $A.P.$ then points represented by z_1, z_2, z_3 are collinear

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

38. Let z_1, z_2, z_3 represent vertices of a triangle.

Statement-1: $\frac{1}{z_1-z_2} + \frac{1}{z_2-z_3} + \frac{1}{z_3-z_1} = 0$, when triangle is equilateral.

Statement-2 : $|z_1|^2 - z_1\bar{z}_0 - \bar{z}_1 z_0 = |z_2|^2 - z_2\bar{z}_0 - \bar{z}_2 z_0 = |z_3|^2 - z_3\bar{z}_0 - \bar{z}_3 z_0$, where z_0 is circumcentre of triangle.

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(4) Statement-1 is true, statement-2 is false.

39. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals-

(1) 128ω (2) -128ω
(3) $128\omega^2$ (4) $-128\omega^2$

40. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral tringle. Then:

(1) $a^2 = b$ (2) $a^2 = 2b$
(3) $a^2 = 3b$ (4) $a^2 = 4b$

41. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $arg(z) - arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\ \omega$ is equal to :

(1) 1 (2) -1
(3) i (4) $-i$

42. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

(1) $x = 4n$, where n is any positive integer
(2) $x = 2n$, where n is any positive integer
(3) $x = 4n + 1$, where n is any positive integer
(4) $x = 2n + 1$, where n is any positive integer

43. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $arg\ zw = \pi$. Then $arg\ z$ equals

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$
(3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{4}$

44. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right)\Big/(p^2 + q^2)$ is equal to

(1) 1 (2) -1
(3) 2 (4) -2

45. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x - 1)^3 + 8 = 0$, are

(1) $-1, 1 + 2\omega, 1 + 2\omega^2$.
(2) $-1, 1 - 2\omega, 1 - 2\omega^2$.
(3) $-1, -1, -1$.
(4) $-1, -1 + 2\omega, -1 - 2\omega^2$.

46. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $arg\ z_1 - arg\ z_2$ is equal to:

(1) $-\frac{\pi}{2}$ (2) 0
(3) $-\pi$ (4) $\frac{\pi}{2}$

47. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$, then z lies on

(1) a parabola (2) a straight line
(3) a circle (4) an ellipse.

48. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is:

(1) 1 (2) -1
(3) $-i$ (4) i

49. If $\omega(\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals

(1) (0, 1) (2) (1, 1)
(3) (1, 0) (4) (–1, 1)

50. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $Re\ z = 1$, then it is necessary that

(1) $\beta \in (0, 1)$ (2) $\beta \in (-1, 0)$
(3) $|\beta| = 1$ (4) $\beta \in (1, \infty)$

51. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals :

(1) $-\theta$ (2) $\frac{\pi}{2} - \theta$ (3) θ (4) $\pi - \theta$

52. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a

(1) Straight line parallel to x-axis
(2) Straight line parallel to y-axis
(3) Circle of radius 2

(4) Circle of radius $\sqrt{2}$

53. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is:

(1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

54. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), the $Re(\omega)$ is

(1) 0 (2) $-\frac{1}{|z+1|^2}$

(3) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ (4) $\frac{\sqrt{2}}{|z+1|^2}$

55. If $|z| = 2$, then the points representing the complex numbers $-1 + 5z$ will lie on a

(1) circle (2) straight line

(3) parabola (4) hyperbola

56. Values of z satisfying the equation $z^2 - (1 + i) zz_1 + iz_1^2 = 0$ (where z_1 is a complex no.) are two vertices of a triangle having one vertex as origin then the area of this triangle is

(1) $\frac{1}{3}|z_1|^2$ (2) $2|z_1|^2$

(3) $\frac{1}{2}|z_1|^2$ (4) $3|z_1|^2$

57. Let z be non real number such that

$\frac{1+z+z^2}{1-z+z^2} \in R$, then value of $7|z|$ is

(1) 1 (2) 3

(3) 5 (4) 7

58. If $\log_{1/2}\left(\frac{|z-1|+4}{3|z-1|-2}\right) > 1$, then the locus of z is

(1) Exterior to circle with center $1 + i0$ and radius 10

(2) Interior to circle with center $1 + i0$ and radius 10

(3) Circle with center $1 + i0$ and radius 10

(4) Circle with center $2 + i0$ and radius 10

59. If z_1 & z_2 are two complex numbers & if $arg \frac{z_1+z_2}{z_1-z_2} = \frac{\pi}{2}$ but $|z_1 + z_2| \neq |z_1 - z_2|$ then the figure formed by the points represented by 0, z_1, z_2 & $z_1 + z_2$ is

(1) A parallelogram but not a rectangle or a rhombous

(2) A rectangle but not a square

(3) A rhombous but not a square

(4) A square

60. Let $\omega = \alpha + i\beta$, $\beta \neq 0$ and $z \neq 1$, If $\frac{\omega - \bar{\omega}z}{1-z}$ is purely real, then the set of values of z is

(1) $\{z: |z| = 1\}$

(2) $\{z : \bar{z} = z\}$

(3) $\{z: |z| \neq 1\}$

(4) $\{z: |z| = 1, z \neq 1\}$

61. A man walks a distance of 3 units from the origin towards the north-east (N 45ºE) direction. From there, he walks a distance of 4 units towards the north-west (N 45º W) direction to reach a point P. Then the position of P in the Argand plane is

(1) $3e^{i\pi/4} + 4i$

(2) $(3 - 4i)e^{i\pi/4}$

(3) $(4 + 3i)e^{i\pi/4}$

(4) $(3 + 4i)e^{i\pi/4}$

62. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on

(1) A line not passing through the origin

(2) $|z| = \sqrt{2}$

(3) The x-axis

(4) The y-axis

Integer Type Questions (63 to 73)

63. If $i^2 = -1$, then the value of $\sum_{n=1}^{200} i^n$ is

64. If $z = 3 - 4i$, then $z^4 - 3z^3 + 3z^2 + 99z - 95$ is equal to

65. If $\frac{z-i}{z+i}$ $(z \neq -i)$ is a purely imaginary number, then $z\bar{z}$ is equal to

66. If ω is the cube root of unity, then $\left|(3+5\omega+3\omega^2)^2+(3+3\omega+5\omega^2)^2\right| =$

67. If $x^2 + x + 1 = 0$ then the numerical value of;

$$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\left(x^4+\frac{1}{x^4}\right)^2+\ldots\ldots+\left(x^{27}+\frac{1}{x^{27}}\right)^2$$

68. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is

69. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

70. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

71. a, b, c are integers, not all simultaneously equal and ω is cube root of unity $(\omega \neq 1)$, then minimum value of $|a + b\omega + c\omega^2|$ is

72. If α and β are imaginary cube roots of unity, then $\alpha^4+\beta^4+\frac{1}{\alpha\beta} =$

73. O is origin and affixes of P, Q, R are respectively z, iz, $z + iz$. If $\Delta PQR = 200$ then the value of $|z|$ is

CHAPTER

06

BINOMIAL THEOREM

Single Option Correct Type Questions (01 to 60)

1. The coefficient of x^{52} in the expansion $\sum_{m=0}^{100} {}^{100}C_m \ (x-3)^{100-m}. 2^m$ is

(1) ${}^{100}C_{47}$ (2) ${}^{100}C_{48}$

(3) $-{}^{100}C_{52}$ (4) $-{}^{100}C_{100}$

2. The co-efficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+\ldots\ldots+(1+x)^{30}$ is

(1) ${}^{51}C_5$ (2) ${}^{9}C_5$

(3) ${}^{31}C_6-{}^{21}C_6$ (4) ${}^{30}C_5+{}^{20}C_5$

3. The value of

$$\frac{C_0}{1\cdot3}-\frac{C_1}{2\cdot3}+\frac{C_2}{3\cdot3}-\frac{C_3}{4\cdot3}+\ldots\ldots+(-1)^n\frac{C_n}{(n+1)\cdot3}$$

is

(1) $\frac{3}{n+1}$ (2) $\frac{n+1}{3}$

(3) $\frac{1}{3(n+1)}$ (4) $\frac{1}{(n+1)}$

4. The value of the expression $\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{K=0}^{10}(-1)^K\frac{{}^{10}C_K}{2^K}\right)$ is

(1) 2^{10} (2) 2^{20}

(3) 1 (4) 2^5

5. If $a_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$, the value of $\sum_{r=0}^{n}\frac{n-2r}{{}^nC_r}$ is

(1) $\frac{n}{2}a_n$ (2) $\frac{1}{4}a_n$

(3) na_n (4) 0

6. In the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$, the term independent of x is

(1) $C_0^2+2C_1^2+\ldots..+(n+1)C_n^2$

(2) $(C_0+C_1+\ldots.+C_n)^2$

(3) $C_0^2+C_1^2+\ldots\ldots.+C_n^2$

(4) $C_0^3+C_1^2+C_2^3+\ldots\ldots+C_n^3$

7. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n\in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then n is

(1) 25 (2) 20

(3) 15 (4) None of these

8. In the expansion of $\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}$

(1) the number of irrational terms is 19

(2) middle term is irrational

(3) the number of rational terms is 2

(4) All of these

9. The last three digits of the number $(27)^{27}$ is

(1) 805 (2) 301

(3) 503 (4) 803

10. The sum of the series $\sum_{r=1}^{n}(-1)^{r-1}.{}^nC_r(a-r)$ is equal to

(1) $n\cdot2^{n-1}+a$ (2) 0

(3) a (4) 2^n+a+1

11. If $(1 + x + x^2)^n = a_0 + a_1x + a_2 x^2 + ... + a_{2n} x^{2n}$, then $a_0 + a_2 + a_4 + ... + a_{2n}$ is

(1) $\frac{3^n - 1}{2}$ (2) $\frac{3^n + 1}{2}$

(3) $\frac{3^n - 2}{2}$ (4) $\frac{3^n - 5}{2}$

12. The sum of the coefficients of all the integral powers of x in the expansion of $\left(1+2\sqrt{x}\right)^{40}$ is

(1) $3^{40} + 1$ (2) $3^{40} - 1$

(3) $\frac{1}{2}(3^{40} - 1)$ (4) $\frac{1}{2}(3^{40} + 1)$

13. The sum $\sum_{r=0}^{n} (r + 1)\, C_r^2$ is equal to

(1) $\frac{(n+2)(2n-1)!}{n!\,(n-1)!}$

(2) $\frac{(n+2)(2n+1)!}{n!\,(n-1)!}$

(3) $\frac{(n+2)(2n+1)!}{n!\,(n+1)!}$

(4) $\frac{(n+2)(2n-1)!}{n!\,(n+1)!}$

14. The number of terms in the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n$, $n \in N$, is

(1) $2n$ (2) $3n$

(3) $2n + 1$ (4) $3n + 1$

15. $(1 + x + x^2 + x^3)^5 = a_0 + a_1x + a_2x^2 + ... + a_{15}x^{15}$, then a_{10} equals to

(1) 99 (2) 101

(3) 100 (4) 110

16. Coefficient of x^{n-1} in the expansion of, $(x + 3)^n + (x + 3)^{n-1}(x + 2) + (x + 3)^{n-2}(x + 2)^2 + ... + (x + 2)^n$ is

(1) ${}^{n+1}C_2(3)$ (2) ${}^{n+1}C_2(7)$

(3) ${}^{n+1}C_2(5)$ (4) ${}^{n}C_2(5)$

17. **STATEMENT - 1 :** The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^m$ is $\frac{(2m)!}{(m!)^2}$.

STATEMENT - 2 : The coefficient of x^b in the expansion of $(1 + x)^n$ is ${}^nC_b$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

18. **STATEMENT - 1 :** If n is even, then ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1} = 2^{2n-1}$.

STATEMENT - 2 : ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{2n-1} = 2^{2n-1}$.

(1) Statement-1 and Statement-2 both are True

(2) Statement-1 and Statement-2 both are False

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

19. Let $a_n = \frac{1000^n}{n!}$ for $n \in N$, then a_n is greatest, when

(1) $n = 997$ (2) $n = 998$

(3) $n = 1002$ (4) $n = 1000$

20. Which of the following is/are correct ?

(1) $101^{50} - 99^{50} < 100^{50}$

(2) $101^{50} + 100^{50} < 99^{50}$

(3) $(1000)^{1000} > (1001)^{999}$

(4) $(1001)^{999} > (1000)^{1000}$

21. If x is positive, the first negative term in the expansion of $(1+x)^{\frac{27}{5}}$ is

(1) 7th term (2) 5th term

(3) 8th term (4) 6th term.

22. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$ is

(1) $(n - 1)$ (2) $(-1)^n (1 - n)$

(3) $(-1)^{n-1}(n - 1)^2$ (4) $(-1)^{n-1} n$

23. If $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to

(1) $\frac{n}{2}$ (2) $\frac{n}{2} - 1$

(3) $n - 1$ (4) $\frac{2n-1}{2}$

24. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in AP, then m and r satisfy the equation

(1) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$.

(2) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$.

(3) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$.

(4) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$.

25. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as

(1) $\frac{x}{2} - \frac{3}{8}x^2$ (2) $-\frac{3}{8}x^2$

(3) $3x + \frac{3}{8}x^2$ (4) $1 - \frac{3}{8}x^2$

26. If the expansion in powers of x of the function $\frac{1}{1-ax \;\; 1-bx}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then a_n is :

(1) $\frac{a^n - b^n}{b-a}$ (2) $\frac{a^{n+1} - b^{n+1}}{b-a}$

(3) $\frac{b^{n+1} - a^{n+1}}{b-a}$ (4) $\frac{b^n - a^n}{b-a}$

27. For natural numbers m, n if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ...$ and $a_1 = a_2 = 10$, then (m, n) is

(1) (35, 20) (2) (45, 35)

(3) (35, 45) (4) (20, 45)

28. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th term is zero, then $\frac{a}{b}$ equals

(1) $\frac{n-4}{5}$ (2) $\frac{5}{n-4}$

(3) $\frac{6}{n-5}$ (4) $\frac{n-5}{6}$

29. **Statement-1 :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n + 2)\, 2^{n-1}$

Statement-2 : $\sum_{r=0}^{n} (r + 1)\,{}^nC_r\, x^r = (1 + x)^n + nx\,(1 + x)^{n-1}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

30. Let $S_1 = \sum_{j=1}^{10} j\,(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement -1 : $S_3 = 55 \times 2^9$.

Statement -2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement-1 and Statement-2 both are True

(2) Statement-1 is true, Statement-2 is false.

(3) Statement -1 is false, Statement -2 is true

(4) Statement-1 and Statement-2 both are False

31. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(1) 144 (2) – 132
(3) – 144 (4) 132

32. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is

(1) An irrational number
(2) An odd positive integer
(3) An even positive integer
(4) A rational number other than positive integers

33. The term independent of x in expansion of $\left(\frac{x+1}{(x^{2/3}-x^{1/3}+1)} - \frac{x-1}{(x-x^{1/2})}\right)^{10}$ is

(1) 4 (2) 120
(3) 210 (4) 31

34. The coefficient of x^n in the expansion of $\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2+.....\right)^2$, is equal to

(1) $\frac{2^n}{n!}$ (2) $\frac{2^n}{n}$
(3) $n!$ (4) $\frac{1}{n!}$

35. The number of zeros at the end of $99^{1001} + 1$, is equal to

(1) 2 (2) 4
(3) 1002 (4) 1004

36. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$ is

(1) $2^{21} - 2^{11}$ (2) $2^{21} - 2^{10}$
(3) $2^{20} - 2^9$ (4) $2^{20} - 2^{10}$

37. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is

(1) 1 (2) 2
(3) –1 (4) 0

38. Coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is:

(1) ${}^{12}C_6 + 3$ (2) ${}^{12}C_6 + 1$
(3) ${}^{12}C_6$ (4) ${}^{12}C_6 + 2$

39. If ${}^{(n-1)}C_r = (k^2 - 3)\, {}^nC_{r+1}$, then an interval in which k lies is

(1) $(2, \infty)$ (2) $(-\infty, -2)$
(3) $\left[-\sqrt{3}, \sqrt{3}\right]$ (4) $\left(\sqrt{3}, 2\right]$

40. The value of

$$\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} - \text{.........} + \binom{30}{20}\binom{30}{30}$$ is

(1) $\binom{60}{20}$ (2) $\binom{30}{10}$
(3) $\binom{30}{15}$ (4) $\binom{30}{21}$

41. For $r = 0, 1, ..., 10$, let A_r, B_r and C_r denote, respectively the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

(1) $B_{10} - C_{10}$
(2) $A_{10}(B^2{}_{10} - C_{10} A_{10})$
(3) 0
(4) $C_{10} - B_{10}$

42. If the 6th term in expansion of $\left(2x^2 + \frac{1}{3x^2}\right)^{10}$ is $\frac{a}{b}$, where a & b are coprime natural numbers then $a + b =$

(1) 896 (2) 27
(3) 923 (4) 869

43. The coefficient of $a^3 b^4 c^5$ in the expansion of $(ab + bc + ca)^6$ is

(1) 80 (2) 50
(3) 60 (4) 70

44. If the sum of the coefficients in expansion of $(a^3x^2-2a^2x+1)^{51}$ vanishes, then possible value of a can be

(1) 1 (2) $\frac{1+\sqrt{5}}{2}$

(3) $\frac{1-\sqrt{5}}{2}$ (4) All of these

45. If the sum of coefficient in the expansion of $(x-2y+3z)^n$ is128, then the greatest coefficient in the expansion of $(1+x)^{2n}$ is

(1) ${}^{14}C_7$ (2) ${}^{7}C_4$

(3) ${}^{7}C_3$ (4) ${}^{16}C_8$

46. The last two digits of 17^{256} is

(1) 18 (2) 81

(3) 71 (4) 17

47. The value of $\left\{\frac{3^{2003}}{28}\right\}$, where {.} denotes the fractional part, is

(1) $\frac{17}{28}$ (2 $\frac{19}{28}$

(3) $\frac{23}{28}$ (4) $\frac{25}{28}$

48. The degree of the polynomial $\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7-\left(\frac{1-\sqrt{4x+1}}{2}\right)^7\right\}$ is

(1) 1 (2) 2

(3) 3 (4) 4

49. If T_0, T_1, T_2, T_3, ... represent the terms in the expansion of $(x+a)^n$, then the value of $(T_0-T_2+T_4-T_6+...)^2+(T_1-T_3+T_5-T_7+...)^2$, $n\in N$ is

(1) $(x^2+a^2)^{n/2}$ (2) $(x^2+a^2)^n$

(3) $(x^2-a^2)^n$ (4) $(a^2-x^2)^n$

50. If the numerically greatest term in the expansion of $(3-5x)^{15}$ when $x=\frac{1}{5}$ is 455×3^n, $n\in N$ then value of $\frac{{}^nC_2}{2}$ is

(1) 66 (2) 33

(3) 22 (4) 55

51. The sum of $1-\frac{1}{8}+\frac{1\cdot3}{8\cdot16}-\frac{1\cdot3\cdot5}{8\cdot16\cdot24}+.......$is

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{\sqrt{5}}{2}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{2}{\sqrt{3}}$

52. The coefficient of x^n in the expansion of $(1-9x+20x^2)^{-1}$ is

(1) 5^n-4^n (2) $5^{n-1}-4^{n-1}$

(3) $5^{n+1}-4^{n+1}$ (4) 0

53. The number of irrational terms in the expansion of $\left(5^{\frac{1}{6}}+2^{\frac{1}{8}}\right)^{100}$ is

(1) 5 (2) 97

(3) 95 (4) 6

54. If the last term in the binomial expansion of $\left(2^{\frac{1}{3}}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$, then the 5th term of expansion is

(1) 210 (2) 420

(3) 105 (4) 425

55. Let $R=(5\sqrt{5}+11)^{2n+1}$ and $f=R-[R]$ where [.] denotes the greatest integer function then value of Rf is

(1) 4^{2n+1} (2) 5^{2n+1}

(3) 4^{n+1} (4) 5^{n+1}

56. If the sum of the co-efficients in the expansion of $(1+2x)^n$ is 6561, then the greatest term in the expansion for $x=1/2$ is

(1) 4th (2) 5th

(3) 6th (4) 7th

57. The expression, $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$ is a polynomial of degree

(1) 5 (2) 6

(3) 7 (4) 8

58. Co-efficient of x^{15} in $(1 + x + x^3 + x^4)^n$ is :

(1) $\sum_{r=0}^{5} {}^nC_{15-3r}\,{}^nC_r$ (2) $\sum_{r=0}^{5} {}^nC_{5r}$

(3) $\sum_{r=0}^{5} {}^nC_{3r}$ (4) $\sum_{r=0}^{3} {}^nC_{3-r}\,{}^nC_{5r}$

59. The term independent of x in the expansion of $(1 + x + 2x^2)\left(3x^2 - \frac{1}{3x^2}\right)^4$ is

(1) 10 (2) 2

(3) 0 (4) 6

60. If n is even natural and coefficient of x^r in the expansion of $\frac{(1+x)^n}{(1-x)}$ is 2^n, $(|x| < 1)$, then

(1) $r \le n/2$ (2) $r \ge (n-2)/2$

(3) $r \le (n+2)/2$ (4) $r \ge n$

Integer Type Questions (61 to 74)

61. If the sum of the coefficients in the expansion of $(2 + 3cx + c^2x^2)^{12}$ vanishes, then number of negative value of c equals to

62. The coefficient of x^n in polynomial $(x + {}^{2n+1}C_0)(x + {}^{2n+1}C_1)........(x + {}^{2n+1}C_n)$ is 2^{an+b}, the value of a is

63. The value of $\binom{50}{0}\binom{50}{1} + \binom{50}{1}\binom{50}{2} + ... + \binom{50}{49}\binom{50}{50}$ is, where ${}^nC_r = \binom{n}{r}$, ${}^aC_b$ then the value of b is $(b < 50)$

64. ${}^nC_0 - 2\cdot 3\,{}^nC_1 + 3\cdot 3^2\,{}^nC_2 - 4\cdot 3^3\,{}^nC_3 + ... + (-1)^n\,(n+1)\,{}^nC_n\,3^n$ is equal to $(-1)^n a^n\left(\frac{bn}{2}+1\right)$ then value of $(a + b)$ is

65. If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$, then k, is equal to

66. Let $f(n) = 10^n + 3\cdot 4^{n+2} + 5$, $n \in N$. The greatest value of the integer which divides $f(n)$ for all n is

67. The sum of series $3\cdot {}^nC_0 - 8\cdot {}^nC_1 + 13\cdot {}^nC_2 - 18v\,{}^nC_3 + ...$ upto $(n+1)$ terms, where $n > 1$, is:

68. Coefficient of sum of odd powers of x in expansion of $(9x^2 + x - 8)^6$ is

69. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q} = 0$, if $p < q$) is maximum when 'm' is

70. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, if the ratio of 7th term from begining to the 7th term from the end is $\frac{1}{6}$ then value of n is

71. If R is remainder when $6^{83} + 8^{83}$ is divided by 49, then $\frac{R}{5} =$

72. The largest real value of x such that $\sum_{k=0}^{4} \frac{3^{4-k}}{(4-k)!}\left(\frac{x^k}{k!}\right) = \frac{32}{3}$ is

73. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3) ......(1 + x^{100})$ is

74. The coefficient of x^8 in the expression $(2 + x)^2(3 + x)^3(4 + x)^4$ must be

CHAPTER

07

PERMUTATIONS AND COMBINATIONS

Single Option Correct Type Questions (01 to 60)

1. The number of numbers from 1000 to 9999 (both inclusive) that do not have all 4 different digits, is:

(1) 4048 (2) 4464
(3) 4518 (4) 4536

2. In a 12 storey house 10 people enter the lift cabin. It is known that they will leave the lift in pre-decided groups of 2, 3 and 5 people at different storeys. The number of ways they can do so if the lift does not stop at the second storey is –

(1) 820 (2) 720
(3) 1430 (4) 640

3. How many nine digit numbers can be formed using the digits 2, 2, 3, 3, 5, 5, 8, 8, 8 so that the odd digits occupy even positions?

(1) 7560 (2) 180
(3) 16 (4) 60

4. In how many ways n books can be arranged in a row so that two specified books are not together ?

(1) $n! - (n-2)!$ (2) $(n-1)!\,(n-2)$
(3) $n! - 2(n-1)$ (4) $(n-2)n!$

5. Out of 16 players of a cricket team, 4 are bowlers and 2 are wicket keepers. A team of 11 players is to be chosen so as to contain at least 3 bowlers and at least 1 wicketkeeper. The number of ways in which the team be selected, is

(1) 2400 (2) 2472
(3) 2500 (4) 960

6. Words are formed by arranging the letters of the word "STRANGE" in all possible manner. Let m be the number of words in which vowels do not come together and 'n' be the number of words in which vowels come together. Then the ratio of m: n is-

(1) 5: 4 (2) 5: 2
(3) 7 : 2 (4) 2 : 5

7. Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4, is:

(1) 22222200 (2) 11111100
(3) 55555500 (4) 20333280

8. Number of ways in which a pack of 52 playing cards be distributed equally among four players so that each may have the Ace, King, Queen and Jack of the same suit, is:

(1) $\frac{36!}{(9!)^4}$ (2) $\frac{36!.4!}{(9!)^4}$
(3) $\frac{36!}{(9!)^4.4!}$ (4) None

9. The number of ways in which the number 94864 can be resolved as a product of two factors is -

(1) 27 (2) 23
(3) 29 (4) 31

10. How many divisors of 21600 are divisible by 10 but not by 15?

(1) 10 (2) 30
(3) 40 (4) None

11. 12 guests at a dinner party are to be seated along a circular table. Supposing that the master and mistress of the house have fixed seats opposite to one another and that there are two specified guests who must always be placed next to one another. The number of ways in which the company can be placed, is :

(1) 20. 10 ! (2) 22. 10 !
(3) 44. 10 ! (4) None

12. The number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated, is :

(1) 4 !. 4 ! (2) $\frac{8\,!}{4\,!}$
(3) 288 (4) None

13. Number of ways in which 2 Indians, 3 Americans, 3 Italians and 4 Frenchmen can be seated on a circle, if the people of the same nationality sit together, is:

(1) $2.\,(4\,!)^2\,(3\,!)^2$ (2) $2.\,(3\,!)^3.\,4\,!$
(3) $2.\,(3\,!)\,(4\,!)^3$ (4) None

14. Number of positive integral solutions of xyz = 21600 is

(1) 1360 (2) 1260
(3) 1460 (4) 1270

15. If chocolates of a particular brand are all identical then the number of ways in which we can choose 6 chocolates out of 8 different brands available in the market, is:.

(1) ${}^{13}C_6$ (2) ${}^{13}C_8$
(3) 8^6 (4) None

16. Number of ways in which 3 persons throw a normal die to have a total score of 11, is

(1) 27 (2) 25
(3) 29 (4) 18

17. The number of triangles that can be formed by 5 points in a line and 3 points on a parallel line is

(1) ${}^8C_3$ (2) ${}^8C_3 - {}^5C_3$
(3) ${}^8C_3 - {}^5C_3 - 1$ (4) None of these

18. The greatest possible number of points of intersection of 8 straight lines and 4 circles is

(1) 32 (2) 64
(3) 76 (4) 104

19. Number of zeros at the end of 45! is -

(1) 10 (2) 4
(3) 5 (4) 6

20. A person writes letters to five friends and addresses on the corresponding envelopes. In how many ways can the letters be placed in the envelopes so that at least four of them are in the wrong envelopes?

(1) 89 (2) 40
(3) 44 (4) 109

21. The digits from 0 to 9 are written on slips of paper and placed in a box. Four of the slips are drawn at random and placed in the order. The number of outcomes possible is

(1) ${}^{10}P_4$ (2) ${}^{10}C_4$
(3) 10^4 (4) 4^{10}

22. How many words can be formed by using all the letters of the word 'MONDAY' if each word starts with a consonant ?

(1) 120 (2) 240
(3) 560 (4) 480

23. The number of natural numbers from 1 to 1000 having none of their digits repeated is

(1) 738 (2) 648
(3) 729 (4) 800

24. The number of words those can be formed by using all letters of the word 'DAUGHTER', if all the vowels must not be together is

(1) 3600 (2) 36000
(3) 40320 (4) 41420

25. A 5 digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 & 5 without repetition, then the total number of ways in which this can be done is -

(1) 36 (2) 256
(3) 108 (4) 216

26. Let P_m stand for ${}^mP_m$. Then the expression $1.\,P_1 + 2.\,P_2 + 3.\,P_3 + ... + n.\,P_n =$

(1) $(n+1)! - 1$ (2) $(n+1)\,! + 1$
(3) $(n+1)\,!$ (4) None

27. The number of 6 digit numbers that end with 21 (eg. 537621), without repetition of digits is

(1) $7.\,{}^{7}C_3$ (2) $7.\,{}^{7}P_3$
(3) $9.\,{}^{7}P_3$ (4) $9.\,{}^{7}C_3$

28. Two variants of a test paper are distributed among 12 students. Number of ways of seating of the students in two rows so that the students sitting side by side do not have identical papers & those sitting in the same column have the same paper is :

(1) $\frac{12!}{6!\,6!}$ (2) $\frac{(12)!}{2^5\,.\,6!}$
(3) $(6!)^2.\,2$ (4) $12! \times 2$

29. If all the letters of the word 'AGAIN' are arranged in all possible ways & put in dictionary order, then the 50^{th} word is

(1) NAAIG (2) NAAGI
(3) NAIGA (4) NAIAG

30. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letter C are separated from one another is:

(1) ${}^{13}C_3.\ \frac{12!}{5!\,3!\,2!}$

(2) $\frac{13\,!}{5\,!\ 3\,!\ 3\,!\ 2\,!}$

(3) $\frac{14\,!}{3\,!\ 3\,!\ 2\,!}$

(4) $11.\ \frac{13\,!}{5!}$

31. There are 3 white, 4 blue and 1 red flowers. All of them are taken out one by one and arranged in a row in the order. The number of different arrangements possible is (flowers of same colours are similar)

(1) 12 (2) 8
(3) 8! (4) 280

32. The number of different possible permutations using all the letters of the word "MISSISSIPPI", if no two $I's$ are together is

(1) 7150 (2) 7350
(3) 7249 (4) ${}^{8}C_4$

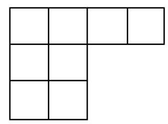
33. In a unique hockey series between India & Pakistan, they decide to play on till a team wins 5 matches. The number of ways in which the series can be won by India, if no match ends in a draw is:

(1) 126 (2) 252
(3) 225 (4) none

34. The number of ways of selecting 11 players from 15 players, if only 6 of these players can bowl and the playing 11 must include atleast 4 bowlers is

(1) 540 (2) 1080
(3) 280 (4) 1170

35. The number of ways in which 5 $X's$ can be placed in the squares of the figure so that no row remains empty is:



(1) 97 (2) 44
(3) 100 (4) 126

36. In a conference 10 speakers are present. If S_1 wants to speak before S_2 & S_2 wants to speak after S_3, then the number of ways all the 10 speakers can give their speeches with the above restriction if the remaining seven speakers have no objection to speak at any number is:

(1) ${}^{10}C_3$ (2) ${}^{10}P_8$
(3) ${}^{10}P_3$ (4) $\frac{10!}{3}$

37. In a cricket match against Pakistan, Azhar wants to bat before Jadeja and Jadeja wants to bat before Ganguli. Number of possible batting orders with the above restrictions, if the remaining eight team members are prepared to bat at any given place, is

(1) $\frac{11\,!}{3\,!}$ (2) ${}^{11}C_3$
(3) $\frac{11\,!}{3}$ (4) None of these

38. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is:

(1) 420 (2) 630
(3) 710 (4) None

39. The number of ways in which 200 different things can be divided into groups of 100 pairs is:

(1) (1. 3. 5...... 199)

(2) $\left(\frac{101}{2}\right)\left(\frac{102}{2}\right)\left(\frac{103}{2}\right)....\left(\frac{200}{2}\right)$

(3) $\frac{200\,!}{2^{100}\,(100)\,!}$

(4) All of these

40. The number of ways in which an insect can move from left bottom corner of a chess board to the right top corner, if it is given that it can move only upside or right, along the lines is-

(1) ${}^{8}C_4$ (2) ${}^{16}C_8$
(3) ${}^{15}C_8$ (4) ${}^{8}C_2$

41. In how many ways 14400 can be resolved into product of two factors?

(1) 16 (2) 32
(3) 64 (4) 128

42. In how many ways the number 10080 can be written as product of two coprime factors?

(1) 2 (2) 4
(3) 8 (4) 16

43. The number of ways in which 5 beads, chosen from 8 different beads be threaded on to a ring, is:

(1) 672 (2) 1344
(3) 336 (4) None

44. The number of ways in which 5 persons can sit at a round table, if two of the persons does not sit together is

(1) 12 (2) 24
(3) 60 (4) 72

45. The number of ways in which four men and three women may sit around a round table if all the women are together is

(1) 144 (2) 720
(3) 120 (4) 24

46. Seven persons including *A, B, C* are seated on a circular table. How many arrangements are possible if *B* is always between *A* and *C*?

(1) 5040 (2) 24
(3) 720 (4) 48

47. In a shooting competition a man can score 0, 2 or 4 points for each shot. Then the number of different ways in which he can score 14 points in 5 shots, is:

(1) 20 (2) 24
(3) 30 (4) none

48. The number of negative integral solutions of equation $x + y + z = -12$ is

(1) 54 (2) 53
(3) 120 (4) None of these

49. The total number of positive integral solutions of $15 < x_1 + x_2 + x_3 \leq 20$ is

(1) 635 (2) 645
(3) 685 (4) None of these

50. The number of ways in which 15 identical apples & 10 identical oranges can be distributed among three persons, each receiving none, one or more is:

(1) 5670 (2) 7200
(3) 8976 (4) None of these

51. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \leq n$ (where n is a positive integer) is

(1) ${}^{n+3}C_3$ (2) ${}^{n+4}C_4$
(3) ${}^{n+5}C_5$ (4) None of these

52. The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12 is:

(1) 8550 (2) 5382
(3) 6062 (4) 8055

53. Number of derangement of all the digits of number 1234567 such that even digits occupy even places and odd digits occupy odd places is

(1) 12 (2) 14

(3) 16 (4) 18

54. **Statement- 1 :** The maximum value of k such that $(50)^k$ divides 100! is 2.

Statement- 2 : If P is any prime number, then power of P in $n!$ is equal to $\left[\frac{n}{P}\right] + \left[\frac{n}{P^2}\right] + \left[\frac{n}{P^3}\right] ....$

where $[\cdot]$ represents greatest integer function.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

55. Consider the number $N = 249480$

	Column-I		Column-II
(A)	Number of ways N is divisible by 3 but not by 5	**(P)**	20
(B)	Number of ways N is divisible by 5 but not by 7	**(Q)**	40
(C)	Number of ways N is divisible by 3 but not by 21	**(R)**	64
(D)	Number of ways N is divisible by 35 but not by 77	**(S)**	60

	A	B	C	D
(1)	R	Q	R	P
(2)	Q	R	P	R
(3)	R	R	P	Q
(4)	P	R	S	Q

56. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1 : The number of different ways the child can buy the six ice-creams, is ${}^{10}C_5$.

Statement-2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging $6A'$s and $4B'$s in a row.

(1) Statement-1 is False, Statement-2 is False

(2) Statement-1 is True, Statement-2 is True

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

57. How many different words can be formed by jumbling the letters in the word "MISSISSIPPI" in which no two S are adjacent ?

(1) $8.\,{}^6C_4.\,{}^7C_4$ (2) $6.\,7\,{}^8C_4$

(3) $6.\,8.\,{}^7C_4$ (4) $7.\,{}^6C_4.\,{}^8C_4$

58. **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

59. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) At least 500 but less than 750

(2) At least 750 but less than 1000

(3) At least 1000

(4) Less than 500

60. The sum of all 4 digit numbers that can be formed by using the digits 2,4,6,8 (repetition of digits not allowed) is

(1) 133320 (2) 533280
(3) 53328 (4) None of these

Integer Type Questions (61 to 75)

61. A bag contains 9 balls marked with digits 1, 2................9. If two balls are drawn from the bag, then number of ways of getting the sum of the digits on balls as odd number is

62. In a football championship, 153 matches were played. Every team played one match with each other. The number of teams participating in the championship is

63. Passengers are to travel by a double decked bus which can accommodate 13 in the upper deck and 7 in the lower deck. The number of ways that they can be divided if 5 refuse to sit in the upper deck and 8 refuse to sit in the lower deck, is

64. Let T_n denotes the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals

65. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is-

66. The number of ways of distributing 8 identical balls in 3 distinct boxes, so that none of the boxes is empty, is -

67. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

68. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :

69. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

70. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is

71. There are 12 points in a plane of which 5 are collinear. The number of distinct quadrilaterals which can be formed with vertices at these points is:

72. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

73. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

74. Number of different squares of any size (side of square be natural no.) which can be made from a rectangle of size 15×8, is

75. A five letter word is to be formed such that the letters appearing in the odd numbered positions are taken from the letters which appear without repetition in the word "MATHEMATICS". Further the letters appearing in the even numbered positions are taken from the letters which appear with repetition in the same word "MATHEMATICS". The number of ways in which the five letter word can be formed is:

CHAPTER 08

SEQUENCE AND SERIES

Single Option Correct Type Questions (01 to 60)

1. If $a_1, a_2, \ldots\ldots, a_n$ are distinct terms of an A.P, then equations satisfied are

(1) $a_1 + 2a_2 + a_3 = 0$

(2) $2a_1 + 2a_2 + a_3 = 0$

(3) $a_1 + 3a_2 - 3a_3 - a_4 = 0$

(4) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

2. If $\log_5 2$, $\log_5(2^x - 5)$ and $\log_5(2^x - 7/2)$ are in A.P., then value of $2x$ is equal to

(1) 6 (2) 9

(3) 3 (4) 1

3. The sum of n terms of the series

$\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots\ldots$ is

(1) $n \log \left(\frac{a}{b}\right)$

(2) $n \log (ab)$

(3) $\frac{n^2}{2} \log \frac{a}{b} + \frac{n}{2} \log (ab)$

(4) $\frac{n^2}{2} \log \frac{a}{b} - \frac{n}{2} \log (ab)$

4. If the ratio of sum of n terms of two A.P.'s is $(3n + 8) : (7n + 15)$, then the ratio of 12^{th} terms is

(1) 16 : 7 (2) 7 : 16

(3) 7 : 12 (4) 12 : 5

5. If a and ℓ be the first and last term of an A.P. and S be the sum of its all terms; then its common difference is

(1) $\frac{\ell^2 + a^2}{2S - \ell - a}$ (2) $\frac{\ell^2 - a^2}{2S - \ell - a}$

(3) $\frac{\ell^2 - a^2}{2S + \ell + a}$ (4) $\frac{\ell^2 - a^2}{2S - \ell + a}$

6. If b_1, b_2, b_3 $(b_i > 0)$ are three successive terms of a G.P. with common ratio r, then value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds is given by

(1) $r \geq 3$ (2) $1 < r < 2$

(3) $r > 3$ (4) $r \in (0,2)$

7. Let 1, 2, 4, 8, ... is a G.P. and 4, 8, 16, 32 is another G.P., then 1 + 4, 2 + 8, 4 + 16, 8 + 32, ... are in

(1) A.P.

(2) G.P.

(3) H.P.

(4) Neither A.P nor GP nor HP

8. If a, b, c are in A.P., then $b + c - a$, $c + a - b$, $a + b - c$ are in

(1) A.P.

(2) G.P.

(3) H.P.

(4) Neither A.P nor GP nor HP

9. If x, y, z are in G.P. then $x^2 + y^2$, $xy + yz$, $y^2 + z^2$ are in

(1) A.P.

(2) G.P.

(3) H.P.

(4) Decreasing order

10. If a, b, c, d are in G.P., then $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in
(1) A.P.
(2) G.P.
(3) H.P.
(4) Neither A.P nor GP nor HP

11. Three positive numbers form a GP. If the middle number is increased by 8, the three numbers form an AP. If the last number is also increased by 64 along with the previous increase in the middle number, the resulting numbers form a GP again. Then
(1) common ratio = 3
(2) first number = $\frac{4}{9}$
(3) common ratio = –5
(4) first number = 5

12. Let S_1, S_2, S_3 ,..........be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm then for which of the following values of n is, the area of S_n less than 1 cm^2?
(1) 7
(2) 8
(3) 6
(4) 5

13. If a, b, c be in H.P., then $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ will be in
(1) A.P.
(2) G.P.
(3) H.P.
(4) Increasing order

14. If $\frac{a+be^y}{a-be^y} = \frac{b+ce^y}{b-ce^y} = \frac{c+de^y}{c-de^y}$, then a, b, c, d are in
(1) A.P.
(2) G.P.
(3) H.P.
(4) Increasing order

15. If $a_1, a_2, a_3, ...a_n$ are in H.P. and $a_1a_2 + a_2a_3 + a_3 a_4 + ...a_{n-1}a_n = ka_1a_n$, then k is equal to
(1) 1 (2) 2
(3) $n + 1$ (4) $n - 1$

16. If in an AP, $t_1 = \log_{10} a$, $t_{n+1} = \log_{10}b$ and $t_{2n+1} = \log_{10}c$ then a, b, c are in
(1) A.P. (2) G.P.
(3) H.P. (4) A.G.P.

17. If $a_1, a_2, a_3, ..., a_{2n+1}$ are in AP then $\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1} + \frac{a_{2n}-a_2}{a_{2n}+a_2} + ...+ \frac{a_{n+2}-a_n}{a_{n+2}+a_n}$ is equal to
(1) $\frac{n(n+1)}{2} \cdot \frac{a_2-a_1}{a_{n+1}}$
(2) $\frac{n(n+1)}{2}$
(3) $(n + 1)(a_2 - a_1)$
(4) $\frac{n(n-1)}{2}\left(\frac{a_2-a_1}{a_{n+1}}\right)$

18. The value of $x + y + z$ is 15 if a, x, y, z, b are in A.P, while the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is $\frac{5}{3}$ if a, x, y, z, b are in H.P., then a and b are
(1) 1, 9 (2) 3, 7
(3) 2, 9 (4) 4, 5

19. If first and $(2n - 1)^{th}$ terms of an A.P., G.P. and H.P. are equal and their nth terms are respectively a, b, c, then
(1) $a = b = c$ (2) $a + c = b$
(3) $ac - b^2 = 0$ (4) $2b = a + c$

20. Let $1^2 + 2^2 + 3^2 + ... + n^2 = g(n)$, then $g(n) - g(n-1)$ is equal to
(1) n^2
(2) $(n - 1)^2$
(3) $n - 1$
(4) n^3

21. If $x_i > 0$, $i = 1, 2, ..., 50$ and $x_1 + x_2 + .. + x_{50} = 50$, then the minimum value of $\frac{1}{x_1} + \frac{1}{x_2} + ...+ \frac{1}{x_{50}}$ equals

(1) 50 (2) $(50)^2$
(3) $(50)^3$ (4) $(50)^4$

22. If $0 < x, y, a, b < 1$, then the sum of the infinite terms of the series $\sqrt{x}\left(\sqrt{a}+\sqrt{x}\right)+\sqrt{x}\left(\sqrt{ab}+\sqrt{xy}\right)+\sqrt{x}\left(b\sqrt{a}+y\sqrt{x}\right)+$ .....is-

(1) $\frac{\sqrt{ax}}{1+\sqrt{b}}+\frac{x}{1+\sqrt{y}}$

(2) $\frac{\sqrt{x}}{1+\sqrt{b}}+\frac{\sqrt{x}}{1+\sqrt{y}}$

(3) $\frac{\sqrt{x}}{1-\sqrt{b}}+\frac{\sqrt{x}}{1-\sqrt{y}}$

(4) $\frac{\sqrt{ax}}{1-\sqrt{b}}+\frac{x}{1-\sqrt{y}}$

23. The sum to n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$...... is equal to

(1) $2^{-n} + n + 1$ (2) $2^{-n} + n - 1$
(3) $2^{n} + n - 1$ (4) $2^{-n} + n - 2$

24. **STATEMENT-1:** 3,6,12 are in G.P., then 9,12,18 are in H.P.

STATEMENT-2: If three consecutive terms of a G.P. are positive and if middle term is added in these terms, then resultant will be in H.P.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

25. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$

STATEMENT -1: The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

STATEMENT-2: The numbers b_1, b_2, b_3, b_4 are in H.P.

(1) Statement-1 and Statement-2 both are True
(2) Statement-1 and Statement-2 both are False
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

26. Let a_1, a_2, a_3,..... be terms of an AP. If $\frac{a_1+a_2+......+a_p}{a_1+a_2+......+a_q}=\frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

(1) $\frac{7}{2}$ (2) $\frac{2}{7}$
(3) $\frac{11}{41}$ (4) $\frac{41}{11}$

27. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

(1) $(1-\sqrt{5})$ (2) $\frac{1}{2}\sqrt{5}$
(3) $\sqrt{5}$ (4) $\frac{1}{2}(\sqrt{5}-1)$

28. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after

(1) 18 months (2) 19 months
(3) 20 months (4) 21 months

29. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is

(1) $\alpha - \beta$

(2) $\frac{\alpha - \beta}{100}$

(3) $\beta - \alpha$

(4) $\frac{\alpha - \beta}{200}$

30. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is

(1) – 150

(2) 150 times its 50^{th} term

(3) 150

(4) zero

31. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is

(1) $\frac{7}{81}(179 - 10^{-20})$

(2) $\frac{7}{9}(99 - 10^{-20})$

(3) $\frac{7}{81}(179 + 10^{-20})$

(4) $\frac{7}{9}(99 + 10^{-20})$

32. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is

(1) $2 - \sqrt{3}$

(2) $2 + \sqrt{3}$

(3) $\sqrt{2} + \sqrt{3}$

(4) $3 + \sqrt{3}$

33. If m is the A. M. of two distinct real numbers l and $n(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals :

(1) $4l^2 mn$ (2) $4 lm^2 n$

(3) $4 lmn^2$ (4) $4 l^2m^2n^2$

34. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is

(1) $\frac{4}{3}$ (2) 1

(3) $\frac{7}{4}$ (4) $\frac{8}{5}$

35. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$, Then

(1) b, c and a are in G.P.

(2) b, c and a are in A.P.

(3) a, b and c are in A.P.

(4) a, b and c are in G.P.

36. If a_1, a_2, a_3, ..., a_n are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + a_3 + ... + a_{n-1} + 2a_n$ is

(1) $n(2c)^{\frac{1}{n}}$ (2) $(n + 1)c^{1/n}$

(3) $2nc^{1/n}$ (4) $(n + 1)(2c)^{1/n}$

37. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. if $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{2\sqrt{3}}$

(3) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (4) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

38. If $\alpha \in \left(0, \frac{\pi}{2}\right)$, then $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to

(1) $2 \tan \alpha$ (2) 1

(3) 2 (4) $\sec^2 \alpha$

39. An infinite G.P. has first term as x and sum upto infinity as 5. Then the range of values of 'x' is

(1) $x \le -10$

(2) $x \ge 10$

(3) $0 < x < 10$

(4) $-10 \le x \le 10$

40. The least value of $\alpha \in R$ for which $4\alpha x^2 + \frac{1}{x} \ge 1$ for all $x > 0$, is

(1) $\frac{1}{64}$ (2) $\frac{1}{32}$

(3) $\frac{1}{27}$ (4) $\frac{1}{25}$

41. The largest term common to the sequences 1, 11, 21, 31, ... to 100 terms and 31, 36, 41, 46, ... to 100 terms is

(1) 381 (2) 471

(3) 281 (4) 521

42. If a, b, c are in AP, then $a^3 + c^3 - 8b^3$ is equal to

(1) $8\,abc$ (2) $-6\,abc$

(3) $2\,abc$ (4) $-4\,abc$

43. If $\log\left(\frac{5c}{a}\right)$, $\log\left(\frac{3b}{5c}\right)$, $\log\left(\frac{a}{3b}\right)$ are in AP, where a, b, c are in GP, then a, b, c are the lengths of sides of

(1) an isosceles triangle

(2) an equilateral triangle

(3) a scalene triangle

(4) none of these

44. If x, $2y$, $3z$ are in AP, where the distinct numbers x, y, z are in GP then common ratio of GP is

(1) 3 (2) $\frac{1}{3}$

(3) 2 (4) $\frac{1}{2}$

45. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, ... where n consecutive terms have the value n, then 1025[th] term is

(1) 2^9 (2) 2^{10}

(3) 2^{11} (4) 2^8

46. If $(1 + x)(1 + x^2)(1 + x^4) \ldots (1 + x^{128}) = \sum_{r=0}^{n} x^r$, then n is equal to

(1) 255 (2) 127

(3) 60 (4) 256

47. The common ratio of a GP having 10[th] term and 1[st] term equal to 1536 and – 3 respectively, is

(1) 2 (2) 1

(3) –2 (4) 2

48. If $\frac{a_2 a_3}{a_1 a_4} = \frac{a_2 + a_3}{a_1 + a_4} = \frac{3(a_2 - a_3)}{a_1 - a_4}$, then a_1, a_2, a_3, a_4 are in

(1) A.P. (2) G.P.

(3) H.P. (4) none of these

49. Let a,b be two positive numbers, where $a > b$ and 4(GM) = 5(HM) for the numbers, then a is equal to-

(1) b (2) $2b$

(3) $4b$ (4) $\frac{1}{4}b$

50. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP then common difference will be-

(1) ± 1 (2) ± 2

(3) ± 3 (4) ± 4

51. If x, y, z, w are non – zero real numbers such that
$(x^2 + y^2 + z^2)(y^2 + z^2 + w^2) \le (xy + yz + zw)^2$
then x, y, z, w are in
(1) A.P. (2) G.P.
(3) H.P. (4) none of these

52. The sum of those integers from 1 to 100 which are not divisible by 3 or 5 is
(1) 2489 (2) 4735
(3) 2317 (4) 2632

53. If a, b & c are in arithmetic progression and a^2, b^2 & c^2 are in harmonic progression, then
(1) $a = b = \frac{c}{2}$
(2) $a, b, -\frac{c}{2}$ are in A.P.
(3) $a, b, -\frac{c}{2}$ are in G.P.
(4) $a, b, -\frac{c}{2}$ are in H.P.

54. a, b, c, d are four different real numbers which are in AP. If $2(a - b) + x(b - c)^2 + (c - a)^3 = 2(a - d) + (b - d)^2 + (c - d)^3$, then
(1) $-8 \le x \le 16$
(2) $x \le -8$
(3) $x \ge 16$
(4) $x \le -8$ or $x \ge 16$

55. The H.M. between two numbers is $\frac{16}{5}$, their A.M. is A and G.M. is G. If $2A + G^2 = 26$, then the numbers are
(1) 6, 8 (2) 4, 8
(3) 2, 8 (4) 1, 8

56. If $x \in R$, the numbers $5^{1+x} + 5^{1-x}$, $a/2$, $25^x + 25^{-x}$ form an A.P. then 'a' must lie in the interval:
(1) [1, 5] (2) [2, 5]
(3) [5, 12] (4) $[12, \infty)$

57. If a, b, c, x are real numbers and equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ has equal roots, then a, b, c are in-
(1) A.P.
(2) G.P.
(3) H.P.
(4) None of these

58. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is
(1) 7 (2) 8
(3) 9 (4) 11

59. The sum of 10 terms of the series $\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \left(x^3+\frac{1}{x^3}\right)^2 + ...$is (where $x \ne \pm 1$)
(1) $\left(\frac{x^{20}-1}{x^2-1}\right)\left(\frac{x^{22}+1}{x^{20}}\right) + 20$
(2) $\left(\frac{x^{18}-1}{x^2-1}\right)\left(\frac{x^{11}+1}{x^9}\right) + 20$
(3) $\left(\frac{x^{18}-1}{x^2-1}\right)\left(\frac{x^{11}-1}{x^9}\right) + 20$
(4) $\left(\frac{x^{20}+1}{x^2-1}\right)\left(\frac{x^{22}-1}{x^{20}}\right) + 20$

60. If $a, a_1, a_2, a_3, ..., a_{2n-1}, b$ are in AP, $a, b_1, b_2, b_3, ...b_{2n-1}, b$ are in GP and $a, c_1, c_2, c_3, ...c_{2n-1}, b$ are in HP, where a, b are positive, then the equation $a_n x^2 - b_n x + c_n = 0$ has its roots
(1) real and unequal
(2) real and equal
(3) imaginary
(4) None of these

Integer Type Questions (61 to 75)

61. If a, b, c are first three terms of a G.P., if H.M. of a and b is 12 and that of b and c is 36, then the value of a is equal to

62. If $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots\ldots=\frac{3\sqrt{\lambda}}{2}$, then the value of λ is

63. If $(1+p)(1+3x+9x^2+27x^3+81x^4+243x^5)=(1-p^6)$ where $p \neq 1$, then the value of $\left|\frac{p}{x}\right|$ is

64. If the roots of the equation $x^3-12x^2+39x-28=0$ are in A.P. and its common difference is d, then the value of $|d|$ is

65. If the average of the numbers $n \sin n°$, where (n = 2, 4, 6, ……, 180), is cot k°, then the value of k is

66. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference –2, then the time taken by him to count all notes is

67. In the quadratic equation $ax^2+bx+c=0$, $a \neq 0$, $\Delta = b^2-4ac$ and $\alpha+\beta$, $\alpha^2+\beta^2$, $\alpha^3+\beta^3$ are in G.P. where α, β are the root of $ax^2+bx+c=0$, then the value of $c.\Delta$ is

68. If $\frac{3+5+7+\ldots\text{upto } n \text{ terms}}{5+8+11+\ldots\text{upto } 10 \text{ terms}}=7$ then value of n is -

69. Let $f(x) = 2x + 1$. If $f(x)$, $f(2x)$, $f(4x)$ are in GP then number of real values of x is

70. Given a G.P. having an even number of terms. If the sum of all the terms be five times the sum of terms occupying odd places, then the common ratio will be -

71. x, y, z are positive then minimum value of $x^{\log y-\log z}+y^{\log z-\log x}+z^{\log x-\log y}$ is

72. The length of three unequal edges of a rectangular solid block are in GP. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. The length of the largest edge is-

73. Solution set for

$$\left(\sqrt{2+\sqrt{2}}\right)^x+\left(\sqrt{2-\sqrt{2}}\right)^x=2\cdot 2^{x/4} \text{ is}$$

74. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ..., 11. If $\frac{a_1^2+a_2^2+\ldots+a_{11}^2}{11}=90$, then the value of $\frac{a_1+a_2+\ldots+a_{11}}{11}$ is equal to

75. If S denotes the sum to infinity and S_n the sum of n terms of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ldots\ldots$ such that $S-S_n<\frac{1}{1000}$, then the least value of n is

CHAPTER

09

STRAIGHT LINES

Single Option Correct Type Questions (01 to 60)

1. The points $A(-4, -1)$, $B(-2, -4)$, $C(4, 0)$ and $D(2, 3)$ are the vertices of

(1) square (2) rectangle

(3) rhombus (4) None of these

2. Line segment joining (5, 0) and ($10\cos\theta$, $10\sin\theta$) is divided by a point P in ratio 2 : 3. If θ varies then locus of P is -

(1) $(x-3)^2 + y^2 = 16$

(2) $(x+3)^2 + y^2 = 16$

(3) $y^2 = 4x$

(4) $y = 5x + 12$

3. Equation of a straight line passing through the origin and making with x – axis an angle twice the size of the angle made by the line $y = (0.2)x$ with the x–axis, is :

(1) $y = (0.4)x$ (2) $y = (5/12)x$

(3) $6y - 5x = 0$ (4) none of these

4. The equation of a line parallel to $2x - 3y = 4$ which makes with the axes a triangle of area 12 units, is-

(1) $3x + 2y = 12$ (2) $2x - 3y = 12$

(3) $2x - 3y = 6$ (4) $3x + 2y = 6$

5. The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$ is :

(1) $5\sqrt{3}$ (2) $4\sqrt{2}$

(3) $3\sqrt{2}$ (4) $2\sqrt{2}$

6. From (1, 4) you travel $5\sqrt{2}$ units by making 135° angle with positive x-axis (anticlockwise) and then 4 units by making 120° angle with positive x-axis (clockwise) to reach Q. The co-ordinates of point Q are

(1) $(+6,\ 9-2\sqrt{3})$ (2) $(-6,\ 9-2\sqrt{3})$

(3) $(-6,\ 9+2\sqrt{3})$ (4) $(+6,\ 9+2\sqrt{3})$

7. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + aby + 1 = 0\ \forall\ a \in R, b > 0$, is:

(1) (2, 4) (2) (0, 2)

(3) [0, 2] (4) $(2, \infty)$

8. Circumcentre of a triangle whose vertex are (0, 0), (4, 0) and (0, 6) is-

(1) $\left(\frac{4}{3}, 2\right)$ (2) (0, 0)

(3) (2, 3) (4) (4, 6)

9. If two vertices joining the hypotenuse of a right angled triangle are (0, 0) and (3, 4), then the length of the median through the vertex having right angle is-

(1) 3 (2) 2

(3) 5/2 (4) 7/2

10. A variable straight line passes through a fixed point (a, b) intersecting the co-ordinates axes at A & B. If 'O' is the origin, then the locus of the centroid of the triangle OAB is :

(1) $bx + ay - 3xy = 0$

(2) $bx + ay - 2xy = 0$

(3) $ax + by - 3xy = 0$

(4) $ax + by - 2xy = 0$

11. The figure formed by the lines $2x + 5y + 4 = 0$, $5x + 2y + 7 = 0$, $2x + 5y + 3 = 0$ and $5x + 2y + 6 = 0$ is

(1) Square (2) Rectangle

(3) Rhombus (4) None of these

12. A light beam emanating from the point $A(3, 10)$ reflects from the straight line $2x + y - 6 = 0$ and then passes through the point $B(4, 3)$. The equation of the reflected beam is :

(1) $3x - y + 1 = 0$ (2) $x + 3y - 13 = 0$

(3) $3x + y - 15 = 0$ (4) $x - 3y + 5 = 0$

13. The equation of a straight line which passes through the point $(-4, 3)$ and is such that the portion of it between the axes is divided by the point in the ratio 5 : 3 internally, is.

(1) $9x - 20y + 96 = 0$

(2) $2x - y + 11 = 0$

(3) $2x + y + 5 = 0$

(4) $3x - 2y + 7 = 0$

14. Two mutually perpendicular straight lines are drawn from the origin forming an isosceles triangle
together with the straight line, $2x + y = a$. Then the area of the triangle is :

(1) $\frac{a^2}{2}$ (2) $\frac{a^2}{3}$

(3) $\frac{a^2}{5}$ (4) None

15. On the portion of the straight line $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co-ordinates:

(1) (2, 3) (2) (3, 2)

(3) (3, 3) (4) none

16. AB is a variable line sliding between the co-ordinate axes in such a way that A lies on X-axis and B lies on Y-axis. If P is a variable point on AB such that $PA = b$, $PB = a$ and $AB = a + b$, then equation of locus of P is

(1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(3) $x^2 + y^2 = a^2 + b^2$

(4) none of these

17. The nearest point on the line $3x + 4y - 1 = 0$ from the origin is

(1) $\left(\frac{7}{25}, \frac{4}{25}\right)$ (2) $\left(\frac{7}{25}, \frac{2}{25}\right)$

(3) $\left(\frac{3}{25}, \frac{4}{25}\right)$ (4) $\left(\frac{1}{25}, \frac{3}{25}\right)$

18. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are $(-3, 1)$ and $(1, 1)$. Then the equations of other side is -

(1) $7x - 4y + 25 = 0$ (2) $4x + 7y = 11$

(3) $7x - 4y - 3 = 0$ (4) All of these

19. The equation of a straight line which passes through the point (2, 1) and makes an angle of $\pi/4$ with the straight line $2x + 3y + 4 = 0$ is

(1) $x + 5y + 3 = 0$ (2) $x - 5y + 1 = 0$

(3) $5x + y - 11 = 0$ (4) $5x - y + 1 = 0$

20. The equations of the perpendicular bisectors of the sides AB and AC of a ΔABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If the point A is $(1, -2)$, then the equation of the line BC is :

(1) $14x + 23y = 40$

(2) $14x - 23y = 40$

(3) $23x + 14y = 40$

(4) $23x - 14y = 40$

21. The point $(a^2, a + 1)$ is a point in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin if :

(1) $a \geq 1$ or $a \leq -3$

(2) $a \in (-3, 0) \cup (1/3, 1)$

(3) $a \in (0, 1)$

(4) none of these

22. The equations of two straight lines which are parallel to $x + 7y + 2 = 0$ and at unit distance from the point $(1, -1)$ are

(1) $x + 7y + 6 \pm 4\sqrt{2} = 0$

(2) $x + 7y + 6 \pm 5\sqrt{2} = 0$

(3) $2x + 7y + 6 \pm 5\sqrt{2} = 0$

(4) $x + y + 6 \pm 5\sqrt{2} = 0$

23. The points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$, are

(1) (3, 1), (–7, 11) (2) (7, 11), (2, 2)

(3) (7, –11), (–3, 7) (4) (1, 3), (–5, 9)

24. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$. The equation of the other line is:

(1) $3x + 3y - 1 = 0$

(2) $x - 3y + 2 = 0$

(3) $5x + 5y - 3 = 0$

(4) none

25. Through the point $P(4, 1)$ a line is drawn to meet the line $3x - y = 0$ at Q where $PQ = \frac{11}{2\sqrt{2}}$. The equation of line is

(1) $x + y = 5, x - 7y + 3 = 0$

(2) $x - y = 5, x - 7y + 3 = 0$

(3) $x + y = 5, x + 7y + 3 = 0$

(4) $x - y = 5, x + 7y + 3 = 0$

26. A triangle ABC with vertices $A(-1, 0)$, $B(-2, 3/4)$ & $C(-3, -7/6)$ has its orthocentre H. Then the orthocentre of triangle BCH will be :

(1) (– 3, – 2) (2) (1, 3)

(3) (– 1, 2) (4) none of these

27. In a triangle ABC, co-ordinates of A are (1, 2) and the equations to the medians through B and C are $x + y = 5$ and $x = 4$ respectively. Then the co-ordinates of B and C will be

(1) (– 2, 7), (4, 3)

(2) (7, –2), (4, 3)

(3) (2, 7), (– 4, 3)

(4) (2, –7), (3, –4)

28. If $a^2 + 9b^2 - 4c^2 = 6ab$, then the family of lines $ax + by + c = 0$ are concurrent at:

(1) $\left(\frac{1}{2}, \frac{3}{2}\right)\left(-\frac{1}{3}, \frac{-3}{2}\right)$

(2) $\left(-\frac{1}{2}, -\frac{3}{2}\right)\left(-\frac{1}{5}, \frac{-3}{2}\right)$

(3) $\left(-\frac{1}{7}, -\frac{3}{2}\right)\left(-\frac{1}{5}, \frac{-3}{2}\right)$

(4) $\left(-\frac{1}{2}, \frac{3}{2}\right)\left(\frac{1}{2}, -\frac{3}{2}\right)$

29. **Statement-1:** The diagonals of the quadrilateral whose sides are $3x + 2y + 1 = 0$, $3x + 2y + 2 = 0$, $2x + 3y + 1 = 0$ and $2x + 3y + 2 = 0$ include an angle $\pi/2$

Statement-2: Diagonals of a parallelogram bisect each other.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

30. **Statement-1:** A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. The absolute minimum value of $OP + OQ$, as L varies, where O is the origin is 18.

Statement-2: A.M. $\geq$ G.M.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

31. Match the column:

	Column-I		Column-II
I	Two vertices of a triangle are (5, –1) and (–2, 3). If orthocentre is the origin, then coordinates of the third vertex are	**P**	(–4, –7)
II	A point on the line $x + y = 4$ which lies at a unit distance from the line $4x + 3y = 10$, is	**Q**	(–7, 11)
III	Orthocentre of the triangle made by the lines $x + y - 1 = 0$, $x - y + 3 = 0$, $2x + y = 7$ is :	**R**	(–1, 2)
IV	If *a*, *b*, *c* are in A.P., then lines $ax + by = c$ are concurrent at :	**S**	(–1, 2)

(1) I-P; II-Q; III-S; IV-S
(2) I-S; II-S; III-Q; IV-P
(3) I-S; II-Q; III-S; IV-P
(4) I-Q; II-S; III-S; IV-R

32. Consider the lines given by
$L_1: x + 3y - 5 = 0$
$L_2: 3x - ky - 1 = 0$
$L_3: 5x + 2y - 12 = 0$
Match the Statements/Expressions in Column I with the Statements / Expressions in Column II

	Column-I		Column-II
I	L_1, L_2, L_3 are concurrent, if	**P**	$k = -9$
II	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	**Q**	$k = -\frac{6}{5}$
III	L_1, L_2, L_3 form a triangle, if	**R**	$k = \frac{5}{6}$
IV	L_1, L_2, L_3 do not form a triangle, if	**S**	$k = 5$

(1) I-S; II-P, Q; III-R; IV-P, Q, S
(2) I-P, Q; II-S; III-R; IV-P, Q, S
(3) I-P, Q S; II-R; III-S; IV-P, Q
(4) I-R; II-S; III-P, Q; IV-P, Q, S

33. A straight line through the point (2,2) intersects the lines $\sqrt{3}\,x + y = 0$ and $\sqrt{3}\,x - y = 0$ at the points *A* and *B*. The equation to the line *AB* so that the triangle *OAB* is equilateral is
(1) $x - 2 = 0$
(2) $y - 2 = 0$
(3) $x + y - 4 = 0$
(4) none of these

34. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of '*c*' is :
(1) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$
(2) $a_1^2 - a_2^2 + b_1^2 - b_2^2$
(3) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$
(4) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

35. Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cos t) and (1, 0), where t is a parameter is :
(1) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$
(2) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
(3) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$
(4) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

36. Let $A(2,-3)$ and $B(-2,1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line:

(1) $2x + 3y = 9$ (2) $2x - 3y = 7$

(3) $3x + 2y = 5$ (4) $3x - 2y = 3$

37. The equation of the straight line passing through the point (4,3) and making intercepts on the co–ordinate axes whose sum is –1, is:

(1) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(2) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(3) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

(4) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

38. The line parallel to the x–axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a,b) \neq (0,0)$ is :

(1) above the x–axis at a distance of (2/3) from it

(2) above the x–axis at a distance of (3/2) from it

(3) below the x–axis at a distance of (2/3) from it

(4) below the x–axis at a distance of (3/2) from it

39. If non–zero numbers a,b,c are in HP, then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is :

(1) $\left(1, -\frac{1}{2}\right)$ (2) $(1, -2)$

(3) $(-1, -2)$ (4) $(-1, 2)$

40. If a vertex of a triangle is (1,1) and the mid–points of two sides through this vertex are (–1,2) and (3,2), then the centroid of the triangle is :

(1) $\left(\frac{1}{3},\frac{7}{3}\right)$ (2) $\left(1, \frac{7}{3}\right)$

(3) $\left(-\frac{1}{3}, \frac{7}{3}\right)$ (4) $\left(-1, \frac{7}{3}\right)$

41. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is

(1) $3x - 4y + 7 = 0$ (2) $4x + 3y = 24$

(3) $3x + 4y = 25$ (4) $x + y = 7$

42. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, x > 0 and $y = 3x$, $x > 0$, then 'a' belongs to :

(1) $(3, \infty)$ (2) $\left(\frac{1}{2}, 3\right)$

(3) $\left(-3, -\frac{1}{2}\right)$ (4) $\left(0, \frac{1}{2}\right)$

43. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of triangle is 1, then the set of values which 'k' can take is given by

(1) {1, 3} (2) {0, 2}

(3) {–1, 3} (4) {–3, – 2}

44. Let $P = (-1, 0)$ $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the $\angle PQR$ is

(1) $x + y = 0$ (2) $x + \frac{\sqrt{3}}{2}y = 0$

(3) $\frac{\sqrt{3}}{2}x + y = 0$ (4) $x + \sqrt{3}\,y = 0$

45. The lines $p(p^2 + 1)\,x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)\,y + 2q = 0$ are perpendicular to a common line in 2-D geometry for:

(1) exactly one value of p

(2) exactly two values of p

(3) more than two values of p

(4) no value of p

46. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any point from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point :

(1) $\left(\frac{5}{4}, 0\right)$ (2) $\left(\frac{5}{2}, 0\right)$

(3) $\left(\frac{5}{3}, 0\right)$ (4) 0, 0

47. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

(1) $\sqrt{17}$ (2) $\frac{17}{\sqrt{15}}$

(3) $\frac{23}{\sqrt{17}}$ (4) $\frac{23}{\sqrt{15}}$

48. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval :

(1) $(0, \infty)$ (2) $[1, \infty)$

(3) $(-1, \infty)$ (4) $(-1, 1]$

49. If $A(2, -3)$ and $B(-2, 1)$ are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is :

(1) $x - y = 1$ (2) $2x + 3y = 1$

(3) $2x + 3y = 3$ (4) $2x - 3y = 1$

50. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at

(–1, –2), then which one of the following is a vertex of this rhombus ?

(1) (–3, –8) (2) $\left(\frac{1}{3},-\frac{8}{3}\right)$

(3) $\left(-\frac{10}{3},-\frac{7}{3}\right)$ (4) (–3, –9)

51. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point :

(1) $\left(2, -\frac{1}{2}\right)$ (2) $\left(1, \frac{3}{4}\right)$

(3) $\left(1, -\frac{3}{4}\right)$ (4) $\left(2, \frac{1}{2}\right)$

52. A straight line through a fixed point (2,3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is

(1) $3x + 2y = xy$ (2) $3x + 2y = 6xy$

(3) $3x + 2y = 6$ (4) $2x + 3y = xy$

53. The centre of circle inscribed in a square formed by lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is

(1) (4, 7) (2) (7, 4)

(3) (9, 4) (4) (4, 9)

54. Let $O(0, 0)$, $P(3, 4)$, $Q(6, 0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are

(1) $\left(\frac{4}{3}, 3\right)$ (2) $\left(3, \frac{2}{3}\right)$

(3) $\left(3, \frac{4}{3}\right)$ (4) $\left(\frac{4}{3}, \frac{2}{3}\right)$

55. A straight line $\boldsymbol{L}$ through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If $\boldsymbol{L}$ also intersects the x-axis, then the equation of $\boldsymbol{L}$ is

(1) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

(2) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(3) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(4) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

56. The equation to the straight line passing through the point $(a \cos^3\theta, a \sin^3\theta)$ and perpendicular to the line $x \sec\theta + y \operatorname{cosec}\theta = a$, is

(1) $x \cos\theta - y \sin\theta = a \cos 2\theta$

(2) $x \cos\theta + y \sin\theta = a \cos 2\theta$

(3) $x \sin\theta + y \cos\theta = a \cos 2\theta$

(4) $x \sin\theta - y \cos\theta = a \cos 2\theta$

57. Four points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) are such that $\sum_{i=1}^{4}(x_i^2 + y_i^2) \le 2\,(x_1x_3 + x_2x_4 + y_1y_2 + y_3y_4)$

Then these points are vertices of

(1) Parallelogram (2) Rectangle

(3) Square (4) Rhombus

58. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines, $3x + 4y = 9$ and $y = mx + 1$ is also an integer is

(1) 2 (2 0

(3) 4 (4) 1

59. If $P\left(1+\frac{t}{\sqrt{2}}, 2+\frac{t}{\sqrt{2}}\right)$ be any point on a line, then the range of values of t for which the point P lies between the parallel lines $x + 2y = 1$ and $2x + 4y = 15$ is

(1) $-\frac{4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6}$

(2) $0 < t < \frac{5\sqrt{2}}{6}$

(3) $-\frac{4\sqrt{2}}{5} < t < 0$

(4) $-\frac{4\sqrt{2}}{3} < t < \frac{\sqrt{2}}{6}$

60. The point $A(4, 1)$ undergoes following transformations successively :

(i) reflection about line $y = x$

(ii) translation through a distance of 3 units in the positive direction of x-axis

(iii) rotation through an angle 105° in anti-clockwise direction about origin O.

Then the final position of point A is

(1) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (2) $\left(-2, 7\sqrt{2}\right)$

(3) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (4) $\left(-2\sqrt{6}, 2\sqrt{2}\right)$

Integer Type Questions (61 to 70)

61. Area of the quadrilateral formed by the lines $|x| + |y| = 2$ is :

62. The area of parallelogram whose two sides are $y = x + 3$, $2x - y + 1 = 0$ and remaining two sides are passing through (0, 0) is (in sq. unit)

63. The value of k so that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0$ may represent a pair of straight lines is

64. The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is

65. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then $|c|$ equals

66. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept – 4. Then the value of $|k|$ is

67. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals

68. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the absolute value of the slope of the line PQ is

69. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is

70. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

CHAPTER

10

CIRCLES

Single Option Correct Type Questions (01 to 60)

1. If the equation $px^2 + (2 - q)\,xy + 3y^2 - 6qx + 30y + 6q = 0$ represents a circle, then the values of p and q are

(1) 2, 2 (2) 3, 1

(3) 3, 2 (4) 3, 4

2. Equation of line passing through midpoint of intercepts made by circle $x^2 + y^2 - 4x - 6y = 0$ on

co-ordinate axes is

(1) $3x + 2y - 12 = 0$ (2) $3x + y - 6 = 0$

(3) $3x + 4y - 12 = 0$ (4) $3x + 2y - 6 = 0$

3. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x–axis in points whose abscissa are roots of the equation:

(1) $x^2 + ax + b = 0$ (2) $x^2 - ax + b = 0$

(3) $x^2 + ax - b = 0$ (4) $x^2 - ax - b = 0$

4. The co-ordinates of point on line $x + y = -13$, nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$

(1) (– 6, – 7) (2) (– 15, 2)

(3) (– 5, – 6) (4) (– 7, – 6)

5. The coordinate of the point on the circle $x^2 + y^2 - 12x - 4y + 30 = 0$, which is farthest from the origin are:

(1) (9, 3) (2) (8, 5)

(3) (12, 4) (4) (10, 5)

6. The equation of the diameter of the circle $(x - 2)^2 + (y + 1)^2 = 16$ which bisects the chord cut off by the circle on the line $x - 2y - 3 = 0$ is

(1) $x + 2y = 0$ (2) $2x + y - 3 = 0$

(3) $3x + 2y - 4 = 0$ (4) $x - y + 1 = 0$

7. The co-ordinates of the middle point of the chord cut off on $2x - 5y + 18 = 0$ by the circle $x^2 + y^2 - 6x + 2y - 54 = 0$ are

(1) (1, 4) (2) (2, 4)

(3) (4, 1) (4) (1, 1)

8. The locus of the midpoint of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:

(1) $x + y = 2$ (2) $x^2 + y^2 = 1$

(3) $x^2 + y^2 = 2$ (4) $x + y = 1$

9. The locus of the centers of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is

(1) $2x - 5y + 11 = 0$ (2) $2x + 5y - 11 = 0$

(3) $2x + 5y + 11 = 0$ (4) None of these

10. If the circle C_1: $x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the coordinates of the centre of C_2 are:

(1) $\left(\pm\frac{9}{5}, \pm\frac{12}{5}\right)$ (2) $\left(\pm\frac{9}{5}, \mp\frac{12}{5}\right)$

(3) $\left(\pm\frac{12}{5}, \pm\frac{9}{5}\right)$ (4) $\left(\pm\frac{12}{5}, \mp\frac{9}{5}\right)$

11. The locus of the centre of the circle which bisects the circumferences of the circles $x^2 + y^2 = 4$ & $x^2 + y^2 - 2x + 6y + 1 = 0$ is:

(1) A straight line (2) A circle

(3) A parabola (4) A hyperbola

12. The circle $x^2+y^2=4$ cuts the circle $x^2+y^2+2x+3y-5=0$ in A & B. Then the equation of the circle on AB as a diameter is:

(1) $13(x^2+y^2)-4x-6y-50=0$

(2) $9(x^2+y^2)+8x-4y+25=0$

(3) $x^2+y^2-5x+2y+72=0$

(4) $x^2+y^2+5x-2y=0$

13. Equation of circle passing through the points (1,1) and (3,3) and whose centre lies on x-axis

(1) $x^2+y^2+8x+6=0$

(2) $x^2+y^2-8x-6=0$

(3) $x^2+y^2-8x+6=0$

(4) $x^2+y^2-8x-8=0$

14. The equation of circle which touches x & y axis and whose perpendicular distance of centre of circle from $3x+4y+11=0$ is 5 is (Given that circle lies in Ist quadrant)

(1) $x^2+y^2+4x+4y+4=0$

(2) $x^2+y^2-4x-4y+4=0$

(3) $x^2+y^2-4x-4y+8=0$

(4) $x^2+y^2-4x-4y-4=0$

15. If $y=2x$ is a chord of the circle $x^2+y^2-10x=0$, then the equation of a circle with this chord as diameter is

(1) $x^2+y^2-2x-4y=0$.

(2) $x^2+y^2-2x+4y=0$.

(3) $x^2+y^2-2x-8y=0$.

(4) $x^2+y^2+2x+4y=0$.

16. Two thin rods AB & CD of lengths $2a$ & $2b$ move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:

(1) $x^2+y^2=a^2+b^2$ (2) $x^2-y^2=a^2-b^2$

(3) $x^2+y^2=a^2-b^2$ (4) $x^2-y^2=a^2+b^2$

17. Equation of circle which pass through the points (1, –2) and (3, – 4) and touch the x-axis is

(1) $x^2+y^2+6x+2y+9=0$

(2) $x^2+y^2+10x+20y+25=0$

(3) $x^2+y^2+6x+4y+9=0$

(4) $x^2+y^2-6x-4y+5=0$

18. If $\left(a,\frac{1}{a}\right),\left(b,\frac{1}{b}\right),\left(c,\frac{1}{c}\right)$ & $\left(d,\frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, $abcd$ is equal to:

(1) 4 (2) 16

(3) 1 (4) 3

19. Two lines through (2, 3) from which the circle $x^2+y^2=25$ intercepts chords of length 8 units have equations

(1) $2x+3y=13,\ x+5y=17$

(2) $y=3,\ 12x+5y=39$

(3) $x=2,\ 9x-11y=51$

(4) $x=5,\ 5x+4y=37$

20. The locus of the mid points of the chords of the circle $x^2+y^2+4x-6y-12=0$ which subtend an angle of $\frac{\pi}{3}$ radians at its circumference is:

(1) $(x-2)^2+(y+3)^2=6.25$

(2) $(x+2)^2+(y-3)^2=6.25$

(3) $(x+2)^2+(y-3)^2=18.75$

(4) $(x+2)^2+(y+3)^2=18.75$

21. From the point A (0, 3) on the circle $x^2+4x+(y-3)^2=0$ a chord AB is drawn & extended to a point M such that $AM=2AB$. The equation of the locus of M is:

(1) $x^2+8x+y^2=0$

(2) $x^2+8x+(y-3)^2=0$

(3) $(x-3)^2+8x+y^2=0$

(4) $x^2+8x+8y^2=0$

22. The circle $x^2+y^2-2x-3ky-2=0$ passes through two fixed points whose coordinates are

(1) $\left(1\pm\sqrt{3},0\right)$

(2) $\left(-1\pm\sqrt{3},0\right)$

(3) $\left(-\sqrt{3}\pm 2,0\right)$

(4) None of thsese

23. **STATEMENT-1:** Number of circles through the three points $A(3, 5)$, $B(4, 6)$, $C(5, 7)$ is 1

STATEMENT-2: Through three non collinear points in a plane, one and only one circle can be drawn.

(1) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(2) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1

(3) STATEMENT-1 is true, STATEMENT-2 is false

(4) STATEMENT-1 is false, STATEMENT-2 is true

24. A rectangle $ABCD$ is inscribed in the circle $x^2 + y^2 + 3x + 12y + 2 = 0$. If the co-ordinates of A and B are $(3, -2)$ and $(-2, 0)$, then the other two vertices of the rectangle are (a, b) and (c, d) then the value of $|a + b + c + d|$ is:

(1) 29 (2) 15

(3) 23 (4) 55

25. The greatest distance of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is-

(1) 10 unit (2) 15 unit

(3) 5 unit (4) 7 unit

26. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

(1) $4 \le x^2 + y^2 \le 64$ (2) $x^2 + y^2 \le 25$

(3) $x^2 + y^2 \ge 25$ (4) $3 \le x^2 + y^2 \le 9$

27. The equation of circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is:

(1) $x^2 + y^2 = a^2$ (2) $x^2 + y^2 = 4a^2$

(3) $x^2 + y^2 = 16a^2$ (4) $x^2 + y^2 = 9a^2$

28. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then-

(1) $2 < r < 8$ (2) $r < 2$

(3) $r = 2$ (4) $r > 2$

29. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is

(1) $(x - p)^2 = 4qy$ (2) $(x - q)^2 = 4py$

(3) $(y - p)^2 = 4qx$ (4) $(y - q)^2 = 4px$

30. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is

(1) $x^2 + y^2 - 2x + 2y - 23 = 0$

(2) $x^2 + y^2 - 2x + 2y + 23 = 0$

(3) $x^2 + y^2 + 2x + 2y - 23 = 0$

(4) $x^2 + y^2 - 2x - 2y - 23 = 0$

31. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is-

(1) $x^2 + y^2 - x - y = 0$

(2) $x^2 + y^2 - x + y = 0$

(3) $x^2 + y^2 + x + y = 0$

(4) $x^2 + y^2 + x - y = 0$

32. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then the line $5x + by - a = 0$ passes through P and Q for -

(1) exactly two values of a

(2) infinitely many values of a

(3) no value of a

(4) exactly one value of a

33. Let C be the circle with centre $(0, 0)$ and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre, is

(1) $x^2 + y^2 = 1$ (2) $x^2 + y^2 = \frac{27}{4}$

(3) $x^2 + y^2 = \frac{9}{4}$ (4) $x^2 + y^2 = \frac{3}{2}$

34. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is :

(1) $x^2 + y^2 + 2x - 2y - 62 = 0$

(2) $x^2 + y^2 - 2x + 2y - 62 = 0$

(3) $x^2 + y^2 - 2x + 2y - 47 = 0$

(4) $x^2 + y^2 + 2x - 2y - 47 = 0$

35. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(1) (3, –4) (2) (–3, 4)
(3) (–3, –4) (4) (3, 4)

36. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for

(1) All except one value of p
(2) All except two values of p
(3) Exactly one value of p
(4) All values of p

37. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to 1:3. Then the circumcentre of the triangle ABC is at the point

(1) (0, 0) (2) $\left(\frac{5}{4}, 0\right)$
(3) $\left(\frac{5}{2}, 0\right)$ (4) $\left(\frac{5}{3}, 0\right)$

38. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $-35 < m < 15$ (2) $15 < m < 65$
(3) $35 < m < 85$ (4) $-85 < m < -35$

39. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if

(1) $2|a| = c$ (2) $|a| = c$
(3) $a = 2c$ (4) $|a| = 2c$

40. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is:

(1) $\frac{10}{3}$ (2) $\frac{3}{5}$
(3) $\frac{6}{5}$ (4) $\frac{5}{3}$

41. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point

(1) (–5, 2) (2) (2, – 5)
(3) (5, – 2) (4) (–2, 5)

42. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally, then the radius of T is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$
(3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

43. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (– 3, 2), then the radius of S is:

(1) $5\sqrt{3}$ (2) 5
(3) 10 (4) $5\sqrt{2}$

44. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is

(1) 3 (2) 2
(3) 3/2 (4) 5

45. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents

(1) Four straight lines, when $c = 0$ and a, b are of the same sign
(2) Two straight lines and a circle, when $a = b$, and c is of sign opposite to that of a
(3) Two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
(4) A circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

46. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

(1) $\left(-\frac{3}{2}, 0\right)$ (2) $\left(-\frac{5}{2}, 2\right)$
(3) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (4) (–4, 0)

47. An acute angle ΔPQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) & (–4,3) respectively then $\angle QPR =$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

48. The equation of a circle which passes through (1, 0) and (0, 1) and has its radius as small as possible is $x^2 + y^2 - gx - fy + c = 0$ where $g, f \in W$ (set of whole numbers) then $g + f + c =$

(1) 0 (2) 1

(3) 2 (4) 3

49. The set of all values of α for which the point $(\alpha-1, \alpha+1)$ lies in the larger segment of the circle $x^2+ y^2 - x - y - 6 = 0$ made by the chord $x + y - 2 = 0$ is

(1) $[-1, 1]$ (2) $(-1, 1)$

(3) $(-1, 0)$ (4) $(0, 1)$

50. The circle $x^2 + y^2 - 6x - 10y + \lambda = 0$ neither touches nor intersect the coordinate axis and the point (1,4) lies inside the circle then maximum integral value of λ can be

(1) 26 (2) 27

(3) 28 (4) 29

51. The number of integral values of λ for which $x^2 + y^2 + \lambda x + (1 - \lambda) y + 5 = 0$ is the equation of a circle whose radius does not exceed 5 are

(1) 16 (2) 29

(3) 28 (4) 27

52. Value of k for which four distinct points $(2k, 3k)$, (1,0), (0,1), (0,0) lies on a circle is

(1) 0 (2) 1

(3) $\frac{5}{13}$ (4) $\frac{13}{7}$

53. If one end of the diameter is (1,2) and other end lies on the line $2x + y = 5$ then the locus of centre of circle is $ax + by - 9 = 0$ then $a + b =$

(1) 5 (2) 6

(3) 7 (4) 3

54. If a line is drawn through a fixed point $P(10,7)$ to cut the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ at A and B then the value of $PA \cdot PB$ is

(1) $5\sqrt{3}$ (2) 75

(3) 49 (4) 7

55. The length of common chord of circles $x^2 + y^2 + 2x + 6y = 0$ & $x^2 + y^2 - 4x - 2y - 6 = 0$ is $\frac{\alpha\sqrt{106}}{\beta}$ where α and β are coprime then

(1) $\alpha - \beta = 3$ (2) $\alpha - \beta = 2$

(3) $\alpha + \beta = 7$ (4) $\alpha + \beta = 5$

56. The equation of the smallest circle passing through the intersection of the line $x + y = 1$ and the circle $x^2 + y^2 = 9$ is

(1) $x^2 + y^2 - x - y - 8 = 0$

(2) $x^2 + y^2 - x - y + 8 = 0$

(3) $x^2 + y^2 + x - y - 8 = 0$

(4) $x^2 + y^2 + x + y + 8 = 0$

57. Equation of chord of circle $x^2 + y^2 - 3x - 4y - 4 = 0$ which passes through the origin such that origin divides it in the ratio 4:1 is $ax + by = 0$ $(a, b \in N)$ then minimum $(a + b) =$

(1) 30 (2) 31 (3) 32 (4) 33

58. $ABCD$ is a square of unit area. A circle is tangent to two sides of $ABCD$ and passes through exactly one of its vertex, the radius of the circle can be

(1) $2-\sqrt{2}$ (2) $\sqrt{2}-1$

(3) $2+\sqrt{3}$ (4) $\frac{1}{\sqrt{2}}$

59. If the chord $y = mx + 1$ of circle $x^2 + y^2 = 1$ subtends an angle 45° at the major segment of the circle then value of m can be

(1) 2 (2) –2

(3) – 1 (4) $\sqrt{2}$

60. If the conics whose equations are $S \equiv x^2\sin^2\theta + 2hxy + y^2\cos^2\theta + 32x + 16y + 19 = 0$ & $S' \equiv x^2\cos^2\theta + 2h'xy + y^2\sin^2\theta + 16x + 32y + 19 = 0$ intersects in four concylic points then

(1) $h + h' = 0$ (2) $h = h'$

(3) $h + h' = 1$ (4) $h + h' = 2$

Integer Type Questions (61 to 70)

61. If common chord of the circle C with centre at (2,1) and radius r and the circle $x^2+y^2-2x-6y+6=0$ is a diameter of 2nd circle then value of r is

62. If l, m, n denote the length of intercepts made by circle $x^2+y^2-8x+10y+16=0$ on x-axis, y-axis and line $y=-x$ respectively, then $\frac{(l^2+10m^2+26n^2)}{4}=$

63. If two distinct chords, drawn from the point (p, q) on the circle $x^2+y^2=px+qy$ (where $pq \neq 0$) are bisected by the x-axis, then $p^2 > Aq^2$, then value of A is:

64. Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2\ CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is

65. If length of common chord of circles $x^2+y^2+2x+3y+1=0$ $x^2+y^2+4x+3y+2=0$ is λ, then value of $[\lambda]$ is
(Where [.] denotes greatest integer function)

66. Complete values of 'k' for which the point (k, 1) lies inside the major segment formed by circle $x^2+y^2-3x+1=0$ & the line $2x-y=2$, is (a, b), then value of ab is equal to

67. If (2, 4) is a point interior to the circle $x^2+y^2-6x-10y+\lambda=0$, and the circle neither touches nor cuts the axes, then the number of integral values of λ is

68. If the tangents are drawn from any point on the line $x+y=3$ to the circle $x^2+y^2=9$. Then all possible chords of contact pass through a fixed point (h, k), then $h+k$ is equal to

69. Equation of circle touching the line $2x-y-1=0$ at (3, 5) & having centre lying on $x+y-5=0$ is $x^2+y^2+2gx+2fy+c=0$, then $\frac{5c}{|g+f|}$ is equal to

70. Let the lines $y+2x=\sqrt{11}+7\sqrt{7}$ and $2y+x=2\sqrt{11}+6\sqrt{7}$ be normal to a circle C: $(x-h)^2+(y-k)^2=r^2$. If the line $\sqrt{11}y-3x=\frac{5\sqrt{77}}{3}+11$ is tangent to the circle C, then the value of $(5h-8k)^2+5r^2$ is equal to

CHAPTER

11

CONIC SECTIONS

Single Option Correct Type Questions (01 to 63)

1. Length of the latus rectum of the parabola 25 $[(x-2)^2+(y-3)^2] = (3x-4y+7)^2$ is:

(1) 4 (2) 2

(3) 1/5 (4) 2/5

2. The points on the parabola $y^2 = 12x$ whose focal distance is 4, are

(1) $\left(2, \sqrt{3}\right), \left(2, -\sqrt{3}\right)$

(2) $\left(1, 2\sqrt{3}\right), \left(1, -2\sqrt{3}\right)$

(3) (1, 2)

(4) None of these

3. A parabola is drawn with its focus at (3, 4) and vertex at the focus of the parabola $y^2 - 12x - 4y + 4 = 0$. The equation of the parabola is:

(1) $x^2 - 6x - 8y + 25 = 0$

(2) $y^2 - 8x - 6y + 25 = 0$

(3) $x^2 - 6x + 8y - 25 = 0$

(4) $x^2 + 6x - 8y - 25 = 0$

4. The length of the side of an equilateral triangle inscribed in the parabola, $y^2 = 4x$ so that one of its angular point is at the vertex is:

(1) $8\sqrt{3}$ (2) $6\sqrt{3}$

(3) $4\sqrt{3}$ (4) $2\sqrt{3}$

5. The ends of latus rectum of parabola $x^2 + 8y = 0$ are

(1) (–4, –2) and (4, 2)

(2) (4, 2) and (–4, 2)

(3) (–4, –2) and (4, –2)

(4) (4, 2) and (4, –2)

6. Vertex of the parabola $9x^2 - 6x + 36y + 9 = 0$ is

(1) $\left(\frac{1}{3}, -\frac{2}{9}\right)$ (2) $\left(-\frac{1}{3}, -\frac{1}{2}\right)$

(3) $\left(-\frac{1}{3}, \frac{1}{2}\right)$ (4) $\left(\frac{1}{3}, \frac{1}{2}\right)$

7. The focus of the parabola is (1, 1) and the tangent at the vertex has the equation $x + y = 1$. Then:

(1) Length of the latus rectum is $2\sqrt{2}$

(2) Equation of the parabola is $(x - y)^2 = 4(x + y - 1)$

(3) The co-ordinates of the vertex are (1/2, 1/2)

(4) All of these

8. Which one of the following equations parametrically represents equation to a parabolic profile?

(1) $x = 3\cos t;\ y = 4\sin t$

(2) $x^2 - 2 = -2\cos t;\ y = 4\cos^2\frac{t}{2}$

(3) $\sqrt{x} = \tan t;\ \sqrt{y} = \sec t$

(4) $x = \sqrt{1-\sin t}\ ;\ y = \sin\frac{t}{2} + \cos\frac{t}{2}$

9. The latus rectum of a parabola whose focal chord is PSQ such that $SP = 3$ and $SQ = 2$ is given by:

(1) 24/5 (2) 12/5

(3) 6/5 (4) None of these

10. A variable chord PQ of the parabola, $y^2 = 4x$ is drawn parallel to the line $y = x$. If the parameters of the points P & Q on the parabola be p & q respectively, then $(p + q)$ equal to.

(1) 1 (2) 1/2

(3) 2 (4) 4

11. The eccentricity of the ellipse $4x^2 + 9y^2 + 8x + 36y + 4 = 0$ is

(1) $\frac{5}{6}$ (2) $\frac{3}{5}$

(3) $\frac{\sqrt{2}}{3}$ (4) $\frac{\sqrt{5}}{3}$

12. If distance between the directrices be thrice the distance between the focii, then eccentricity of ellipse is

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{4}{5}$

13. The eccentricity of the ellipse which meets the straight line $\frac{x}{7} + \frac{y}{2} = 1$ on the axis of x and the straight line $\frac{x}{3} - \frac{y}{5} = 1$ on the axis of y and whose axes lie along the axes of coordinates is

(1) $\frac{\sqrt{6}}{7}$ (2) $\frac{4\sqrt{6}}{7}$

(3) $\frac{2\sqrt{6}}{5}$ (4) $\frac{2\sqrt{6}}{7}$

14. Equation of the ellipse whose foci are (2, 2) and (4, 2) and the major axis is of length 10 is

(1) $\frac{(x+3)^2}{4} + \frac{(y+2)^2}{5} = 1$

(2) $\frac{(x+3)^2}{24} + \frac{(y+2)^2}{25} = 1$

(3) $\frac{(x+3)^2}{25} + \frac{(y+2)^2}{24} = 1$

(4) $\frac{(x-3)^2}{25} + \frac{(y-2)^2}{24} = 1$

15. The length of the axes of the conic $9x^2 + 4y^2 - 6x + 4y + 1 = 0$, are

(1) $\frac{1}{2}, 9$ (2) $3, \frac{2}{5}$

(3) $1, \frac{2}{3}$ (4) 3, 2

16. The equation $\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0$ represents an ellipse, if -

(1) $r > 2$ (2) $2 < r < 5$

(3) $r > 5$ (4) $r \in \{2, 5\}$

17. The eccentricity of an ellipse in which distance between their focii is 10 and that of focus and corresponding directrix is 15 is

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{2}}$

18. If focus and corresponding directrix of an ellipse are (3, 4) and $x + y - 1 = 0$ and eccentricity $\frac{1}{2}$ is then the co-ordinates of extremities of major axis are

(1) (2, 3), (4, 7) (2) (6, 7), (2, 3)

(3) (1, 3), (2, 3) (4) (4, 7), (6, 7)

19. Which of the following pair, may represent the eccentricities of two conjugate hyperbolas, for all $\alpha \in (0, \pi/2)$?

(1) $\sin\alpha, \cos\alpha$

(2) $\tan\alpha, \cot\alpha$

(3) $\sec\alpha, \operatorname{cosec}\alpha$

(4) $1 + \sin\alpha, 1 + \cos\alpha$

20. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the focii, is

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$

(3) $\frac{2}{\sqrt{3}}$ (4) None of these

21. The equation of the hyperbola whose conjugate axis is 5 and the distance between the focii is 13, is

(1) $25x^2 - 144y^2 = 900$

(2) $144x^2 - 25y^2 = 900$

(3) $144x^2 + 25y^2 = 990$

(4) $25x^2 + 144y^2 = 900$

22. The length of the transverse axis of a hyperbola (in standard equation) is 7 and it passes through the point (5, –2). The equation of the hyperbola is

(1) $\frac{49}{4}x^2 - \frac{196}{51}y^2 = 1$

(2) $\frac{4}{49}x^2 - \frac{196}{51}y^2 = 1$

(3) $\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1$

(4) None of these

23. The vertices of a hyperbola are at (0, 0) and (10, 0) and one of its foci is at (13, 0). The equation of the hyperbola is

(1) $\frac{x^2}{25} - \frac{y^2}{144} = 1$

(2) $\frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$

(3) $\frac{x^2}{25} - \frac{(y-5)^2}{144} = 1$

(4) $\frac{(y-5)^2}{25} - \frac{x^2}{144} = 1$

24. An ellipse and a hyperbola have the same centre origin, the same foci and the minor-axis of the one is the same as the conjugate axis of the other. If e_1, e_2 be their eccentricities respectively, then $\frac{1}{e_1^2} + \frac{1}{e_2^2} =$

(1) 1 (2) 2

(3) 4 (4) None of these

25. If e and e' are the eccentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, then the point $\left(\frac{1}{e}, \frac{1}{e'}\right)$ lies on the circle :

(1) $x^2 + y^2 = 1$ (2) $x^2 + y^2 = 2$

(3) $x^2 + y^2 = 3$ (4) $x^2 + y^2 = 4$

26. The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ have in common

(1) Centre only

(2) Centre, foci and directries

(3) Centre, foci and vertices

(4) Centre and vertices only

27. If (5, 12) and (24, 7) are the focii of a conic passing through the origin then the eccentricity of conic can be

(1) $\sqrt{386}/14$ (2) $\sqrt{386}/13$

(3) $\sqrt{386}/25$ (4) $\sqrt{386}/38$

28. If (a sec θ, b tan θ) and (a secϕ, b tan ϕ) are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then tan $\frac{\theta}{2}$ tan $\frac{\phi}{2}$ equals to

(1) $\frac{e-1}{e+1}$ (2) $\frac{e+1}{e-1}$

(3) $\frac{1+e}{1-e}$ (4) None of these

29. The line $y = x$ intersects the hyperbola $\frac{x^2}{9} - \frac{y^2}{25} = 1$ at the points P and Q. The eccentricity of ellipse with PQ as major axis and minor axis of length $\frac{5}{\sqrt{2}}$ is

(1) $\frac{\sqrt{5}}{3}$ (2) $\frac{5}{\sqrt{3}}$

(3) $\frac{5}{9}$ (4) $\frac{2\sqrt{2}}{3}$

30. If latus rectum of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is double ordinate of parabola $y^2 = 4ax$ value of a is

(1) $\frac{64}{25}$ (2) $\frac{25}{64}$

(3) $\frac{64}{50}$ (4) $\frac{64}{75}$

31. The focus of the parabola $x^2 + 2y - 3x + 5 = 0$ is

(1) $\left(\frac{3}{2}, -\frac{15}{8}\right)$ (2) $\left(\frac{3}{2}, \frac{15}{8}\right)$

(3) $\left(-\frac{3}{2}, \frac{15}{8}\right)$ (4) $\left(-\frac{3}{2}, -\frac{15}{8}\right)$

32. Length of the focal chord of the parabola $y^2 = 4ax$ at a distance p from the vertex is:

(1) $\frac{2a^2}{p}$ (2) $\frac{a^3}{p^2}$

(3) $\frac{4a^3}{p^2}$ (4) $\frac{p^2}{a}$

33. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then

(1) $4al + n = 0$

(2) $4al + 4am + n = 0$

(3) $4am + n = 0$

(4) None of these

34. The locus of the midpoint of the focal radii of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

(1) Latus rectum is half the latus rectum of the original parabola

(2) Vertex is $(a/2, 0)$

(3) Directrix is y-axis

(4) All of these

35. If the distance between a focus and corresponding directrix of an ellipse be 8 and the eccentricity be $\frac{1}{2}$, then length of the semi minor axis is

(1) $\frac{8}{\sqrt{3}}$ (2) $\frac{4}{\sqrt{3}}$

(3) $8\sqrt{3}$ (4) $4\sqrt{3}$

36. $\frac{x^2}{r^2 - r - 6} + \frac{y^2}{r^2 - 6r + 5} = 1$ will represents the ellipse, if r lies in the interval

(1) $(-\infty, \infty)$ (2) $(3, \infty)$

(3) $(5, \infty)$ (4) $(1, \infty)$

37. If the midpoint of a chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is $(0, 3)$, then length of the chord is

(1) $\frac{32}{3}$ (2) $\frac{32}{5}$

(3) $\frac{8}{5}$ (4) $\frac{16}{5}$

38. The centre of ellipse $5x^2 + 5y^2 - 2xy + 8x + 8y + 2 = 0$ is

(1) $(-1, 2)$ (2) $(-2, 1)$

(3) $(-2, -2)$ (4) $(-1, -1)$

39. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. The equation of the hyperbola if its eccentricity is 2, is

(1) $3x^2 - y^2 - 4 = 0$

(2) $3x^2 - y^2 - 16 = 0$

(3) $3x^2 - y^2 - 12 = 0$

(4) $-3x^2 + y^2 + 20 = 0$

40. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point of intersection of the lines, $7x + 13y - 87 = 0$ & $5x - 8y + 7 = 0$ & the latus rectum is $32\sqrt{2}/5$. The value of ab is

(1) $10\sqrt{2}$ (2) $5\sqrt{2}$

(3) $10\sqrt{3}$ (4) 200

41. If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2\alpha + y^2 = 25$, then a value of α is

(1) $\pi/2$ (2) $\pi/3$

(3) $\pi/4$ (4) $\pi/6$

42. If hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ passes through the focus of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then eccentricity of hyperbola is.

(1) $\sqrt{3}$ (2) $\sqrt{2}$

(3) $\sqrt{5}$ (4) 2

43. The co-ordinates of the focii of the hyperbola $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{16} = 1$ are

(1) (6, 2) and (–6, 2)
(2) (– 6, 2) and (4, 2)
(3) (6, 2) and (– 4, 2)
(4) (6, – 2) and (–4, 2)

44. A rectangular hyperbola circumscribes a triangle *ABC*, then it will always pass through its

(1) Orthocenter (2) Circumcentre
(3) Centroid (4) Incentre

45. **STATEMENT-1:** Latus rectum of the parabola $y^2 = 8x$ subtends a right angle at the vertex of the parabola.

STATEMENT-2: Every chord of the parabola $y^2 = 4ax$ passing through the point $(4a, 0)$ subtends a right angle at the vertex of the parabola.

(1) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

46. Let *P* be the point (1, 0) and *Q* be a point on the curve $y^2 = 8x$. The locus of midpoint of *PQ* is

(1) $x^2 - 4y + 2 = 0$ (2) $x^2 + 4y + 2 = 0$
(3) $y^2 + 2x + 2 = 0$ (4) $y^2 - 4x + 2 = 0$

47. An ellipse has *OB* as semi minor axis, *F* and *F′* as foci and the angle *FBF′* is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{4}$
(3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

48. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and eccentricity is $\frac{1}{2}$, then the length of the semi major axis is

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$
(3) $\frac{4}{3}$ (4) $\frac{5}{3}$

49. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

(1) $xy = \frac{3}{4}$ (2) $xy = \frac{35}{16}$
(3) $xy = \frac{64}{105}$ (4) $xy = \frac{105}{64}$

50. In an ellipse, the distances between its foci is 6 and minor axis is 8. Then its eccentricity is

(1) $\frac{1}{2}$ (2) $\frac{4}{5}$
(3) $\frac{1}{\sqrt{5}}$ (4) $\frac{3}{5}$

51. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?

(1) Eccentricity
(2) Directrix
(3) Abscissae of vertices
(4) Abscissae of foci

52. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ can be

(1) $3x^2 + 5y^2 - 23 = 0$
(2) $5x^2 + 3y^2 - 48 = 0$
(3) $3x^2 + 5y^2 - 15 = 0$
(4) $5x^2 + 3y^2 - 32 = 0$

53. The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by

(1) $x^2 - 3y^2 = 3$ (2) $3x^2 - y^2 = 3$
(3) $-x^2 + 3y^2 = 3$ (4) $-3x^2 + y^2 = 3$

54. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ as semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is

(1) $4x^2+y^2=4$ (2) $x^2+4y^2=8$

(3) $4x^2+y^2=8$ (4) $x^2+4y^2=16$

55. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is

(1) $x^2+y^2-6y-7=0$

(2) $x^2+y^2-6y+7=0$

(3) $x^2+y^2-6y-5=0$

(4) $x^2+y^2-6y+5=0$

56. Let O be the vertex and Q be any point on the parabola, $x^2=8y$. If the point P divides the line segment OQ internally in the ratio 1:3, then the locus of P is

(1) $x^2=y$ (2) $y^2=x$

(3) $y^2=2x$ (4) $x^2=2y$

57. A parabola has its vertex and focus in the first quadrant and axis along the line $y=x$. If the distances of the vertex and focus from the origin are respectively $\sqrt{2}$ and $2\sqrt{2}$, then an equation of the parabola is

(1) $(x+y)^2=x-y+2$

(2) $(x-y)^2=x+y-2$

(3) $(x-y)^2=8(x+y-2)$

(4) $(x+y)^2=8(x-y+2)$

58. A hyperbola, having the transverse axis of length 2 sin θ, is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is

(1) $x^2\operatorname{cosec}^2\theta-y^2\sec^2\theta=1$

(2) $x^2\sec^2\theta-y^2\operatorname{cosec}^2\theta=1$

(3) $x^2\sin^2\theta-y^2\cos^2\theta=1$

(4) $x^2\cos^2\theta-y^2\sin^2\theta=1$

59. Consider a branch of the hyperbola $x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$

with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(1) $1-\sqrt{\frac{2}{3}}$ (2) $\sqrt{\frac{3}{2}}-1$

(3) $1+\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{3}{2}}+1$

60. Let (x, y) be any point on the parabola $y^2=4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1:3. Then the locus of P is

(1) $x^2=y$ (2) $y^2=2x$

(3) $y^2=x$ (4) $x^2=2y$

61. The ellipse E_1: $\frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(1) $\frac{\sqrt{2}}{2}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $\frac{3}{4}$

62. Let $16x^2-3y^2-32x-12y-44=0$ be a hyperbola, then which of the following does not hold?

(1) Length of the transverse axis is $2\sqrt{3}$

(2) Length of latus rectum is $\frac{32}{\sqrt{3}}$

(3) Eccentricity is $\sqrt{\frac{19}{3}}$

(4) Equation of a directrix is $x=\frac{3}{\sqrt{19}}$

63. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, such that ΔOPQ is equilateral, O being the centre, then the eccentricity e satisfies

(1) $e\in\left(1,\frac{2}{\sqrt{3}}\right)$ (2) $e=\frac{11}{10}$

(3) $e=\frac{\sqrt{3}}{2}$ (4) $e>\frac{2}{\sqrt{3}}$

Integer Type Questions (64 to 74)

64. The focal distance of a point on the parabola $y^2 = 16x$ whose ordinate is twice the abscissa, is

65. The length of chord intercepted on the line $2x + y = 2$ by the parabola $y^2 = 4x$, is

66. The length of the chord of the parabola, $y^2 = 12x$ passing through the vertex & making an angle of 60° with the axis of x is:

67. Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with focii at S and S'. If A be the area of triangle PSS', then the maximum value of A is

68. If e and e' are the eccentricities of the ellipse $5x^2 + 9y^2 = 45$ and the hyperbola $5x^2 - 4y^2 = 45$ respectively then $ee' =$

69. The circle $x^2 + y^2 = 5$ meets the parabola $y^2 = 4x$ at P & Q. Then the length PQ is equal to

70. The equation of the hyperbola with vertices (3, 0) and (–3, 0) and semi-latus rectum 4, is given by is $4x^2 - 3y^2 = 4k$, then $k =$

71. For each point (x, y) on an ellipse, the sum of the distance from (x, y) to the points (2, 0) and (–2, 0) is 8. Then the positive value of α so that $(\alpha, 3)$ lies on the ellipse is

72. If P is a point on the hyperbola $16x^2 - 9y^2 = 144$ whose focii are S_1 and S_2 then $|PS_1 - PS_2|$ is equal to

73. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

74. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then ten times the area of the triangle with vertices at A, M and the origin O is

CHAPTER

12

VECTORS & 3-D GEOMETRY

Single Option Correct Type Questions (01 to 57)

1. The position vector of a point C with respect to B is $\hat{i}+\hat{j}$ and that of B with respect to A is $\hat{i}-\hat{j}$. The position vector of C with respect to A is

(1) $2\hat{i}$ (2) $-2\hat{i}$

(3) $2\hat{j}$ (4) $-2\hat{j}$

2. Points $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$, $3\hat{i}-4\hat{j}-4\hat{k}$ are the vertices of a triangle, then the triangle is

(1) equilateral

(2) isosceles

(3) right angled

(4) obtuse angle triangle

3. If A, B, C, D be any four points and E and F be the middle points of AC and BD respectively, then $\overrightarrow{AB}+\overrightarrow{CB}+\overrightarrow{CD}+\overrightarrow{AD}$ is equal to-

(1) $3\,\overrightarrow{EF}$ (2) $3\,\overrightarrow{FE}$

(3) $4\,\overrightarrow{EF}$ (4) $4\,\overrightarrow{FE}$

4. A, B, C are vertices of a ΔABC having position vectors 3i – j + 2k, $5\hat{i}+2\hat{j}+4\hat{k}$ and $-\hat{i}-\hat{j}+6\hat{k}$ respectively. D is point on side BC such that $\frac{BD}{DC}=\frac{2}{1}$ and E is the midpoint of side AC. If AD and BE intersect at point P, then $PB:PE$ is equal to

(1) 1 : 4 (2) 2 : 3

(3) 3 : 2 (4) 4 : 1

5. If the sum of the squares of the distances of a point from the three coordinate axes be 36, then its distance from the origin is

(1) 6 (2) $3\sqrt{2}$

(3) 2 (4) $6\sqrt{2}$

6. The vertices of a triangle are A (1, 1, 2), B(4, 3, 1) and C(2, 3, 5). A vector representing the internal bisector of the angle A is

(1) $\hat{i}+\hat{j}+2\hat{k}$ (2) $2\hat{i}-2\hat{j}+\hat{k}$

(3) $2\hat{i}+2\hat{j}-\hat{k}$ (4) $2\hat{i}+2\hat{j}+\hat{k}$

7. The locus of a point P which moves such that $PA^2 - PB^2 = 2k^2$ where A and B are (3, 4, 5) and (– 1, 3, – 7) respectively is

(1) $8x + 2y + 24z - 9 + 2k^2 = 0$

(2) $8x + 2y + 24z - 2k^2 = 0$

(3) $8x + 2y + 24z + 9 + 2k^2 = 0$

(4) $8x - 2y + 24z - 2k^2 = 0$

8. If the lengths of the edges of a rectangular parallelopiped are 3, 2, 1 then the angle between a pair of diagonals is given by

(1) $\cos^{-1}\frac{6}{7}$

(2) $\cos^{-1}\frac{3}{7}$

(3) $\cos^{-1}\frac{2}{7}$

(4) All of these

9. Angle between diagonals of a parallelogram whose side are represented by $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} - \hat{k}$ is

(1) $\cos^{-1}\left(\frac{1}{3}\right)$ (2) $\cos^{-1}\left(\frac{1}{2}\right)$

(3) $\cos^{-1}\left(\frac{4}{9}\right)$ (4) $\cos^{-1}\left(\frac{5}{9}\right)$

10. Let $\vec{a} = \hat{i}$ be a vector which makes an angle of 120º with a unit vector $\vec{b}$ in *XY* plane, then the unit vector $(\vec{a} + \vec{b})$ is -

(1) $-\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ (2) $-\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

(3) $\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ (4) $\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$

11. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is equal to:

(1) $2\sqrt{5}$ (2) $2\sqrt{2}$

(3) $10\sqrt{5}$ (4) $5\sqrt{2}$

12. The vertices of a triangle *ABC* are *A* (1, –2, 2), *B* (1, 4, 0) and *C* (–4, 1, 1) respectively. If *M* be the foot of perpendicular drawn from *B* on *AC*, then $\overrightarrow{BM}$ equals

(1) $\frac{10}{7}(-2\hat{i} - 3\hat{j} + \hat{k})$

(2) $10(-2\hat{i} - 3\hat{j} + \hat{k})$

(3) $2\hat{i} + 3\hat{j} - \hat{k}$

(4) $-2\hat{i} - 3\hat{j} + \hat{k}$

13. If the vector product of a constant vector $\overrightarrow{OA}$ with a variable vector $\overrightarrow{OB}$ in a fixed plane *OAB* be a constant vector, then locus of *B* is:

(1) a straight line perpendicular to $\overrightarrow{OA}$

(2) a circle with centre *O* radius equal to $|\overrightarrow{OA}|$

(3) a straight line parallel to $\overrightarrow{OA}$

(4) a circle with centre *O* radius equal to $|\overrightarrow{AB}|$

14. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then the vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are:

(1) null vectors

(2) linearly independent

(3) perpendicular

(4) parallel

15. Coordinate of a point on the line $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-2}$ which is at a distance of 6 unit from the point (2, –1, 3) is

(1) (–4, –3, 1) (2) (4, 3, 1)

(3) (0, –5, 7) (4) (0, 5, –7)

16. The length of perpendicular from (2, –1, 5) to the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$ and the coordinates of the foot are -

(1) $\sqrt{14}$, (1,2,–3) (2) $\sqrt{14}$, (1,–2,3)

(3) $\sqrt{14}$, (1,2,3) (4) $\sqrt[3]{14}$ (1,2,3)

17. Equation of the acute angle bisector of the angle between the lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$ & $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{-1}$ is

(1) $\frac{x-1}{2} = \frac{y-2}{2}$; $z - 3 = 0$

(2) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$

(3) $x - 1 = 0$; $\frac{y-2}{1} = \frac{z-3}{1}$

(4) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-3}{3}$

18. Consider the lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ the equation of the line which

(1) bisects the angle between the lines is $\frac{x}{3}=\frac{y}{3}=\frac{z}{8}$

(2) bisects the angle between the lines is $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(3) passes through origin and is perpendicular to the given lines is $x = y = -z$

(4) passes through origin and is parallel to the given lines is $x = y = -z$

19. If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, $\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent then

(1) $h = -2, k = -6$ (2) $h = \frac{1}{2}, k = 2$

(3) $h = 6, k = 2$ (4) $h = 2, k = \frac{1}{2}$

20. A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system $\vec{a}$, has components $p + 1$ and 1, then

(1) $p = 0$

(2) $p = 1$ or $p = -\frac{1}{3}$

(3) $p = -1$ or $p = \frac{1}{3}$

(4) $p = 1$ or $p = -1$

21. The cosine of the angle between any two diagonals of a cube is -

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$

22. If the unit vectors $\vec{e}_1$ and $\vec{e}_2$ are inclined at an angle 2θ and $|\vec{e}_1-\vec{e}_2| < 1$, then for $\theta \in [0, \pi]$, θ may lie in the interval :

(1) $\left[0, \frac{\pi}{6}\right)$ (2) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

(3) $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ (4) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

23. $(\vec{r}\,.\,\hat{i})(\hat{i}\times\vec{r})+(\vec{r}\,.\,\hat{j})(\hat{j}\times\vec{r})+(\vec{r}\,.\,\hat{k})(\hat{k}\times\vec{r})$ is equal to

(1) $\vec{0}$ (2) $\vec{r}$

(3) $2\vec{r}$ (4) $3\vec{r}$

24. If a line has a vector equation $\vec{r} = 2\hat{i} + 6\hat{j} + \lambda\left(\hat{i} - 3\hat{j}\right)$, then which of the following statements does not hold good?

(1) the line is parallel to $2\hat{i} + 6\hat{j}$

(2) the line passes through the point $3\hat{i} + 3\hat{j}$

(3) the line passes through the point $\hat{i} + 9\hat{j}$

(4) the line is parallel to *XY*-plane

25. A line passes through a point A with position vector $3\hat{i} + \hat{j} - \hat{k}$ and is parallel to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. If P is a point on this line such that $AP = 15$ units, then the position vector of the point P is/are

(1) $13\hat{i} + 4\hat{j} - 9\hat{k}$

(2) $13\hat{i}+4\hat{j}+9\hat{k}$

(3) $7\hat{i} - 6\hat{j} + 11\hat{k}$

(4) $-7\hat{i} + 6\hat{j} - 11\hat{k}$

26. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is :

(1) $-\hat{i} + \hat{j} + 2\hat{k}$ (2) $3\hat{i} - \hat{j} + \hat{k}$

(3) $3\hat{i} + \hat{j} - \hat{k}$ (4) $\hat{i} - \hat{j} - \hat{k}$

27. The perpendicular distance of an angular point of a cube from diagonal which does not pass that angular point is (where a is length of side of cube)

(1) $\sqrt{2}\, a$ (2) $\frac{1}{\sqrt{2}} a$

(3) $\frac{\sqrt{3}}{2} a$ (4) $\sqrt{\frac{2}{3}}\, a$

28. If $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$ are two lines, then the equation of acute angle bisector of two lines is

(1) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + t(\hat{j} - \hat{k})$

(2) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} - 3\hat{k})$

(3) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) - t(\hat{j} + \hat{k})$

(4) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) - t(\hat{j} - \hat{k})$

29. Points X and Y are taken on the sides QR and RS, respectively of a parallelogram $PQRS$, so that $QX = 4XR$ and $RY = 4YS$. The line XY cuts the line PR at Z. The ratio $PZ : ZR$ is

(1) 4 : 21 (2) 3 : 4 (3) 21 : 4 (4) 4 : 3

30. **Statement-1** : If I is incentre of $\Delta\, ABC$ then $|\overrightarrow{BC}|\overrightarrow{IA} + |\overrightarrow{CA}|\overrightarrow{IB} + |\overrightarrow{AB}|\overrightarrow{IC} = \vec{0}$

Statement-2 : In a triangle, if position vector of vertices are $\vec{a}, \vec{b}, \vec{c}$, then position vector of centroid is $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$

(1) Statement-1 is False, Statement-2 is False
(2) Statement-1 is True, Statement-2 is True
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

31. **Statement 1:** If α, β, γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Statement 2 : If l, m, n are the direction cosines of a line then $l^2 + m^2 + n^2 = 1$.

(1) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

32. **Statement 1 :** The shortest distance between the skew lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$ is 9

Statement 2 : Two lines are skew lines if there exists no plane passing through them.

(1) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

33. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{AB}$, $\overrightarrow{AC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overrightarrow{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is-

(1) $\frac{(AB)(AC)}{AB + AC}$ (2) $\frac{1}{AB} + \frac{1}{AC}$

(3) $\frac{1}{AD}$ (4) $\frac{AB^2 AC^2}{(AB)^2 + (AC)^2}$

34. The value of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are-

(1) – 2 and – 1 (2) – 2 and 1
(3) 2 and – 1 (4) 2 and 1

35. The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ are perpendicular to each other, if-

(1) $aa' + cc' = 1$ (2) $\frac{a}{a'} + \frac{c}{c'} = -1$
(3) $\frac{a}{a'} + \frac{c}{c'} = 1$ (4) $aa' + cc' = -1$

36. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for-

(1) exactly two values of θ
(2) more than two values of θ
(3) no value of θ
(4) exactly one value of θ

37. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis & y-axis then the angle that the line makes with the positive direction of the z-axis is-

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$
(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

38. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then, the angle between $\vec{a}$ and $\vec{c}$ is-

(1) 0 (2) $\frac{\pi}{4}$
(3) $\frac{\pi}{2}$ (4) π

39. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then,

(1) $a = 2, b = 8$ (2) $a = 4, b = 6$
(3) $a = 6, b = 4$ (4) $a = 8, b = 2$

40. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(1) – 5 (2) 5
(3) 2 (4) – 2

41. The projections of a vector on the three coordinate axes are 6, –3, 2 respectively. The direction cosines of the vector are

(1) 6, –3, 2 (2) $\frac{6}{5}, -\frac{3}{5}, \frac{2}{5}$
(3) $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$ (4) $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

42. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is

(1) $2\hat{i} - \hat{j} + 2\hat{k}$ (2) $\hat{i} - \hat{j} - 2\hat{k}$
(3) $\hat{i} + \hat{j} - 2\hat{k}$ (4) $-\hat{i} + \hat{j} - 2\hat{k}$

43. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ) =

(1) (2, – 3) (2) (–2, 3)
(3) (3, – 2) (4) (–3, 2)

44. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equal

(1) 45° (2) 60°
(3) 75° (4) 30°

45. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector $\vec{d}$ is equal to

(1) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{d}}\right)\vec{c}$ (2) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

(3) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (4) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

46. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is :

(1) $\vec{a}$ (2) $\vec{c}$

(3) $\vec{0}$ (4) $\vec{a}+\vec{c}$

47. **Statement-1 :** The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement-2: The line: $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(1) Statement-1 is False, Statement-2 is False

(2) Statement-1 is True, Statement-2 is True

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

48. The length of the perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is:

(1) $\sqrt{29}$ (2) $\sqrt{33}$

(3) $\sqrt{53}$ (4) $\sqrt{66}$

49. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to :

(1) – 1 (2) $\frac{2}{9}$

(3) $\frac{9}{2}$ (4) 0

50. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

51. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB}=\vec{q}$, $\overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by

(1) $\vec{r}=3\vec{q}-\frac{3(\vec{p}\,.\,\vec{q})}{(\vec{p}\,.\,\vec{p})}\vec{p}$

(2) $\vec{r}=-\vec{q}+\left(\frac{\vec{p}\,.\,\vec{q}}{\vec{p}\,.\,\vec{p}}\right)\vec{p}$

(3) $\vec{r}=\vec{q}-\left(\frac{\vec{p}\,.\,\vec{q}}{\vec{p}\,.\,\vec{p}}\right)\vec{p}$

(4) $\vec{r}=-3\vec{q}+\frac{3(\vec{p}\,.\,\vec{q})}{(\vec{p}\,.\,\vec{p})}\vec{p}$

52. If the vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

(1) $\sqrt{18}$ (2) $\sqrt{72}$

(3) $\sqrt{33}$ (4) $\sqrt{45}$

53. The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2=m^2+n^2$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

54. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement-1: $\overrightarrow{PQ} \times \left(\overrightarrow{RS}+\overrightarrow{ST}\right) \neq \overrightarrow{0}$

because

Statement-2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \overrightarrow{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \overrightarrow{0}$

(1) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

55. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$. Which one of the following is correct?

(1) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} = \vec{0}$
(2) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} \neq \vec{0}$
(3) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{a}\times\vec{c} \neq \vec{0}$
(4) $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$ are mutually perpendicular

56. From a point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x$, $z = 1$ and $y = -x$, $z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value of λ is

(1) $\sqrt{2}$ (2) 1
(3) –1 (4) $-\sqrt{2}$

57. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$\overrightarrow{OP}.\overrightarrow{OQ} + \overrightarrow{OR}.\overrightarrow{OS} = \overrightarrow{OR}.\overrightarrow{OP} + \overrightarrow{OQ}.\overrightarrow{OS} = \overrightarrow{OQ}.\overrightarrow{OR} + \overrightarrow{OP}.\overrightarrow{OS}$

Then the triangle PQR has S as its
(1) centroid
(2) orthocentre
(3) incentre
(4) circumcenter

Integer Type Questions (58 to65)

58. Let $\vec{p}$ is the position vector of the orthocentre and $\vec{g}$ is the position vector of the centroid of the triangle ABC, where circumcentre is the origin. If $\vec{p} = k\vec{g}$, then k is equal to

59. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ and $\vec{a} + P\vec{b}$ is normal to $\vec{c}$, then P is equal

60. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$. If $|\vec{u}|=3, |\vec{v}|=4, |\vec{w}|=5$ then $\sqrt{|\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}|}$ is

61. Given two vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, $\vec{b} = -2\hat{i} + 2\hat{j} - \hat{k}$ and $\lambda = \dfrac{\text{the projection of } \vec{a} \text{ on } \vec{b}}{\text{the projection of } \vec{b} \text{ on } \vec{a}}$, then the value of 3λ is

62. Let $\vec{a}, \vec{b}, \vec{c}$ be three non zero vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ then $\lambda(\vec{b}\times\vec{a})+\vec{b}\times\vec{c}+\vec{c}\times\vec{a} = \vec{0}$, where λ is equal to

63. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then the value of $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$ is equal to

64. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30º. Then $\vec{a}.\vec{c}$ is equal to

65. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, The value of $|(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})|$ is

CHAPTER

13

LIMITS

Single Option Correct Type Questions (01 to 65)

1. The value of $\ell\text{im}_{x\to 2}\left\{\frac{x}{2}\right\}$, where {.} represents fractional part function, is

(1) 0
(2) 1
(3) –1
(4) Limit does not exists

2. The value of $\ell\text{im}_{x\to\pi}$ sgn [tan x], where [.] represents greatest integer function, is

(1) 0
(2) 1
(3) –1
(4) Limit does not exists

3. $\ell\text{im}_{x\to 1}\,(1-x+[x-1]+[1-x])$ is equal to (where [ ] denotes greatest integer function)

(1) 0 (2) 1
(3) – 1 (4) does not exist

4. Let $f(x)=\frac{|x+\pi|}{\sin x}$, then $\ell\text{im}_{x\to-\pi} f(x)=$

(1) – 1 (2) 1
(3) does not exist (4) 0

5. $\ell\text{im}_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ equal to

(1) $\frac{2}{3}$ (2) $\frac{2}{\sqrt{3}}$
(3) 2 (4) $\frac{2}{3\sqrt{3}}$

6. $\ell\text{im}_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$ is equal to

(1) $\sqrt{2}$ (2) $\frac{\sqrt{2}}{8}$
(3) 0 (4) $\frac{1}{\sqrt{2}}$

7. The value of $\ell\text{im}_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$ $(a > b)$ equals

(1) $\frac{1}{4a}$ (2) $\frac{1}{a\sqrt{a-b}}$
(3) $\frac{1}{2a\sqrt{a-b}}$ (4) $\frac{1}{4a\sqrt{a-b}}$

8. $\ell\text{im}_{x\to 0}\frac{\sqrt{x}\tan x}{(e^x-1)^{3/2}}$ equals

(1) 0 (2) 1
(3) 1/2 (4) 2

9. $\ell\text{im}_{x\to 0^+}\frac{(4^x-1)^3}{\sin\left(\frac{x}{p}\right)\ell\text{n}\left(1+\frac{x^2}{3}\right)}$ is equal to

(1) $9\,p\,(\ell\text{n}\,4)$ (2) $3\,p\,(\ell\text{n}4)^3$
(3) $12\,p\,(\ell\text{n}\,4)^3$ (4) $27\,p\,(\ell\text{n}\,4)^2$

10. $\ell\text{im}_{x\to 0}\frac{x(1-\sqrt{1-x^2})}{\sqrt{1+x^2}\,(\sin^{-1}x)^3}$ equals

(1) 0 (2) 1
(3) 1/2 (4) 1/4

11. $\lim_{x\to 0}\dfrac{e^{\alpha x}-e^{\beta x}}{\sin\alpha x-\sin\beta x}$ equals

(1) 0 (2) $\alpha-\beta$
(3) –1 (4) 1

12. The value of $\lim_{x\to 0}\dfrac{\sec 4x-\sec 2x}{\sec 3x-\sec x}$ is

(1) 1 (2) 0
(3) 3/2 (4) ∞

13. $\lim_{x\to 2}\dfrac{(x+6)^{1/3}-2}{2-x}=$

(1) $\dfrac{1}{12}$ (2) $-\dfrac{1}{12}$
(3) $\dfrac{1}{2}$ (4) Does not exist

14. The value of $\lim_{x\to\infty}\dfrac{\sqrt{3x^2-1}-\sqrt{2x^2-1}}{4x+3}$ is

(1) $\dfrac{1}{4}\left(\sqrt{3}-\sqrt{2}\right)$ (2) $\dfrac{1}{4}\left(\sqrt{3}+\sqrt{2}\right)$
(3) $\left(\sqrt{3}-\sqrt{2}\right)$ (4) $\dfrac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$

15. The value of

$\lim_{n\to\infty}\left(\dfrac{1}{1-n^4}+\dfrac{8}{1-n^4}+.....+\dfrac{n^3}{1-n^4}\right)$ is

(1) 1 (2) 0
(3) $-\dfrac{1}{4}$ (4) $-\dfrac{1}{2}$

16. $\lim_{n\to\infty}\dfrac{1+5+5^2+.......+5^{n-1}}{1-25^n}=$

(1) 0 (2) –1
(3) 1 (4) ∞

17. If $n\in N$, then $\lim_{n\to\infty}\dfrac{(n+2)!+(n+3)!}{(n+4)!}=$

(1) 0 (2) ∞
(3) 1 (4) $\dfrac{1}{2}$

18. $\lim_{n\to\infty}\left(\sqrt{(x+a)(x+b)}-x\right)=$

(1) $\dfrac{a-b}{2}$ (2) $\dfrac{a-b}{3}$
(3) $\dfrac{a+b}{2}$ (4) $\dfrac{a+b}{3}$

19. $\lim_{x\to\infty}\left(\dfrac{(x+1)^{10}+(x+2)^{10}+.....+(x+100)^{10}}{x^{10}+10^{10}}\right)$

(1) 10^2 (2) 10^3
(3) ∞ (4) 10^4

20. $\lim_{x\to 0}\dfrac{1+\sin x-\cos x+\log_e(1-x)}{x^3}=$

(1) 1/2 (2) –1/2
(3) 0 (4) 1

21. $\lim_{x\to 1}\dfrac{1+\log_e x-x}{1-2x+x^2}=$

(1) 1 (2) – 1
(3) –1/2 (4) 1/2

22. The value of $\lim_{h\to 0}\left(\dfrac{1}{h(8+h)^{1/3}}-\dfrac{1}{2h}\right)$ is

(1) 1/12 (2) –4/3
(3) –16/3 (4) –1/48

23. If $\lim_{x\to 0}\dfrac{axe^x-b\,\ell n(1+x)+cxe^{-x}}{x^2\sin x}=2$, then $a+b+c=$

(1) 12 (2) 24
(3) 36 (4) –12

24. The value of $\lim_{x\to 1}\sec\dfrac{\pi}{2x}\log_e x$ is

(1) $\pi/2$ (2) $2/\pi$
(3) $-\pi/2$ (4) $-2/\pi$

25. $\lim_{x\to a}\dfrac{a^x-x^a}{x^x-a^a}=-1$, then a equals -

(1) 0 (2) 1
(3) e (4) – 1

26. $\lim_{x\to 0}\dfrac{(a+x)^2\sin(a+x)-a^2\sin a}{x}=$

(1) $a^2\cos a+2a\sin a$

(2) $a(\cos a + 2\sin a)$
(3) $a^2(\cos a + 2\sin a)$
(4) $a^2\cos a - 2a\sin a$

27. The value of $\lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{5/x} =$

(1) e^5 (2) e^2
(3) e (4) e^3

28. If α and β be the roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha}(1+ax^2+bx+c)^{\frac{1}{x-\alpha}}$ is equal to

(1) $a(\alpha-\beta)$ (2) $\ln|a(\alpha-\beta)|$
(3) $e^{a(\alpha-\beta)}$ (4) $e^{a|\alpha-\beta|}$

29. $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x =$

(1) 1 (2) 2
(3) e^2 (4) e

30. The value of $\lim_{x\to 0^+}(\operatorname{cosec} x)^{1/\ell n x}$ is

(1) 1 (2) –1
(3) e (4) $1/e$

31. The value of $\lim_{n\to\infty}\frac{[1\cdot 2x]+[2\cdot 3x]+.....+[n\cdot(n+1)x]}{n^3}$ (where [.] denotes the greatest integer function) is

(1) x (2) $\frac{x}{2}$
(3) $2x$ (4) $\frac{x}{3}$

32. $\lim_{x\to 0}\frac{\sin(6x^2)}{\ell n\cos(2x^2-x)}$ is equal to

(1) 12 (2) –12
(3) 6 (4) –6

33. $\lim_{x\to 0^+}\frac{\cos^{-1}(1-x)}{\sqrt{x}}$ is equal to

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$
(3) 1 (4) 0

34. $\lim_{n\to\infty} n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{\pi}{4n}\right)$ has the value equal to:

(1) π/3 (2) π/4
(3) π/6 (4) π/2

35. $\lim_{x\to 1}\frac{\left(\sum_{k=1}^{100}x^k\right)-100}{x-1}$ is equal to

(1) 0 (2) 5050
(3) 4550 (4) –5050

36. If $f(x) = \begin{cases} x \text{ when } x\in Q \\ -x \text{ when } x\notin Q\end{cases}$, then $\lim_{x\to 0} f(x)$ equals-

(1) 0 (2) 1
(3) –1 (4) Does not exist

37. $\lim_{x\to\pi/2}\left[\frac{x-\frac{\pi}{2}}{\cos x}\right]$ is equal to (where [.] represents greatest integer function)

(1) –1 (2) 0
(3) –2 (4) Does not exist

38. $\lim_{x\to a^-}\left(\frac{|x|^3}{a}-\left[\frac{x}{a}\right]^3\right)$ $(a < 0)$, where $[x]$ denotes the greatest integer less than or equal to x is

(1) $a^2 + 1$ (2) $-a^2 - 1$
(3) a^2 (4) $-a^2$

39. If $\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}\,(bx-\sin x)} = 1$, then the constants '$a$' and '$b$' are (where $a > 0$)

(1) $b = 1, a = 36$ (2) $a = 1, b = 6$
(3) $a = 1, b = 36$ (4) $b = 1, a = 6$

40. If $\lim\limits_{x\to 0}\left[1+x\ln(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

(1) $\pm\frac{\pi}{4}$ (2) $\pm\frac{\pi}{3}$

(3) $\pm\frac{\pi}{6}$ (4) $\pm\frac{\pi}{2}$

41. If $\ell = \lim\limits_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}$

$= \lim\limits_{x\to 0}\frac{1+a\cos x}{x^2} - \lim\limits_{x\to 0}\frac{b\sin x}{x^3}$, then

(1) $(a, b) = (-1, 0)$

(2) a & b are any real numbers

(3) $(a, b) = (1, 0)$

(4) $(a, b) = (0, 1)$

42. Let $f(x) = \frac{x^2 - 9x + 20}{x - [x]}$ (where [.] represents greatest integer function), then $\lim\limits_{x\to 5} f(x) =$

(1) 0 (2) 1

(3) 2 (4) Does not exist

43. $\lim\limits_{x\to 1/\sqrt{2}}\frac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)} =$

(1) $\frac{1}{\sqrt{2}}$ (2) $-\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

44. The value of $\lim\limits_{x\to\infty}(x+e^x)^{2/x}$ is-

(1) 1 (2) 2

(3) e (4) e^2

45. $\lim\limits_{x\to\infty}\frac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}}-\left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}$, $n \in N$ is equal to

(1) 0 (2) $\ell n\frac{2}{3}$

(3) $\ell n\frac{3}{2}$ (4) $\ell n\frac{1}{2}$

46. The value of $\lim\limits_{x\to\infty}\frac{x\ell n\left(1+\frac{\ell n\ x}{x}\right)}{\ell nx}$ is

(1) 1

(2) 0

(3) –1

(4) limit does not exist

47. $\lim\limits_{x\to 0}\left(\frac{1^x+2^x+3^x+.......+n^x}{n}\right)^{1/x} =$

(1) $(n!)^n$ (2) $(n!)^{1/n}$

(3) $n!$ (4) $\ln(n!)$

48. $\lim\limits_{x\to 1}(\log_5 5x)^{\log_x 5} =$

(1) 1 (2) e

(3) –1 (4) Does not exist

49. $\lim\limits_{\theta\to 0}\left(\left[\frac{n\sin\theta}{\theta}\right]+\left[\frac{n\tan\theta}{\theta}\right]\right)$, where [ ] is the greatest integer function and $n \in N$, is

(1) 2n (2) $2n + 1$

(3) $2n - 1$ (4) Does not exist

50. **STATEMENT-1 :** $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{\frac{1}{x}} = e$

STATEMENT-2:

$\lim\limits_{x\to a}(1+f(x))^{g(x)} = e^{\lim\limits_{x\to a} f(x).g(x)}$, if $\lim\limits_{x\to a} f(x) = 0$ and $\lim\limits_{x\to a} g(x) = \infty$.

(1) Both statement 1 and 2 are true

(2) Both statements 1 and 2 are false

(3) Statement-1 is true, Statement-2 is False

(4) Statement-1 is False, Statement-2 is true

51. **STATEMENT-1 :** $\lim_{x\to\infty} \frac{2x^4+3x^3+7x}{3x^4+2x^2+3x} = \frac{2}{3}$.

STATEMENT-2 : If $P(x)$ and $Q(x)$ are two polynomials with rational coefficients, then

$\lim_{x\to\infty} \frac{P(x)}{Q(x)}$

$= \frac{\text{coefficient of highest power of } x \text{ in } P(x)}{\text{coefficient of highest power of } x \text{ in } Q(x)}$

(1) Both statement 1 and 2 are true

(2) Both statements 1 and 2 are false

(3) Statement-1 is true, Statement-2 is False

(4) Statement-1 is False, Statement-2 is true

52. Match the column-I and column –II

	Column-I		**Column-II**
I	$\lim_{x\to 0} \frac{\tan x - \sin x}{x^3} =$	**P**	$\frac{1}{2}$
II	$\lim_{x\to\infty} \sqrt{\frac{x-\sin x}{x+\cos^2 x}} =$	**Q**	1
III	Let $G(x) = -\sqrt{25-x^2}$. If $\lim_{x\to 1} \frac{G(x)-G(1)}{x-1} = \frac{1}{2\sqrt{\lambda}}$ then value of $\lambda =$	**R**	6
IV	If $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2} = e^{1/\lambda}$ then value of $\lambda =$	**S**	3

(1) I-P, II-R, III-Q, IV-S

(2) I-R, II-S, III-Q, IV-S

(3) I-P, II-Q, III-R, IV-S

(4) I-P, II-R, III-S, IV-R

53. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(1) $\frac{1}{2}(\alpha-\beta)^2$ (2) $-\frac{a^2}{2}(\alpha-\beta)^2$

(3) 0 (4) $\frac{a^2}{2}(\alpha-\beta)^2$

54. Let $f : \boldsymbol{R} \to \boldsymbol{R}$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)}$ is equal to

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$

(3) 3 (4) 1

55. $\lim_{x\to 2} \left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ is equal to

(1) does not exist (2) equals $\sqrt{2}$

(3) equals $-\sqrt{2}$ (4) equals $\frac{1}{\sqrt{2}}$

56. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) $-\frac{1}{4}$ (2) $\frac{1}{2}$

(3) 1 (4) 2

57. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to

(1) $-\pi$ (2) π

(3) $\pi/2$ (4) 1

58. Let $p = \lim_{x\to 0^+} \left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$, then $\log p$ is equal to:

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) 2

59. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

(1) $\frac{1}{24}$ (2) $\frac{1}{16}$

(3) $\frac{1}{8}$ (4) $\frac{1}{4}$

60. For each $t \in R$ let $[t]$ be the greatest integer less than or equal to t. Then

$\lim\limits_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

(1) is equal to 120
(2) does not exist (in R)
(3) is equal to 0
(4) is equal to 15

61. The integer 'n' for which $\ell im_{x \to 0} \frac{(\cos x - 1)\ (\cos x - e^x)}{x^n}$ is a finite non-zero number, is

(1) 1 (2) 2

(3) 3 (4) 4

62. Let $f: R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim\limits_{x \to 0} \left(\frac{f(1+x)}{f(1)}\right)^{\frac{1}{x}}$ equals

(1) 1 (2) e

(3) e^2 (4) e^3

63. If $\ell im_{x \to 0} \frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is a non-zero real number, then a is equal to

(1) 0 (2) $\frac{n+1}{n}$

(3) n (4) $n + \frac{1}{n}$

64. For $x > 0$, $\ell im_{x \to 0} \left((\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x}\right)$ is equal to

(1) 0 (2) –1

(3) 1 (4) 2

65. If $\lim\limits_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b\right) = 4$, then

(1) $a = 1, b = 4$ (2) $a = 1, b = -4$

(3) $a = 2, b = -3$ (4) $a = 2, b = 3$

Integer Type Questions (66 to 75)

66. If $f(x) = \begin{cases} 4x, & x < 0 \\ 1, & x = 0 \\ 3x^2, & x > 0 \end{cases}$, then $\ell im_{x \to 0} f(x)$ equal

67. $\lim\limits_{x \to 4} \left(\frac{x^{3/2} - 8}{x - 4}\right) =$

68. $\lim\limits_{x \to 0} \frac{e^{\tan x} - e^x}{\tan x - x}$ is equal to

69. $\ell im_{n \to \infty} \frac{5^{n+1} + 3^n - 2^{2n}}{5^n + 2^n + 3^{2n+3}} =$

70. $\ell im_{n \to \infty} \sqrt{\frac{3x - \sin x}{3x + \cos^2 x}} =$

71. $\ell im_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{x \sin x} =$

72. $\ell im_{x \to \pi/2} \tan x \log_e \sin x$ is equal to

73. If $f(x) = \begin{cases} x^2 + 4, & x \le 2 \\ x + 2, & x > 2 \end{cases}$ and

$g(x) = \begin{cases} x^2, & x \le 2 \\ 8, & x > 2 \end{cases}$ then $\ell im_{x \to 2} f(x)\, g(x)$ equals-

74. $\ell im_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}} =$

75. If $\ell im_{x \to 0} \frac{a + b\sin x - \cos x + ce^x}{x^3}$ exists, then the value of $a + b + c$ is

CHAPTER 14

CONTINUITY, DIFFERENTIABILITY, MOD

Single Option Correct Type Questions (01 to 50)

1. If $f(x) = (\tan x \cot \alpha)^{1/(x-\alpha)}$, $x \neq \alpha$ is continuous at $x = \alpha$, then the value of $f(\alpha)$ is-

(1) $e^{2\sin 2\alpha}$ (2) $e^{2\text{cosec } 2\alpha}$

(3) $e^{\text{cosec } 2\alpha}$ (4) $e^{\sin 2\alpha}$

2. If function $f(x) = \left(\frac{\sin x}{\sin \alpha}\right)^{1/(x-\alpha)}$ for $x \neq \alpha$ where, $\alpha \neq m\pi$ ($m \in I$) is continuous at $x = \alpha$ then -

(1) $f(\alpha) = e^{\tan \alpha}$ (2) $f(\alpha) = e^{\cot \alpha}$

(3) $f(\alpha) = e^{2 \cot \alpha}$ (4) $f(\alpha) = \cot \alpha$

3. If $f(x) = \begin{cases} x^2 + 2 & , x \geq 2 \\ 1 - x & , x < 2 \end{cases}$ and $g(x) = \begin{cases} 2x & , x > 1 \\ 3 - x & , x \leq 1 \end{cases}$, then $f(g(x))$ is

(1) continuous $x \in R - \{2\}$

(2) continuous at $x \in R - \{1\}$

(3) continuous at $x \in R$

(4) continuous at $x \in R - \{0, 1, 2\}$

4. If $f(x)$ is continuous function and $g(x)$ is discontinuous function, then correct statement is-

(1) $f(x) + g(x)$ is a continuous function

(2) $f(x) - g(x)$ is a continuous function

(3) $f(x) + g(x)$ is a discontinuous function

(4) $f(x)\, g(x)$ is a continuous function

5. If $y = \frac{1}{t^2 + t - 2}$ where $t = \frac{1}{x-1}$, then the number of points of discontinuities of $y = f(x)$, $x \in R$ is

(1) 1 (2) 2

(3) 3 (4) infinite

6. If $f(x) = \begin{cases} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, then $f(x)$ is

(1) continuous as well differentiable at $x = 0$

(2) continuous but not differentiable at $x = 0$

(3) neither differentiable at $x = 0$ nor continuous at $x = 0$

(4) none of these

7. If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then indicate the correct alternative(s):

(1) $f(x)$ is continuous but not differentiable at $x = 0$

(2) $f(x)$ is differentiable at $x = 0$

(3) $f(x)$ is not differentiable at $x = 0$

(4) $f(x)$ is discontinous at $x = 0$

8. If $f(x) = \begin{cases} \frac{x}{1 + e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then the function $f(x)$ is differentiable for-

(1) $x \in R^+$ (2) $x \in R$

9. If $f(x) = \begin{cases} x\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then correct statement is-

(1) f is continuous at all points except at $x = 0$

(2) f is continuous at every point but not differentiable at $x = 0$

(3) f is differentiable at every point

(4) f is differentiable only at the origin

10. Let $f(x) = \begin{cases} x^n \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then $f(x)$ is continuous but not differentiable at $x = 0$ if -

(1) $n \in (0, 1]$ (2) $n \in [0, \infty)$

(3) $n \in (-\infty, 0)$ (4) $n = 0$

11. The set of all points, where the function $\sqrt[3]{x^2 |x|}$ is differentiable is

(1) $(-\infty, 0) \cup (0, \infty)$ (2) $(-\infty, \infty)$

(3) $(0, \infty)$ (4) $(-\infty, 0)$

12. If $f(x) = \cos^{-1}(\cos x)$, then at the points, where f is differentiable, $f'(x)$ equals

(1) 1 (2) – 1

(3) $sgn(\sin x)$ (4) $-sgn(\sin x)$

13. If $f(x)$ is differentiable everywhere, then:

(1) $|f|$ is differentiable everywhere

(2) $|f|^2$ is differentiable everywhere

(3) $f|f|$ is not differentiable at some point

(4) $f + |f|$ is differentiable everywhere

14. If $f(x) = \sum_{k=1}^{n} a_k |x|^k$, where a_i's are real constants, then $f(x)$ is

(1) continuous at $x=0$ for all a_i only if $a_{2k} = 0$

(2) differentiable at $x = 0$ for all $a_i \in R$

(3) differentiable at $x = 0$ for all $a_{2k-1} = 0$

(4) none of these

15. If $f(x)$ be a differentiable function for all positive numbers such that $f(x.\,y) = f(x) + f(y)$ and $f(e) = 1$ then $\lim_{x \to 0} \dfrac{f(x+1)}{2x}$

(1) 2 (2) 1

(3) 1/2 (4) –1

16. A function $f(x)$ is defined as

$f(x) = \dfrac{\cos(\sin x) - \cos x}{x^2}$, $x \neq 0$ and $f(0) = a$

then $f(x)$ is continuous at $x = 0$ if 'a' equals

(1) 0 (2) 4

(3) 5 (4) 6

17. Let $f(x) = \dfrac{1 - \sin x}{(\pi - 2x)^2} \cdot \dfrac{\ell n(\sin x)}{\ell n(1 + \pi^2 - 4\pi x + 4x^2)}$, $x \neq \dfrac{\pi}{2}$. The value of $f\left(\dfrac{\pi}{2}\right)$ so that the function is continuous at $x = \pi/2$ is:

(1) 1/16 (2) 1/32

(3) – 1/64 (4) 1/128

18. Which of the following function(s) defined below do not have single point continuity?

(1) $f(x) = \begin{bmatrix} 1 & if & x \in Q \\ 0 & if & x \notin Q \end{bmatrix}$

(2) $g(x) = \begin{bmatrix} x & if & x \in Q \\ 1-x & if & x \notin Q \end{bmatrix}$

(3) $h(x) = \begin{bmatrix} x & if & x \in Q \\ 0 & if & x \notin Q \end{bmatrix}$

(4) $k(x) = \begin{bmatrix} x & if & x \in Q \\ -x & if & x \notin Q \end{bmatrix}$

19. If $f(x) = x + \{-x\} + [x]$, where [.] is the integral part & { . } is the fractional part function, then the number of points of discontinuity of f in [– 2, 2] is/are

(1) 3

(2) 4

(3) 5

(4) 7

20. Let $f(x) = [x] + \sqrt{x-[x]}$, where [.] denotes the greatest integer function. Then

(1) $f(x)$ is discontinuous on R^+
(2) $f(x)$ is continuous on R
(3) $f(x)$ is discontinuous on $R - I$
(4) discontinuous at $x = 1$

21. If $f(x) = \begin{cases} ax^2 - b & if\ |x| < 1 \\ -\dfrac{1}{|x|} & if\ |x| \geq 1 \end{cases}$ is derivable at $x = 1$, then the values of $a + b$ is

(1) 0 (2) 1
(3) 2 (4) 3

22. If $f(x) = \dfrac{x}{\sqrt{x+1}-\sqrt{x}}$ be a real valued function, then

(1) $f(x)$ is continuous, but $f'(0)$ does not exist
(2) $f(x)$ is differentiable at $x = 0$
(3) $f(x)$ is not continuous at $x = 0$
(4) $f(x)$ is not differentiable at $x = 0$

23. Function $f(x) = \begin{cases} x\tan^{-1}(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ at $x = 0$ is-

(1) Discontinuous (2) Continuous
(3) Differentiable (4) None of these

24. Let $f(x) = \begin{cases} \sin 2x, & 0 < x \leq \pi/6 \\ ax+b, & \pi/6 < x < 1 \end{cases}$. If $f(x)$ and $f'(x)$ are continuous, then-

(1) $a = 1, b = \dfrac{1}{\sqrt{2}} + \dfrac{\pi}{6}$

(2) $a = \dfrac{1}{\sqrt{2}}, b = \dfrac{1}{\sqrt{2}}$

(3) $a = 1, b = \dfrac{\sqrt{3}}{2} - \dfrac{\pi}{6}$

(4) $a = 1, b = \dfrac{\sqrt{3}}{2} + \dfrac{\pi}{6}$

25. For what triplets of real numbers (a, b, c) with $a \neq 0$ the function

$$f(x) = \begin{cases} x & , \quad x \leq 1 \\ ax^2 + bx + c & , \quad otherwise \end{cases}$$

is differentiable for all real x?

(1) $\{(a, 1-2a, a) \mid a \in R, a \neq 0\}$
(2) $\{(a, 1-2a, c) \mid a, c \in R, a \neq 0\}$
(3) $\{(a, b, c) \mid a, b, c \in R, a + b + c = 1\}$
(4) $\{(a, 1-2a, 0) \mid a \in R, a \neq 0\}$

26. If the derivative of the function $f(x) = \begin{cases} ax^2 + b, & x < -1 \\ bx^2 + ax + 4, & x \geq -1 \end{cases}$ is everywhere continuous, then-

(1) $a = 2, b = 3$ (2) $a = 3, b = 2$
(3) $a = -2, b = -3$ (4) $a = -3, b = -2$

27. $[x]$ denotes the greatest integer less than or equal to x. If $f(x) = [x]\,[\sin \pi x]$ in $(-1, 1)$, then $f(x)$ is:

(1) continuous at $x = 0$
(2) continuous in $(-1, 0)$
(3) differentiable in $(-1,1)$
(4) none of these

28. The number of points at which the function $f(x) = \max.\{a - x, a + x, b\}, -\infty < x < \infty, 0 < a < b$ cannot be differentiable is:

(1) 1 (2) 2
(3) 3 (4) 4

29. Let $f: R \to R$ be a function defined by $f(x) = \max.\{x, x^3\}$. The set of all points where $f(x)$ is not differentiable is

(1) $\{-1, 1\}$ (2) $\{-1, 0\}$
(3) $\{0, 1\}$ (4) $\{-1, 0, 1\}$

30. Let $f(x)$ be defined in $[-2, 2]$ by $f(x) = \begin{cases} \max\left(\sqrt{4-x^2}, \sqrt{1+x^2}\right), & -2 \leq x \leq 0 \\ \min\left(\sqrt{4-x^2}, \sqrt{1+x^2}\right), & 0 < x \leq 2 \end{cases}$, then $f(x)$

(1) is continuous at all points
(2) is not continuous at more than one point
(3) is not differentiable only at one point
(4) is not differentiable at more than one point

31. $f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is:

(1) continuous but not differentiable at $x = 1$
(2) differentiable at $x = 1$
(3) neither continuous nor differentiable at $x =$
(4) continuous everywhere

32. Lef f: $R \to R$ be any function. Define g: $R \to R$ by $g(x) = |f(x)|$ for all x. Then g is

(1) onto if f is onto
(2) one-one if f is one-one
(3) continuous if f is continuous
(4) differentiable if f is differentiable

33. Let $f''(x)$ be continuous at $x = 0$ and $f''(0) = 4$ then value of $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

(1) 11 (2) 2
(3) 12 (4) 9

34. Let f: $R \to R$ be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, $f(0) = 0$ and $f'(0) = 3$, then

(1) $\frac{f(x)}{x}$ is differentiable in R
(2) $f(x)$ is continuous but not differentiable in R
(3) $f(x)$ is continuous in R
(4) None of these

35. Suppose that f is a differentiable function with the property that $f(x + y) = f(x) + f(y) + xy$ and $\lim_{h\to 0}\frac{1}{h}f(h) = 3$, then

(1) f is a linear function
(2) $f(x) = 3x + x^2$
(3) $f(x) = 3x + \frac{x^2}{2}$
(4) $f(x) = 3x - \frac{x^2}{2}$

36. If a differentiable function f satisfies $f\left(\frac{x+y}{3}\right) = \frac{4-2(f(x)+f(y))}{3}$ $\forall$ $x, y \in R$, then $f(x)$ is equal to

(1) $\frac{1}{7}$ (2) $\frac{2}{7}$
(3) $\frac{8}{7}$ (4) $\frac{4}{7}$

37. **Statement–1** $f(x) = \{\tan x\} - [\tan x]$ is continuous at $x = \frac{\pi}{3}$, where [.] and {.} represent greatest integral function and fractional part function respectively.

Statement – 2 If $y = f(x)$ & $y = g(x)$ are continuous at $x = a$ then $y = f(x) \pm g(x)$ are continuous at $x = a$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

38. The set of points, where $f(x) = \frac{x}{1+|x|}$ is differentiable, is

(1) $(-\infty, -1) \cup (-1, \infty)$
(2) $(-\infty, \infty)$
(3) $(0, \infty)$
(4) $(-\infty, 0) \cup (0, \infty)$

39. Let f: $R \to R$ be a function defined by $f(x) = \text{Min}\{x + 1, |x| + 1\}$. Then which of the following is true?

(1) $f(x) \geq 1$ for all $x \in R$
(2) $f(x)$ is not differentiable at $x = 1$
(3) $f(x)$ is differentiable everywhere
(4) $f(x)$ is not differentiable at $x = 0$

40. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & x \neq 1 \\ 0 & , x = 1 \end{cases}$ Then which one of the following is true?

(1) f is differentiable at $x = 0$ and at $x = 1$

(2) f is differentiable at $x = 0$ but not at $x = 1$

(3) f is differentiable at $x = 1$ but not at $x = 0$

(4) f is neither differentiable at $x = 0$ nor at $x = 1$

41. Let $f(x) = x|x|$ and $g(x) = \sin x$

Statement-1 gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 gof is twice differentiable at $x = 0$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

42. The value of p and q for which the function $f(x)$

$$= \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \dfrac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$$ is continuous

for all x in R, are:

(1) $p = \frac{1}{2}, q = -\frac{3}{2}$ (2) $p = \frac{5}{2}, q = \frac{1}{2}$

(3) $p = -\frac{3}{2}, q = \frac{1}{2}$ (4) $p = \frac{1}{2}, q = \frac{3}{2}$

43. Define $F(x)$ as the product of two real functions $f_1(x) = x, x \in R$, and

$f_2(x) = \begin{cases} \sin\frac{1}{x}, & If \quad x \neq 0 \\ 0, & If \quad x = 0 \end{cases}$ as follows :

Statement - 1: $F(x)$ is continuous on R.

Statement - 2: $f_1(x)$ and $f_2(x)$ are continuous on R.

(1) Statement-1 is true, Statement-2 is true

(2) Statement-1 is false, Statement-2 is false

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

44. If function $f(x)$ is differentiable at $x = a$, then $\lim\limits_{x \to a} \dfrac{x^2 f(a) - a^2 f(x)}{x - a}$ is

(1) $-a^2 f'(a)$

(2) $af(a) - a^2 f'(a)$

(3) $2af(a) - a^2 f'(a)$

(4) $2af(a) + a^2 f'(a)$

45. If $f: R \to R$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

(1) continuous for every real x.

(2) discontinuous only at $x = 0$.

(3) discontinuous only at non-zero integral values of x.

(4) continuous only at $x = 0$.

46. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

(1) $g'(0) = \cos(\log 2)$

(2) $g'(0) = -\cos(\log 2)$

(3) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(4) g is not differentiable at $x = 0$

47. The domain of the derivative of the function

$f(x) = \begin{cases} \tan^{-1} x & , \quad if\ |x| \leq 1 \\ \frac{1}{2}(|x| - 1) & , \quad if\ |x| > 1 \end{cases}$ is

(1) $R - \{0\}$

(2) $R - \{1\}$

(3) $R - \{-1\}$

(4) $R - \{-1, 1\}$

48. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x\to 0} \frac{f(x^2)-f(x)}{f(x)-f(0)}$ is

(1) 1 (2) 0

(3) –1 (4) 2

49. Let $g(x) = \frac{(x-1)^n}{\log \cos^m (x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the lefthand derivative of $|x - 1|$ at $x = 1$. If $\lim_{x\to 1^+} g(x) = p$, then

(1) $n = 1, m = 1$ (2) $n = 1, m = -1$

(3) $n = 2, m = 2$ (4) $n > 2, m = n$

50. Let $f(x) = \begin{cases} x^2 \left|\cos \frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in R$, then f is

(1) differentiable both at $x = 0$ and at $x = 2$

(2) differentiable at $x = 0$ but not differentiable at $x = 2$

(3) not differentiable at $x = 0$ but differentiable at $x = 2$

(4) differentiable neither at $x = 0$ nor at $x = 2$

Integer Type Questions (51 to 58)

51. If $f(x) = \frac{1}{(1 - x)}$ and $g(x) = f[f\{f(x)\}]$, then the number of values of x where $g(x)$ is discontinuous is

52. If $f(x)$ be a differentiable function such that $f(x + y) = f(x) + f(y)$ and $f(1) = 2$ then $f'(2)$ is equal to

53. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals:

54. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0} \frac{1}{h} f(1 + h) = 5$, then $f'(1)$ equals

55. The function $f: R - \{0\} \to R$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$ can be made continuous at $x = 0$ by defining $f(0)$ as

56. If $y = (1+x)(1+x^2)(1+x^4).......(1+x^{2n})$ then $\frac{dy}{dx}$ at $x = 0$ is

57. If $f(x) =$

$$\begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$$

Then $f'(0)$ is equal to :

58. Let $y = \cfrac{x}{a + \cfrac{x}{b + \cfrac{x}{a + \cfrac{x}{b + ...}}}}$, if $\frac{dy}{dx} = \frac{b}{a(b + \lambda y)}$ then λ is

CHAPTER 15

INVERSE TRIGONOMETRIC FUNCTIONS

Single Option Correct Type Questions (01 to 60)

1. Domain of $f(x) = \cos^{-1} x + \cot^{-1} x + \text{cosec}^{-1} x$ is

(1) $[-1, 1]$ (2) R

(3) $(-\infty, -1] \cup [1, \infty)$ (4) $\{-1, 1\}$

2. $\text{cosec}^{-1} (\cos x)$ is real if

(1) $x \in [-1, 1]$

(2) $x \in R$

(3) x is an odd multiple of $\frac{\pi}{2}$

(4) $x = n\pi, n \in z$

3. Range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is

(1) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (2) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

(3) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (4) $\left\{0, \frac{\pi}{4}\right\}$

4. $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] =$

(1) $\frac{\sqrt{3}}{2}$ (2) $-\frac{\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

5. $\cos[\cos^{-1}(\sqrt{3}/2) + \sin^{-1}(1/\sqrt{2})]$ is equal to

(1) $(\sqrt{3} + 1)/2\sqrt{2}$

(2) $(\sqrt{3} - 1)/2\sqrt{2}$

(3) $(-\sqrt{3} + 1)/2\sqrt{2}$

(4) $(-\sqrt{3} - 1)/2\sqrt{2}$

6. $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$ is equal to-

(1) $-\frac{17\pi}{15}$ (2) $\frac{17\pi}{15}$

(3) $\frac{2\pi}{15}$ (4) $\frac{13\pi}{15}$

7. The value of $\sin^{-1} (\sin 10)$ is-

(1) 10 (2) $10 - 3\pi$

(3) $3\pi - 10$ (4) $10 - \frac{7\pi}{2}$

8. $\cos^{-1}\sqrt{\frac{1+\cos x}{2}}$; $\forall\ 0 < x < \pi$ is-

(1) x (2) $\frac{x}{2}$

(3) $2x$ (4) $\frac{2}{x}$

9. $\sin^{-1}\sin\frac{13\pi}{15}$ is equal to-

(1) $-\frac{17\pi}{15}$ (2) $\frac{17\pi}{15}$

(3) $\frac{13\pi}{15}$ (4) $\frac{2\pi}{15}$

10. If $x < 0$ then value of $\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right)$ is

(1) $\frac{\pi}{2}$ (2) $-\frac{\pi}{2}$

(3) 0 (4) π

11. If $3\cos^{-1}(x^2 - 7x + 25/2) = \pi$, then $x =$

(1) Only 3 (2) Only 4
(3) 3 or 4 (4) 2

12. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)$,

$(x \neq 0)$ is equal to-

(1) x (2) $2x$
(3) $2/x$ (4) $\frac{x}{2}$

13. The value of $\sin^2\left(\cos^{-1}\frac{1}{2}\right) + \cos^2\left(\sin^{-1}\frac{1}{3}\right)$ is-

(1) $\frac{17}{36}$ (2) $\frac{59}{36}$
(3) $\frac{36}{59}$ (4) $\frac{36}{17}$

14. $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$ is equal to-

(1) $\sqrt{\frac{x^2+2}{x^2+3}}$ (2) $\sqrt{\frac{x^2+2}{x^2+1}}$
(3) $\sqrt{\frac{x^2+1}{x^2+2}}$ (4) $\sqrt{\frac{x^2-1}{x^2+1}}$

15. The value of $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$ is

(1) $\frac{17}{6}$ (2) $\frac{7}{17}$
(3) $\frac{3}{4}$ (4) $\frac{\sqrt{5}}{3}$

16. The numerical value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$ is

(1) $\frac{-7}{17}$ (2) $\frac{7}{17}$
(3) $\frac{3}{4}$ (4) $\frac{17}{7}$

17. The value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is

(1) $\frac{17}{6}$ (2) $\frac{7}{17}$
(3) $\frac{3}{4}$ (4) $\frac{\sqrt{7}}{4}$

18. The value of $\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1}\left(\frac{1}{2}\right)\right)\right)\right)$ is

(1) $\frac{17}{6}$ (2) $\frac{7}{17}$
(3) $\frac{3}{4}$ (4) $\frac{\sqrt{5}}{3}$

19. $\tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$ (where $a, b, c > 0$) $=$

(1) $\tan^{-1}a - \tan^{-1}b$ (2) $\tan^{-1}a - \tan^{-1}c$
(3) $\tan^{-1}b - \tan^{-1}c$ (4) $\tan^{-1}c - \tan^{-1}a$

20. If $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ then $x =$

(1) -1 (2) $\frac{1}{6}$
(3) $-1, \frac{1}{6}$ (4) 1

21. If $a \leq \tan^{-1}x + \cot^{-1}x + \sin^{-1}x \leq b$. Then -

(1) $a = 0, b = \pi$ (2) $b = \frac{\pi}{2}$
(3) $a = \frac{\pi}{4}$ (4) $a = \frac{\pi}{2}, b = \pi$

22. The interval of values of x satisfying the inequality $\cot^{-1}x < -\sqrt{3}$ is

(1) $\left(-\infty, -\frac{\pi}{6}\right]$ (2) $\left(-\infty, -\frac{\pi}{3}\right]$
(3) $(-\infty, \infty)$ (4) no solution

23. If $\cos^{-1}x > \sin^{-1}x$, then -

(1) $x < 0$ (2) $-1 < x < 0$
(3) $0 \leq x < \frac{1}{\sqrt{2}}$ (4) $-1 \leq x < \frac{1}{\sqrt{2}}$

24. $\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$ is equal to-

(1) $\tan^{-1}(2^n)$ (2) $\tan^{-1}(2^n)-\frac{\pi}{4}$

(3) $\tan^{-1}(2^{n+1})$ (4) $\tan^{-1}(2^{n+1})-\frac{\pi}{4}$

25. The sum $\sum_{n=1}^{\infty}\tan^{-1}\frac{4n}{n^4-2n^2+2}$ is equal to:

(1) $3\pi/4$ (2) $4\tan^{-1}1$

(3) $\pi/2$ (4) π

26. If $x^2+y^2+z^2=r^2$, then $\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)=$

(1) π (2) $\pi/6$

(3) 0 (4) $\pi/2$

27. If $f(x)=\cot^{-1}x$, $f: R^+\to\left(0,\frac{\pi}{2}\right)$ and $g(x)=2x-x^2$, $g: R\to R$. Then the range of the function $f(g(x))$ wherever defined is

(1) $\left(0,\frac{\pi}{2}\right)$ (2) $\left(0,\frac{\pi}{4}\right)$

(3) $\left[\frac{\pi}{4},\frac{\pi}{2}\right)$ (4) $\left(\frac{\pi}{4}\right)$

28. If $x\ge 0$ and $\theta=\sin^{-1}x+\cos^{-1}x-\tan^{-1}x$, then

(1) $\frac{\pi}{2}\le\theta\le\frac{3\pi}{4}$ (2) $0\le\theta\le\frac{\pi}{4}$

(3) $0\le\theta<\frac{\pi}{2}$ (4) $\frac{\pi}{4}\le\theta\le\frac{\pi}{2}$

29. If $\sum_{i=1}^{2n}\sin^{-1}x_i=n\pi$, then $\sum_{i=1}^{2n}x_i=$

(1) n (2) $n/2$

(3) $2n$ (4) $\frac{n(n+1)}{2}$

30. The complete solution set of the inequality $[\cot^{-1}x]^2-6[\cot^{-1}x]+9\le 0$, where [.] denotes greatest integer function, is

(1) $(-\infty,\cot 3]$ (2) $[\cot 3,\cot 2]$

(3) $[\cot 3,\infty)$ (4) $(0,\pi)$

31. $\sin^{-1}x>\cos^{-1}x$ holds for

(1) $x\in\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$ (2) $x\in\left(0,\frac{1}{\sqrt{2}}\right)$

(3) $x\in\left(\frac{1}{\sqrt{2}},1\right]$ (4) all values of x

32. If $\cos[\tan^{-1}\{\sin(\cot^{-1}\sqrt{3})\}]=y$, then

(1) $y=\frac{4}{5}$ (2) $y=\frac{2}{\sqrt{5}}$

(3) $y=-\frac{2}{\sqrt{5}}$ (4) $y^2=\frac{10}{11}$

33. The value of $\cos\left[\frac{1}{2}\cos^{-1}\left\{\cos\left(-\frac{14\pi}{5}\right)\right\}\right]$ is

(1) $\cos\left(-\frac{7\pi}{5}\right)$ (2) $\sin\left(\frac{\pi}{10}\right)$

(3) $\cos\left(\frac{\pi}{5}\right)$ (4) $\cos\left(\frac{3\pi}{5}\right)$

34. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x)+\sin^{-1}(\sin x)\}]$, where $x\in\left(\frac{\pi}{2},\pi\right)$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$

(3) $-\frac{\pi}{4}$ (4) $-\frac{\pi}{2}$

35. The value of $\cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$, $\frac{\pi}{2}<x<\pi$, is:

(1) $\pi-\frac{x}{2}$ (2) $\frac{\pi}{2}+\frac{x}{2}$

(3) $\frac{x}{2}$ (4) $2\pi-\frac{x}{2}$

36. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

(1) 0 (2) $\frac{1}{\sqrt{5}}$

(3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{3}}{2}$

37. The equation $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has

(1) no solution
(2) unique solution
(3) infinite number of solutions
(4) two real solution

38. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then x equals

(1) – 1 (2) 1

(3) 0 (4) $\sqrt{3}$

39. The value of $\tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is

(1) $\frac{6}{17}$ (2) $\frac{7}{16}$

(3) $\frac{5}{7}$ (4) $\frac{17}{6}$

40. The value of x satisfying equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ is

(1) $\pm\frac{1}{\sqrt{3}}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{3}$ (4) $\pm\frac{1}{\sqrt{2}}$

41. The value of x satisfying equation $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, $(x > 0)$ is

(1) $\pm\frac{1}{\sqrt{3}}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\pm\frac{1}{\sqrt{2}}$

42. The solution of the equation $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$ is

(1) $x = 2$ (2) $x = -4$
(3) $x = 4$ (4) x = 3

43. If $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = 4°$, then :

(1) $x = \tan 2°$ (2) $x = \tan 4°$
(3) $x = \tan (1/4)°$ (4) $x = \tan 8°$

44. The smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0 \le x \le 1$ are

(1) 0, π (2) 0, $\frac{\pi}{4}$

(3) $-\frac{\pi}{4}, \frac{\pi}{4}$ (4) $\frac{\pi}{4}, \frac{\pi}{2}$

45. The value of x satisfying equation $\cos(2\sin^{-1}x) = \frac{1}{3}$ is

(1) $\pm\frac{1}{\sqrt{3}}$ (2) $\pm\frac{1}{\sqrt{5}}$

(3) $\frac{1}{3}$ (4) $\pm\frac{1}{3}$

46. If $\cos^{-1}x = \tan^{-1}x$, then

(1) $\sin(\cos^{-1}x) = \frac{\sqrt{5}+1}{2}$

(2) $x^2 = \frac{\sqrt{5}+1}{2}$

(3) $\sin(\cos^{-1}x) = \frac{\sqrt{5}-1}{2}$

(4) $x^2 = \frac{\sqrt{5}-1}{4}$

47. If $\tan(x + y) = 33$ and $x = \tan^{-1} 3$, then y will be

(1) 0.3 (2) $\tan^{-1}(1.3)$

(3) $\tan^{-1}(0.3)$ (4) $\tan^{-1}\left(\frac{1}{18}\right)$

48. **Statement-1 :** $\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3) = 11$.

Statement-2 : $\tan^2\theta + \sec^2\theta = 1 = \cot^2\theta + \text{cosec}^2\theta$.

(1) Statement-1 is false and statement-2 is true
(2) Statement-1 is true and statement-2 is false
(3) Both statements are true
(4) Both statements are false

49. **Statement-1 :** If $a > 0, b > 0$,

$\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2} . \Rightarrow x = \sqrt{ab}$.

Statement-2 : If $m, n \in N, n \geq m$, then

$\tan^{-1}\left(\frac{m}{n}\right) + \tan^{-1}\left(\frac{n-m}{n+m}\right) = \frac{\pi}{4}$

(1) Statement-1 is false and statement-2 is true
(2) Statement-1 is true and statement-2 is false
(3) Both statements are true
(4) Both statements are false

50. Match the column

Let (x, y) be such that

$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(b\,xy) = \frac{\pi}{2}$

	Column-I		Column-II
I	If $a = 1$ and $b = 0$, then (x, y)	**P**	lies on the circle $x^2 + y^2 = 1$
II	If $a = 1$ and $b = 1$, then (x, y)	**Q**	lies on $(x^2 - 1)(y^2 - 1) = 0$
III	If $a = 1$ and $b = 2$, then (x, y)	**R**	lies on $y = x$
IV	If $a = 2$ and $b = 2$, then (x, y)	**S**	lies on $(4x^2 - 1)(y^2 - 1) = 0$

(1) I-P; II-Q; III-P; IV-S
(2) I-Q; II-P; III-P; IV-S
(3) I-R; II-S; III-P; IV-Q
(4) I-P; II-P; III-Q; IV-S

51. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is :

(1) [2,3] (2) [2,3)
(3) [1,2] (4) [1,2)

52. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(1) $2\sin 2\alpha$ (2) 4
(3) $4\sin^2\alpha$ (4) $-4\sin^2\alpha$

53. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ is defined, is

(1) $[0, \pi]$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(3) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (4) $\left[0, \frac{\pi}{2}\right)$

54. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is-

(1) 1 (2) 3
(3) 4 (4) 5

55. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is

(1) $\frac{3x-x^3}{1-3x^2}$ (2) $\frac{3x+x^3}{1-3x^2}$
(3) $\frac{3x-x^3}{1+3x^2}$ (4) $\frac{3x+x^3}{1+3x^2}$

56. Domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$ for real valued 'x' is:

(1) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (4) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

57. If $0 < x < 1$, then $\sqrt{1+x^2}$ $[\{x \cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2} =$

(1) $\frac{x}{\sqrt{1+x^2}}$ (2) x

(3) $x\sqrt{1+x^2}$ (4) $\sqrt{1+x^2}$

58. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

(1) $\frac{23}{25}$ (2) $\frac{25}{23}$

(3) $\frac{23}{24}$ (4) $\frac{24}{23}$

59. The equation $2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}$ has

(1) no solution
(2) only one solution
(3) two solutions
(4) three solutions

60. The complete solution set of $\sin^{-1}(\sin 5) > x^2 - 4x$ is

(1) $|x-2| < \sqrt{9-2\pi}$

(2) $|x-2| > \sqrt{9-2\pi}$

(3) $|x| < \sqrt{9-2\pi}$

(4) $|x| > \sqrt{9-2\pi}$

Integer Type Questions (61 to 71)

61. $\sin\left[\frac{\pi}{6}+\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

62. If $\sum_{i=1}^{20}\sin^{-1}x_i = 10\pi$ then $\sum_{i=1}^{20}x_i$ is equal to

63. If $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \frac{3\pi}{2}$. Then $\alpha\beta + \beta\gamma + \gamma\alpha$ is

64. The value of x satisfying equation $\cot^{-1}x + \tan^{-1}3 = \frac{\pi}{2}$ is

65. If $\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$, then x is equal to

66. $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3) =$

67. $\cot\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right) =$

68. If $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \theta$, then $\cot\theta =$

69. If $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$, $n \in N$, then the maximum value of 'n' is

70. If $\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{4}$, then $\tan\theta$ is equal to

71. The number of solution(s) of the equation, $\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$, is

CHAPTER 16

MATRICES & DETERMINANTS

Single Option Correct Type Questions (01 to 57)

1. If $AB = 0$ for the matrices

$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and

$B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $\theta - \phi$ is

(1) an odd multiple of $\frac{\pi}{2}$

(2) an odd multiple of π

(3) an even multiple of $\frac{\pi}{2}$

(4) 0

2. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ and AB is equal to kB, then the value of k is

(1) 2 (2) 3

(3) 0 (4) 6

3. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then value of X^n is, (where n is natural number)

(1) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$

(2) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$

(3) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$

(4) $\begin{bmatrix} 2n+1 & -4n \\ n & -(2n-1) \end{bmatrix}$

4. Suppose A is a matrix such that $A^2 = A$ and $(I + A)^{10} = I + kA$, then $k =$

(1) 1023 (2) 1024

(3) 1047 (4) 2048

5. Which of the following is incorrect?

(1) $A^2 - B^2 = (A + B)(A - B)$

(2) $(A^T)^T = A$

(3) $(AB)^n = A^nB^n$, where A, B commute

(4) $(A - I)(I + A) = O \Leftrightarrow A^2 = I$

6. Let S be the set of all real matrices,

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a + d = 2$ and $A^T = A^2 - 2A$. Then S

(1) has exactly two elements.

(2) has exactly four elements.

(3) is an empty set.

(4) has exactly one element.

7. If A is a square matrix, then

(1) AA' is symmetric matrix

(2) AA' is skew - symmetric matrix

(3) $A'A$ is skew - symmetric matrix

(4) A^2 is symmetric matrix

8. If A is a skew - symmetric matrix and n is an even positive integer, then A^n is

(1) a symmetric matrix

(2) a skew-symmetric matrix

(3) a diagonal matrix

(4) a scalar matrix

9. If A is a non–singular matrix and A^T denotes the transpose of A, then:

(1) $|A| \neq |A^T|$

(2) $|A . A^T| \neq |A|^2$

(3) $|A^T A| \neq |A^T|^2$

(4) $|A| + |A^T| \neq 0$

10. If A and B are two invertible matrices such that $AB = C$ and $|A| = 2$, $|C| = -2$, then det(B) is

(1) 1 (2) 8

(3) 0 (4) –1

11. If D is a determinant of order three and Δ is a determinant formed by the cofactors of dsseterminant D ; then

(1) $\Delta = D^2$

(2) $\Delta = D^3$

(3) if $D = 9$, then Δ is perfect cube

(4) $\Delta = D$

12. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then

(1) $a - d = 0$ (2) $k = -|A|$

(3) $k = |A|$ (4) $k = a + d$

13. Let A be set of all determinants of order 3 with entries 0 or 1, B be the subset of A consisting of all determinants with value 1 and C be the subset of A consisting of all determinants with value –1. Then

STATEMENT -1 : The number of elements in set B is equal to number of elements in set C.

STATEMENT-2 : $(B \cap C) \subseteq A$

(1) Both statements are true

(2) Both statements are false

(3) Statement-1 is false and statement-2 is true.

(4) Statement-1 is true and statement-2 is false

14. **Statement 1 :** If $A = \begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$ then det(A) is real.

Statement 2 : If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a_{ij} being complex numbers then det(A) is always real.

(1) Both statements are true

(2) Both statements are false

(3) Statement-1 is false and statement-2 is true.

(4) Statement-1 is true and statement-2 is false

15. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$. If $10A^{10} +$ adj$(A^{10}) = B$, then $b_1 + b_2 + b_3 + b_4$ is equal to

(1) 91 (2) 92

(3) 111 (4) 112

16. Matrix $\begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible if

(1) $\alpha = 1$

(2) a, b, c are in A.P.

(3) a, b, c are in G.P.

(4) a, b, c are in H.P.

17. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then value of A^{-1} is equal to:

(1) A (2) A^2

(3) A^3 (4) A^4

18. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then $(5A - I)(A - I) =$

(1) A^2 (2) A^3

(3) A^4 (4) A

19. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b \in N$.

Then,

(1) there exist more than one but finite number of B's such that $AB = BA$

(2) there exists exactly one B such that $AB = BA$

(3) there exists infinitely many B's such that $AB = BA$

(4) there cannot exist any B such that $AB = BA$

20. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true ?

(1) $AB = BA$

(2) either A or B is a zero matrix

(3) either A or B is an identity matrix

(4) $A = B$

21. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals -

(1) 5^2 (2) 1

(3) $\frac{1}{5}$ (4) 5

22. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true ?

(1) If det $(A) = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers

(2) If det $(A) \neq \pm 1$, then A^{-1} exists but all its entries are non-integers

(3) If det $(A) = \pm 1$, then A^{-1} exists and all its entries are integers

(4) If det $(A) = \pm 1$, then A^{-1} need not exist

23. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement-I: If $A \neq I$ and $A \neq -I$, then det $(A) = -1$.

Statement-II: If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

(1) Statement-I is false, statement-II is true.

(2) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I.

(3) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I.

(4) Statement-I is true, statement-II is false.

24. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) 5 (2) 6

(3) at least 7 (4) less than 4

25. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. $Tr(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement -1 : Tr$(A) = 0$

Statement -2 : $|A| = 1$

(1) Statement -1 is false, Statement-2 is false

(2) Statement-1 is true, Statement-2 is false.

(3) Statement -1 is false, Statement -2 is true.

(4) Statement -1 is true, Statement -2 is true

26. **Statement 1 :**

If $A = \begin{bmatrix} a^2+x^2 & ab-cx & ac+bx \\ ab+xc & b^2+x^2 & bc-ax \\ ac-bx & bc+ax & c^2+x^2 \end{bmatrix}$ and

$B = \begin{bmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{bmatrix}$, then $|A| = |B|^2$.

Statement 2 : If A^c is cofactor matrix of a square matrix A of order n then $|A^c| = |A|^{n-1}$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

27. Let A and B be two symmetric matrices of order 3.
Statement-1 : $A(BA)$ and $(AB)A$ are symmetric matrices.
Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.
(1) Statement-1 is false, Statement-2 is false
(2) Statement-1 is true, Statement-2 is true
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

28. If $\omega \neq 1$ is the complex cube root of unity and matrix H = $\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to -
(1) 0 (2) $-H$
(3) H^2 (4) H

29. Let A and B be two 2 × 2 matrices.
Statement - 1 : $A(\text{adj } A) = |A|\, I_2$
Statement - 2 : $\text{adj}(AB) = (\text{adj } A)(\text{adj } B)$
(1) Statement-1 is true, Statement-2 is true
(2) Statement-1 is false, Statement-2 is false
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

30. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :
(1) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$
(3) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (4) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

31. If A is a 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals :
(1) B^{-1} (2) $(B^{-1})'$
(3) $I + B$ (4) I

32. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to :
(1) (2, – 1) (2) (–2, 1)
(3) (2, 1) (4) (–2, – 1)

33. Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = 0$.
Statement – I : $A^{-1} = \frac{1}{7}(5I - A)$.
Statement – II : The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$.
(1) Both statements are true
(2) Both statements are false
(3) Statement-I is false and statement-II is true.
(4) Statement-I is true and statement-II is false

34. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is
(1) 1 (2) – 1
(3) 4 (4) no real values

35. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $X = P^TQ^{2005}P$, then X is equal to
(1) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
(2) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$
(3) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$
(4) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

36. If for the matrix $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$; $|A^3| = 125$, then the value of α is

(1) ± 1 (2) ± 4
(3) ± 3 (4) ± 2

37. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix X $= \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(1) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (2) $PX = X$
(3) $PX = 2X$ (4) $PX = -X$

38. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If Q = $[q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals

(1) 52 (2) 103
(3) 201 (4) 205

39. Number of all possible skew symmetric matrices whose elements are taken from 0, 0, 0, 1, 1, 1, 1, –1, –1,–1,–1, are

(1) 8 (2) 9
(3) 10 (4) 11

40. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $(a, b, c, d \neq 0)$ is a matrix such that $A^2 = A$ then $|A|$ must be equal to

(1) 1 (2) 0
(3) –1 (4) $abcd$

41. If $A = \begin{bmatrix} 1 & a \\ b & -1 \end{bmatrix}$ is a matrix such that $AA' = A'A$ then

(1) $a = -b$ (2) $a \neq b$
(3) $a = b$ (4) $ab = 1$

42. The largest value of a third order determinant whose elements are 0 or 2 only is

(1) 16 (2) 0
(3) 24 (4) 2

43. If $\Delta = \begin{vmatrix} abc & b^2c & c^2b \\ abc & c^2a & ca^2 \\ abc & a^2b & b^2a \end{vmatrix} = 0$ $(a, b, c \in R$ and $a \neq b \neq c \neq 0)$ then value of $3(a+b+c)$ is

(1) 3 (2) 2
(3) 0 (4) 1

44. If $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t$ $= \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda^2 + 1 & 2 - \lambda & \lambda - 3 \\ \lambda^2 - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$ then value of p is

(1) –2 (2) –3
(3) –4 (4) –5

45. The number of diagonal matrix A of order n for which $A^3 = A$ is

(1) 1 (2) 0
(3) 2^n (4) 3^n

46. Let $f(x) = \begin{bmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{bmatrix}$ then value of $\int_0^{\pi/2} (f(x) + f'(x))dx$ is

(1) $\dfrac{\pi}{2}$ (2) π
(3) $-\dfrac{\pi}{2}$ (4) 0

47. Let A and B be two $n \times n$ matrices such that $A + B = AB$ then

(1) $AB = I_n$
(2) $A = I_n$ or $B = I_n$
(3) $AB = BA$
(4) $A = B$

48. If A and B are two square matrices such that $B = -A^{-1}BA$ then $(A+B)^2 =$

(1) 0

(2) $A^2 + 2AB + B^2$

(3) $A + B$

(4) $A^2 + B^2$

49. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = k\,I$, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det (Q) = $\frac{k^2}{2}$, then

(1) $\alpha = 0, k = 8$

(2) $4\alpha + k + 8 = 0$

(3) $\det(P\,adj\,(Q)) = 2^9$

(4) $\det(Q\,adj\,(P)) = 2^{13}$

50. For all values of $\theta \in \left[0, \frac{\pi}{2}\right]$, the determinant of the matrix $\begin{bmatrix} -2 & \tan\theta + \sec^2\theta & 3 \\ -\sin\theta & \cos\theta & \sin\theta \\ -3 & -4 & 3 \end{bmatrix}$ always lies in the interval

(1) [4, 6]
(2) [3, 5]
(3) (4, 6)
(4) $\left(\frac{5}{2}, \frac{19}{4}\right)$

51. The solution of the matrix equation

$$\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$$ is

(1) $x = 1, y = 3, z = 2$

(2) $x = 3, y = 2, z = 1$

(3) $x = 2, y = 3, z = 1$

(4) $x = 1, y = 2, z = 3$

52. If A and B are two square matrices such that $AB = A$ & $BA = B$, then A & B are

(1) Idempotent matrices

(2) Involutory matrices

(3) Orthogonal matrices

(4) Nilpotent matrices

53. If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(1) $\alpha = \pm\frac{1}{\sqrt{2}}$
(2) $\beta = \pm\frac{1}{\sqrt{6}}$
(3) $\gamma = \pm\frac{1}{\sqrt{3}}$
(4) all of these

54. The matrix A = $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is

(1) idempotent matrix

(2) involutory matrix

(3) nilpotent matrix

(4) symmetric matrix

55. Let A be a 2 × 2 matrix.

Statement-1 : adj(adj (A)) = A.

Statement-2 : $|\text{adj } A| = |A|$

(1) Both statements are true

(2) Both statements are false

(3) Statement-1 is false and statement-2 is true.

(4) Statement-1 is true and statement-2 is false

56. **Statement - 1 :** Determinant of a skew-symmetric matrix of order 3 is zero

Statement - 2 : For any matrix A,

$\det(A)^T = \det(A)$ and $\det(-A) = -\det(A)$.

Where det (B) denotes the determinant of matrix B. Then :

(1) Both statements are true

(2) Both statements are false

(3) Statement-1 is false and statement-2 is true.

(4) Statement-1 is true and statement-2 is false

57. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

Integer Type Questions (58 to 67)

58. If $A = [a_{ij}]_{3\times3}$ is a scalar matrix with $a_{11} = a_{22} = a_{33} = 2$ and $A(adjA) = kI_3$ then k is

59. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj $A = A A^T$, then $5a + b$ is equal to

60. If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$, then $\lim_{x\to0} \frac{f(x)}{x^2}$ is :

61. Let $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ and a, b, c are the roots of $x^3 + 3x^2 + 4x + 1 = 0$ then value of Δ is

62. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that $AB = B$ and $a + d = 3$ then $|A| =$

63. If A is a diagonal matrix of order 3 × 3 and commutative with every square matrix of order 3 under multiplication and tr(A) = 12, then value of $|A|^{\frac{1}{2}}$ is

64. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ be a matrix and $A^8 = \lambda A + \mu I$, $\lambda, \mu \in I$ then $\lambda + \mu =$

65. Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$ If det $(adj\ A)$ + det $(adj\ B) = 10^6$, then $[k]$ is equal to

(**Note** : adj M denotes the adjoint of a square matrix M and $[k]$ denotes the largest integer less than or equal to k])

66. The total number of distinct $x \in R$ for which

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10 \text{ is}$$

67. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to

CHAPTER

17

APPLICATION OF DERIVATIVES

Single Option Correct Type Questions (01 to 63)

1. Water is poured into an inverted conical vessel of which the radius of the base is 2 *m* and height 4 *m*, at the rate of 77 *litre/minute*. The rate at which the water level is rising at the instant when the depth is 70 *cm* is: (use $\pi = 22/7$)

(1) 10 *cm/min* (2) 20 *cm/min*
(3) 40 *cm/min* (4) 60 *cm/min*

2. A man 1.5 *m* tall walks away from a lamp post 4.5 *m* high at a rate of 4 *km/hr*. How fast is the farther end of shadow moving on the pavement?

(1) 4 *km/hr* (2) 2 *km/hr*
(3) 6 *km/hr* (4) 5 *km/hr*

3. The function $y = \dfrac{2x^2 - 1}{x^4}$ is

(1) Always increasing
(2) Always decreasing
(3) Neither increasing nor decreasing
(4) None of these

4. If $f(x) = 2x^3 - 9x^2 + 12x - 6$, then in which interval $f(x)$ is monotonically increasing

(1) (1, 2)
(2) $(-\infty, 1)$
(3) $(2, \infty)$
(4) $(-\infty, 1)$ and $(2, \infty)$

5. Let *f* be the function $f(x) = \cos x - \left(1 - \frac{x^2}{2}\right)$ then $f(x)$ is strictly increasing in the interval

(1) $(-\infty, \infty)$ (2) $(-2, \infty)$
(3) $[0, \infty)$ (4) $(0, \infty)$

6. For what value of '*a*' the function $f(x) = x + \cos x - a$ increases

(1) 0 (2) 1
(3) – 1 (4) Any value

7. Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function in the set of real numbers *R*. Then *a* & *b* satisfy the condition

(1) $a^2 - 3b - 15 > 0$
(2) $a^2 - 3b + 15 < 0$
(3) $a^2 + 3b - 15 < 0$
(4) $a > 0$ & $b > 0$

8. The values of '*a*' for which the function $f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$ decreases for all real values of *x* is

(1) $(-\infty, -3]$ (2) $(-\infty, -3)$
(3) $(-\infty, -2)$ (4) $(-\infty, -3] \cup [0, \infty)$

9. Let the function $f(x) = \sin x + \cos x$, be defined in $[0, 2\pi]$, then $f(x)$

(1) increases in $(\pi/4, \pi/2)$
(2) decreases in $[\pi/4, 5\pi/4]$
(3) increases in $[0, \pi/4] \cup [\pi, 2\pi]$
(4) decreases in $[0, \pi/4) \cup (\pi/2, 2\pi]$

10. If $f(x) = \log(x - 2) - \dfrac{1}{x}$, then

(1) $f(x)$ is *M.I.* for $x \in (2, \infty)$
(2) $f(x)$ is *M.I.* for $x \in [-1, 2]$
(3) $f(x)$ is *M.D.* for $x \in (2, \infty)$
(4) $f(x)$ is *M.D.* for $x \in [-1, 2]$

11. The interval in which the function $f(x) = x^3$ increases less rapidly than $g(x) = 6x^2 + 15x + 5$ is

(1) $(-\infty, -1)$ (2) $(-5, 1)$

(3) $(-1, 5)$ (4) $(5, \infty)$

12. Fosr what value of x, $x^2 \, ln(1/x)$ is maximum-

(1) $e^{-1/2}$ (2) $e^{1/2}$

(3) e (4) e^{-1}

13. Function $f(x) = x \, lnx$ has local maxima at $x =$

(1) $x = e$

(2) $x = \frac{1}{e}$

(3) $x = 1$

(4) No local maxima

14. The function $f(x) = a \sin x + \frac{1}{3} \sin 3x$ has a maximum at $x = \pi/3$, then a equals.

(1) -2 (2) 2

(3) -1 (4) 1

15. If $f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + ... + 100x^{100}$ is a polynomial in a real variable x, then $f(x)$ has

(1) neither a maximum nor a minimum

(2) only one maximum

(3) only one minimum

(4) one maximum and one minimum

16. $f(x) = \begin{cases} \tan^{-1} x, & |x| < \frac{\pi}{4} \\ \frac{\pi}{2} - |x|, & |x| \geq \frac{\pi}{4} \end{cases}$, then

(1) $f(x)$ has no point of local maxima

(2) $f(x)$ has only one point of local maxima

(3) $f(x)$ has exactly two points of local maxima

(4) $f(x)$ has exactly two points of local minima

17. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5 & , x \leq 1 \\ -2x + \log_2\left(b^2 - 2\right) & , x > 1 \end{cases}$ the set of values of b for which $f(x)$ has greatest value at $x = 1$ is given by:

(1) $1 \leq b \leq 2$

(2) $b = \{1, 2\}$

(3) $b \in (-\infty, -1)$

(4) $\left[-\sqrt{130}, -\sqrt{2}\right) \cup \left(\sqrt{2}, \sqrt{130}\right]$

18. The minimum value of the function defined by $f(x) = \max(x, x + 1, 2 - x)$ is

(1) 0 (2) 1/2

(3) 1 (4) 3/2

19. The absolute maximum and minimum values of $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$, $x \in [0, 3]$ are respectively

(1) $-25, 39$ (2) $25, -39$

(3) $8, -8$ (4) $8, 10$

20. The absolute minimum and maximum values of $f(x) = \sin x + \frac{1}{2} \cos 2x$, $x \in \left[0, \frac{\pi}{2}\right]$ are respectively

(1) $\frac{3}{4}, \frac{1}{2}$ (2) $0, \frac{1}{2}$

(3) $-\frac{1}{2}, \frac{3}{4}$ (4) $\frac{1}{2}, \frac{3}{4}$

21. The ratio between the height of a right circular cone of maximum volume inscribed in a given sphere and the diameter of the sphere is

(1) 2:3 (2) 3:4

(3) 1:3 (4) 1:4

22. The semi vertical angle of a right circular cone of maximum volume of a given slant height is

(1) $\cos^{-1} \sqrt{2}$

(2) $\sin^{-1}. \sqrt{2}$

(3) $\tan^{-1} \sqrt{3}$

(4) $\tan^{-1} \sqrt{2}$

23. The volume of the largest cylinder that can be inscribed in a sphere of radius ' r ' cm is (in cubic units)

(1) $\frac{4\pi r^3}{3\sqrt{3}}$ (2) $\frac{4\pi r^3}{3\sqrt{2}}$

(3) $\frac{4\pi r^2}{3\sqrt{2}}$ (4) $\frac{4\pi r^3}{2\sqrt{3}}$

24. Consider $f(x) = \cos \pi x + 10x + 3x^2 + x^3$, $x \in [-2, 3]$ then the absolute minimum value of $f(x)$ is

(1) 0 (2) $3 - 2\pi$

(3) – 2 (4) – 15

25. If $a = (100)^{1/100}$ and $b = (101)^{1/101}$ then

(1) $a = b$ (2) $a > b$

(3) $a < b$ (4) none of these

26. The function $\frac{|x-1|}{x^2}$ is monotonically decreasing in

(1) $(1, \infty)$ (2) $(0, 2)$

(3) $(0, 1)$ and $(2, \infty)$ (4) $(-\infty, \infty)$

27. If $f(x) = \frac{(\sin^{-1} x + \tan^{-1} x)}{\pi} + 2\sqrt{x}$, then the range of $f(x)$ is

(1) $[-1, 1]$ (2) $[0, 48]$

(3) $\left[0, \frac{15}{4}\right]$ (4) $\left[0, \frac{11}{4}\right]$

28. If $f(x)$ is strictly increasing real function defined on R and c is a real constant, then number of solutions of $f(x) = c$ is always equal to

(1) 1 (2) 2

(3) 0 (4) 0 or 1

29. Given that f is a real valued differentiable function such that $f(x) f'(x) < 0$ for all real x, it follows that

(1) $f(x)$ is an increasing function

(2) $f(x)$ is a decreasing function

(3) $|f(x)|$ is an increasing function

(4) $|f(x)|$ is a decreasing function

30. If $g(x)$ is monotonically increasing and $f(x)$ is monotonically decreasing for $x \in R$ and if $(gof)(x)$ is defined for $x \in R$, then

(1) $(gof)(x - 1) < (gof)(x + 1)$.

(2) $(gof)(x + 1) = (gof)(x - 1)$.

(3) $(gof)(x + 1) < (gof)(x - 1)$.

(4) None of these

31. If $f : [1, 10] \rightarrow [1, 10]$ is a non-decreasing function and $g: [1, 10] \rightarrow [1, 10]$ is a non-increasing function. Let $h(x) = f(g(x))$ with $h(1) = 1$, then $h(2)$

(1) lies in (1, 2)

(2) is more than 2

(3) is equal to 1

(4) is not defined

32. Water is dropped at the rate of 2 m^3/sec. into a cone of semi-vertical angle 45°. The rate at which periphery of water surface changes when height of the water in the cone is 2 meter, is

(1) 1 *m/sec.* (2) 2 *m/sec.*

(3) 3 *m/sec.* (4) 4 *m/sec.*

33. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number, then minimum value of $f(x)$

(1) does not exist

(2) is not attained even though f is bounded

(3) is equal to 1

(4) is equal to –1

34. If $f(x) = \begin{cases} x^2 ; & x \ge 0 \\ ax ; & x < 0 \end{cases}$. Then set of real values of 'a' such that $f(x)$ is strictly monotonically increasing at $x = 0$ is

(1) $a \in R^+$ (2) $a \in R$

(3) $a \in R - \{0\}$ (4) $a \in \phi$

35. The point of inflection for the curve $y = x^{\frac{5}{3}}$ is

(1) (1, 1) (2) (0, 0)

(3) (1, 0) (4) (0, 1)

36. If $f(x) = \begin{cases} -\sqrt{1-x^2} & , \quad 0 \le x \le 1 \\ -x & , \quad x > 1 \end{cases}$, then

(1) Maximum of $f(x)$ exist at $x = 1$
(2) Maximum of $f(x)$ doesn't exists
(3) Maximum of $f^{-1}(x)$ exist at $x = -1$
(4) Minimum of $f^{-1}(x)$ exist at $x = 1$

37. If $f(x) = a\ ln\ |x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

(1) $a = 2, b = -1$ (2) $a = 2, b = -½$
(3) $a = -2, b = 1/2$ (4) None of these.

38. The set of values of p for which all the points of extremum of the function $f(x) = x^3 - 3px^2 + 3(p^2 - 1)x + 1$ lie in the interval $(-2, 4)$, is

(1) $(-3, 5)$ (2) $(-3, 3)$
(3) $(-1, 3)$ (4) $(-1, 4)$

39. A running track of 440 m is to be laid out enclosing a football field, the shape of which is a rectangle with semi circle at two opposite end. If the area of the rectangular portion is to be maximum, then the length of its sides is

(1) $120\,m, \dfrac{220}{\pi}\,m$ (2) $110\,m, \dfrac{\pi}{200}\,m$
(3) $110\,m, \dfrac{220}{\pi}\,m$ (4) $125\,m, \dfrac{220}{\pi}\,m$

40. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone is

(1) one third that of the cone
(2) $1/\sqrt{2}$ times that of the cone
(3) 2/3 that of the cone
(4) 1/2 times that of the cone

41. The maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b is

(1) $2(ab)$ (2) $\dfrac{1}{2}(a+b)^2$
(3) $\dfrac{1}{2}(a^2+b^2)$ (4) None of these.

42. **STATEMENT-1:** e^{π} is bigger than π^{e}.
STATEMENT-2: $f(x) = x^{1/x}$ is *a* increasing function when $x \in [e, \infty)$

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

43. **Statement-1:** A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q, then minimum area of ΔOPQ is 32
Statement-2: Area of triangle formed by a straight line passes through a fixed point (p, q) and coordinate axes will be minimum then (p, q) is midpoint of intercept between coordinate axes

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

44. Let $f(x) = x^{50} - x^{20}$
STATEMENT-1: Global maximum of $f(x)$ in [0, 1] is 0.
STATEMENT-2: $x = 0$ is a stationary point of $f(x)$.

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

45. **STATEMENT-1:** If $f(x)$ is increasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.

STATEMENT-2: If $f(x)$ is decreasing function with concavity upwards, then concavity of $f(x)^{-1}$ is also upwards.

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is false, statement-2 is false
(3) Statement-1 is true, statement-2 is true
(4) Statement-1 is true, statement-2 is false.

46. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(1) $(\pi/4, \pi/2)$ (2) $(-\pi/2, \pi/4)$
(3) $(0, \pi/2)$ (4) $(-\pi/2, \pi/2)$

47. Suppose the cubic $x^3 - px + q = 0$ has three distinct real roots where $p > 0$ and $q > 0$. Then, which one of the following holds?

(1) Minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$
(2) Minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
(3) Minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
(4) Maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

48. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$

(1) $P(-1)$ is the minimum and $P(1)$ is the maximum of P
(2) $P(-1)$ is not minimum but $P(1)$ is the maximum of P
(3) $P(-1)$ is the minimum and $P(1)$ is not the maximum of P
(4) neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P

49. Let $f: R \to R$ be defined by $f(x) = \begin{cases} k-2x, & if \quad x \le -1 \\ 2x+3, & if \quad x > -1 \end{cases}$

If f has a local minimum at $x = -1$, then a possible value of k is

(1) 0
(2) $-\frac{1}{2}$
(3) -1
(4) 1

50. The values of a for which $f(x) = x^3 + 3(a-7)x^2 + 3(a^2-9)x - 1$ have a positive point of maxima is,

(1) $(-\infty, -3)$
(2) $\left(3, \frac{29}{7}\right)$
(3) $(-\infty, -3) \cup (3, \infty)$
(4) $(-\infty, -3) \cup \left(3, \frac{29}{7}\right)$

51. Let f be a function defined by

$$f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Statement - 1: $x = 0$ is point of minima of f

Statement - 2: $f'(0) = 0$.

(1) Statement-1 is false, statement-2 is false
(2) Statement-1 is true, statement-2 is true
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true

52. A spherical balloon is filled with 4500 π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is

(1) $\frac{9}{7}$ (2) $\frac{7}{9}$
(3) $\frac{2}{9}$ (4) $\frac{9}{2}$

53. Let $f(x) = (1 + b^2) x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

(1) [0, 1] (2) $\left(0, \frac{1}{2}\right]$

(3) $\left[\frac{1}{2}, 1\right]$ (4) (0, 1]

54. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]

(1) lies between 1 and 2

(2) lies between 2 and 3

(3) lies between –1 and 0

(4) does not exist.

55. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then:

(1) $\alpha = 2, \beta = -\frac{1}{2}$ (2) $\alpha = 2, \beta = \frac{1}{2}$

(3) $\alpha = -6, \beta = \frac{1}{2}$ (4) $\alpha = -6, \beta = -\frac{1}{2}$

56. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then

(1) $(4 - \pi) x = \pi r$

(2) $x = 2r$

(3) $2x = r$

(4) $2x = (\pi + 4) r$

57. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is:

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$

(3) 3 (4) – 3

58. The length of a longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing is

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{2}$

(3) $\frac{3\pi}{2}$

(4) π

59. If f is differentiable and strictly increasing in a neighborhood of '0', then

$$\lim_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)} =$$

(1) 0 (2) 1

(3) – 1 (4) 2

60. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$

(1) $f(x)$ is a strictly increasing function

(2) $f(x)$ has a local maxima

(3) $f(x)$ is a strictly decreasing function

(4) $f(x)$ is bounded

61. Let the function $g: (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2 \tan^{-1} (e^u) - \frac{\pi}{2}$. Then, g is

(1) even and is strictly increasing in $(0, \infty)$

(2) odd and is strictly decreasing in $(-\infty, \infty)$

(3) odd and is strictly increasing in $(-\infty, \infty)$

(4) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

62. Let $f(x)$ and $g(x)$ be two continuous function defined from R to R such that $f(x_1) > f(x_2)$ & $g(x_1) < g(x_2)$ $\forall\, x_1 > x_2$ then solution set of $f(g(\alpha^2 - 2\alpha)) > f(g(3\alpha - 4))$ is

(1) (1, 4) (2) [1, 4]

(3) {1, 4} (4) ϕ

63. If $f''(x) > 0\ \forall x \in R$ and $f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4)$ for $0 < x < \frac{\pi}{2}$ then $g(x)$ is increasing in

(1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

(3) $\left(0, \frac{\pi}{3}\right)$ (4) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Integer Type Questions (64 to 73)

64. A kite is 300 *m* high and there are 500 *m* of cord out. If the wind moves the kite horizontally at the rate of 5 *km/hr*. directly away from the person who is flying it, the rate at which the cord is being paid is (in *Km/hr*)

65. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^{x} + 1 = 0$ is

66. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$, then the square of the largest area (in sq. unit) of such a rectangle is

67. If x be real, then the minimum value of $f(x) = 3^{x+1} + 3^{-(x+1)}$ is

68. $\lim_{x \to 0} \left[\frac{\sin x \tan x}{x^2}\right]$, (where $x \in \left(0, \frac{\pi}{2}\right)$ and [.] denotes the greatest integer function) =

69. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at x is equal to:

70. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is:

71. The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3 & , \quad -3 < x \le -1 \\ x^{2/3} & , \quad -1 < x < 2 \end{cases}$$ is

72. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is

73. A point (a, b) on ellipse $4x^2 + 3y^2 = 12$ in first quadrant such that the area enclosed by the lines $y = x$, $y = b$, $x = a$ and x-axis is maximum then value of $2a + b$ is

CHAPTER

18

INDEFINITE INTEGRATION

Single Option Correct Type Questions (01 to 66)

1. The value of $\int \frac{\ell n\left(\frac{x-1}{x+1}\right)}{x^2-1}\,dx$ is equal to

(1) $\frac{1}{2}\,ln^2\,\frac{x-1}{x+1}+C$

(2) $\frac{1}{4}\,ln^2\,\frac{x-1}{x+1}+C$

(3) $\frac{1}{2}\,ln^2\,\frac{x+1}{x-1}+C$

(4) $\frac{1}{4}\,ln^2\,\frac{x+1}{x-1}+C$

2. The value of $\int \frac{\ell n\,(\tan x)}{\sin x\ \cos x}\,dx$ is equal to

(1) $\frac{1}{2}\,ln^2\,(\cot x)+C$

(2) $\frac{1}{2}\,ln^2\,(\sec x)+C$

(3) $\frac{1}{2}\,ln^2\,(\sin x)+C$

(4) $\frac{1}{2}\,ln^2\,(\cos x)+C$

3. If $f(x)=\int \frac{2\sin x-\sin 2x}{x^3}\,dx$, where x ¹ 0, then $\underset{x\to 0}{Limit}\ f$¢$(x)$ has the value

(1) 0 (2) 1

(3) 2 (4) None of these

4. If $\int (x^9+x^6+x^3)(2x^6+3x^3+6)^{1/3}\,dx=\frac{1}{a}$ $(2x^9+3x^6+6x^3)^{4/3}+c$, then 'a' is equal to

(1) $\frac{1}{6}$ (2) $\frac{1}{24}$

(3) 24 (4) 6

5. $\int 2^{mx}.\,3^{nx}\,dx$ when m, n Î N is equal to:

(1) $\frac{2^{mx}+3^{nx}}{m\,\ell n\,2+n\,\ell n\,3}+c$

(2) $\frac{(m\,n)\,.\,2^x\,.\,3^x}{m\,\ell n\,2+n\,\ell n\,3}+c$

(3) $\frac{2^{mx}\,.\,3^{nx}}{\ell n\left(2^m.3^n\right)}+c$

(4) none of these

6. $\int \frac{dx}{\sin x\,.\,\sin(x+\alpha)}$ is equal to

(1) cosec a $ln\left|\frac{\sin x}{\sin(x+\alpha)}\right|+C$

(2) cosec a $ln\left|\frac{\sin(x+\alpha)}{\sin x}\right|+C$

(3) cosec a $ln\left|\frac{\sec(x+\alpha)}{\sec x}\right|+C$

(4) cosec a $ln\left|\frac{\sec x}{\sec(x+\alpha)}\right|+C$

7. $\int (\sin 2x - \cos 2x)\ dx = \frac{1}{\sqrt{2}} \sin\ (2x - a) + b$, then

(1) $a = \frac{5\pi}{4}$, b Î R (2) $a = -\frac{5\pi}{4}$, b Î R

(3) $a = \frac{\pi}{4}$, b Î R (4) none of these

8. $\int [1 + \tan x \,.\, \tan(x + \alpha)]\, dx$ is equal to

(1) cos a. $ln \left| \frac{\sin x}{\sin(x+\alpha)} \right| + C$

(2) tan a. $ln \left| \frac{\sin x}{\sin(x+\alpha)} \right| + C$

(3) cot a. $ln \left| \frac{\sec(x+\alpha)}{\sec x} \right| + C$

(4) cot a. $ln \left| \frac{\cos(x+\alpha)}{\cos x} \right| + C$

9. $\int$ 4 sin x cos $\frac{x}{2}$ cos $\frac{3x}{2}$ dx is equal to

(1) $\cos x - \frac{1}{2} \cos 2x + \frac{1}{3} \cos 3x + c$

(2) $\cos x - \frac{1}{2} \cos 2x - \frac{1}{3} \cos 3x + c$

(3) $\cos x + \frac{1}{2} \cos 2x + \frac{1}{3} \cos 3x + c$

(4) $\cos x + \frac{1}{2} \cos 2x - \frac{1}{3} \cos 3x + c$

10. $\int \frac{x^2 + \cos^2 x}{1 + x^2}$ cosec^2 $x\, dx$ is equal to:

(1) $-\tan^{-1} x + \cot x + c$

(2) $2\tan^{-1} x + c$

(3) $-\tan^{-1} x - \frac{\text{cos ec x}}{\text{sec x}} + c$

(4) None of these

11. $\int \sqrt{\frac{x-1}{x+1}} \cdot \frac{1}{x^2}\, dx$ is equal to

(1) $\sin^{-1} \frac{1}{x} + \frac{\sqrt{x^2-1}}{x} + c$

(2) $\frac{\sqrt{x^2-1}}{x} + \cos^{-1} \frac{1}{x} + c$

(3) $\sec^{-1} x - \frac{\sqrt{x^2-1}}{x} + c$

(4) $\tan^{-1} \sqrt{x^2+1} - \frac{\sqrt{x^2-1}}{x} + c$

12. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\, dx = Ax + B\ ln\ |9e^{2x} - 4| + C$, then

(1) $A + 18B = 16$ (2) $18B - A = 20$

(3) $A - 18B = 17$ (4) $A + 18B = 32$

13. If 0 < x < p, then $\int \frac{dx}{\sqrt{\sin^3 x \sin(x - \alpha)}}$ is equal to

(1) $\sqrt{\cos\alpha + \sin\alpha \cot x} + c$

(2) 2 cosec a $\sqrt{\cos\alpha - \sin\alpha \cot x} + c$

(3) $-2 \cos ec\alpha \sqrt{\cos\alpha + \sin\alpha \cot x} + c$

(4) None of these

14. If $\int \frac{\cos 4x + 1}{\cot x - \tan x}\, dx = A \cos 4x + B$; where A & B are constants, then

(1) A = - 1/4 & B may have any real value

(2) A = - 1/8 & B may have any real value

(3) A = - 1/2 & B = - 1/4

(4) none of these

15. If $\int \tan^4 x\, dx = a \tan^3 x + b \tan x + f(x)$, then

(1) $a = \frac{1}{3}$

(2) $b = -1$

(3) f(x) = x + c, c Î R

(4) All of these

16. $\int \frac{dx}{1+\sin 2x-\cos 2x}$ is equal to

(1) $ln\,|\,(1+\cot x)\,|+c$

(2) $\sin^2 x+\cos x+c$

(3) $-\frac{1}{2}\,ln\,|\,(1+\cot x)|+c$

(4) None of these

17. $\int \frac{\sin^2 x}{\cos^6 x}dx$ is equal to

(1) $\tan x+\frac{\tan^5 x}{5}+c$

(2) $\frac{\tan^3 x}{3}+\frac{\tan^5 x}{5}+c$

(3) $\frac{\sin^3 x}{3}+\frac{\sin^5 x}{5}+c$

(4) $\tan x+\frac{\tan^3 x}{3}+c$

18. **STATEMENT-1:** $\int (\sin x)^5 \cos x\, dx$

$=\frac{\sin^6 x}{6}+C$

STATEMENT-2: $\int (f(x))^n f'(x)\, dx$

$=\frac{(f(x))^{n+1}}{n+1}+C,\ n \hat{I}\ I$

(1) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(2) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

19. **STATEMENT-1:** If $x > 0,\ x\ ^1\ 1$ then

$\int (\log_x e-(\log_x e)^2)\,dx\ = x\log_x e + C$

STATEMENT-2: $\int (f(x)+f'(x))e^x dx\ = e^x f(x) + C$

(1) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(2) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

20. $\int \frac{dx}{x(x^n+1)}$ is equal to

(1) $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+c$

(2) $\frac{1}{n}\log\left|\frac{x^n+1}{x^n}\right|+c$

(3) $\log\left|\frac{x^n}{x^n+1}\right|+c$

(4) None of these

21. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-a)+c$, then value of (A, B) is

(1) (sin a, cos a) (2) (cos a, sin a)

(3) (– sin a, cos a) (4) (– cos a, sin a)

22. $\int \frac{dx}{\cos x-\sin x}$ is equal to

(1) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+c$

(2) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+c$

(3) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+c$

(4) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+c$

23. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ is equal to

(1) $\frac{1}{2}\, ln \left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right| + c$

(2) $\frac{1}{2}\, ln \left|\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)\right| + c$

(3) $ln \left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right| + c$

(4) $ln \left|\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)\right| + c$

24. If the integral $\int \frac{5\tan x}{\tan x - 2}\, dx = x + a\, ln\, |\sin x - 2\cos x| + k$, then a is equal to:

(1) -1 (2) -2

(3) 1 (4) 2

25. If $\int f(x)\, dx = \psi(x)$, then $\int x^5 f(x^3)\, dx$ is equal to

(1) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)\, dx\right] + C$

(2) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)\, dx + C$

(3) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\, dx + C$

(4) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)\, dx\right] + C$

26. The integral $\int\left(1 + x - \frac{1}{x}\right)e^{x+\frac{1}{x}}\, dx$ is equal to

(1) $(x+1)\, e^{x+\frac{1}{x}} + c$

(2) $-x\, e^{x+\frac{1}{x}} + c$

(3) $(x-1)\, e^{x+\frac{1}{x}} + c$

(4) $x\, e^{x+\frac{1}{x}} + c$

27. Let $I_n = \int \tan^n x\, dx$, $(n > 1)$. If $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to

(1) $\left(-\frac{1}{5}, 1\right)$ (2) $\left(\frac{1}{5}, 0\right)$

(3) $\left(\frac{1}{5}, -1\right)$ (4) $\left(-\frac{1}{5}, 0\right)$

28. Let $f(x) = \int e^x (x-1)(x-2)\, dx$ then f decreases in the interval:

(1) (- ¥, 2) (2) (- 2, - 1)

(3) (1, 2) (4) (2, + ¥)

29. $\int \frac{x^2 - 1}{x^3\sqrt{2x^4 - 2x^2 + 1}}\, dx$ is equal to

(1) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$

(2) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + C$

(3) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$

(4) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

30. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for n ³ 2 and $g(x) = \underbrace{(f \circ f \circ .... \circ f)}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2} g(x)\, dx$ equals

(1) $\frac{1}{n(n-1)}\left(1 + nx^n\right)^{1-\frac{1}{n}} + K$

(2) $\frac{1}{(n-1)}\left(1 + nx^n\right)^{1-\frac{1}{n}} + K$

(3) $\frac{1}{n(n+1)}\left(1 + nx^n\right)^{1+\frac{1}{n}} + K$

(4) $\frac{1}{(n+1)}\left(1 + nx^n\right)^{1+\frac{1}{n}} + K$

31. Let $F(x)$ be an indefinite integral of $\sin^2 x$.
Statement-1: The function $F(x)$ satisfies $F(x + p) = F(x)$ for all real x.
Statement-2: $\sin^2(x + p) = \sin^2 x$ for all real x.
(1) Statement-1 is False, Statement-2 is False
(2) Statement-1 is True, Statement-2 is True
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

32. Let $I = \int \frac{e^x}{e^{4x}+e^{2x}+1}\,dx$, $J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}\,dx$. Then, for an arbitrary constant C, the value of $J - I$ is equal to
(1) $\frac{1}{2}\, ln\left|\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right| + C$
(2) $\frac{1}{2}\, ln\left|\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right| + C$
(3) $\frac{1}{2}\, ln\left|\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right| + C$
(4) $\frac{1}{2}\, ln\left|\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right| + C$

33. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}\,dx$ equals (for some arbitrary constant K)
(1) $\frac{-1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\} + K$
(2) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\} + K$
(3) $\frac{-1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\} + K$
(4) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\} + K$

34. $\int e^x\left(\ell n(4x+1)+\frac{16}{(4x+1)^2}\right)dx$ is equal to
(1) $e^x\left(\ell n(4x+1)-\frac{4}{4x+1}\right)+C$
(2) $e^x(\ell n(4x+1))+C$
(3) $e^x\left(\ell n(4x+1)+\frac{1}{4x+1}\right)+C$
(4) $e^x\left(\ell n(4x+1)+\frac{4}{4x+1}\right)+C$

35. $\int\left(1+2x^2+\frac{1}{x}\right).e^{x^2-\frac{1}{x}}dx$ is equal to
(1) $(2x + 1)\, e^{\left(x^2-\frac{1}{x}\right)} + C$
(2) $(2x - 1)\, e^{\left(x^2-\frac{1}{x}\right)} + C$
(3) $xe^{\left(x^2-\frac{1}{x}\right)} + C$
(4) $-xe^{\left(x^2-\frac{1}{x}\right)} + C$

36. $\int e^{\sin^2 x}.\sin x(\cos x+\cos^3 x)\,dx$ is equal to
(1) $\frac{1}{2}e^{\sin^2 x}(3-\sin^2 x)+C$
(2) $e^{\sin^2 x}(3\cos^2 x+2\sin^2 x)+C$
(3) $e^{\sin^2 x}(2\cos^2 x+3\sin^2 x)+C$
(4) $\frac{1}{2}e^{\sin^2 x}\left(1-\frac{1}{2}\cos^2 x\right)+C$

37. If $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6} dx = \alpha \ell n \left| \left(\frac{x^\beta}{x^\beta + 1} \right) \right| + C$, then value of a and b are respectively are

(1) $\frac{5}{2}$ and 2 (2) $\frac{2}{5}$ and $\frac{5}{2}$

(3) $\frac{5}{2}$ and $\frac{2}{5}$ (4) 2 and $\frac{5}{2}$

38. $\int x^3 d(\tan^{-1} x)$ is equal to

(1) $\frac{x^2}{2} + \frac{1}{2} \ell n(1 + x^2) + C$

(2) $-\frac{x^2}{2} - \frac{1}{2} \ell n(1 + x^2) + C$

(3) $-\frac{x^2}{2} + \frac{1}{2} \ell n(1 + x^2) + C$

(4) $\frac{x^2}{2} - \frac{1}{2} \ell n(1 + x^2) + C$

39. If $I = \int e^{-x} \ell n(e^x + 1)\, dx$, then I equals

(1) $x + (e^{-x} + 1)\ \ell n(e^x + 1) + C$

(2) $x + (e^{x} + 1)\ \ell n(e^x + 1) + C$

(3) $x - (e^{-x} + 1)\ \ell n(e^x + 1) + C$

(4) none of these

40. $\int \frac{(f(x) g'(x) - f'(x) g(x))}{f(x).g(x)} (\ell n(g(x)) - \ell n(f(x)))\, dx$ is equal to

(1) $\ell n\left(\frac{g(x)}{f(x)}\right) + C$

(2) $\frac{g(x)}{f(x)} \ell n\left(\frac{g(x)}{f(x)}\right) + C$

(3) $\frac{1}{2}\left(\ell n\left(\frac{g(x)}{f(x)}\right)\right)^2 + C$

(4) $\ell n\left(\frac{g(x)}{f(x)}\right)^2 + C$

41. If $f(x) = \int \frac{x^2 + \sin^2 x}{1 + x^2} . \sec^2 x\, dx$ and $f(0) = 0$, then $f(1)$ is equal to

(1) $\frac{\pi}{4} - \tan 1$ (2) $\frac{\pi}{4} - \tan \frac{\pi}{4}$

(3) $\tan 1 - \frac{\pi}{4}$ (4) $\frac{\pi}{4} - 1$

42. $\int e^{\tan^{-1} x} (1 + x + x^2)\, d(\cot^{-1} x)$ is equal to

(1) $-e^{\tan^{-1} x} + C$ (2) $e^{\tan^{-1} x} + C$

(3) $xe^{\tan^{-1} x} + C$ (4) $-xe^{\tan^{-1} x} + C$

43. If $I_n = \int (\ell nx)^n\, dx$, then $I_5 + 5I_4$ is equal to

(1) $\frac{(\ell nx)^5}{x} + C$ (2) $x(\ell nx)^2 + C$

(3) $x(\ell nx)^5 + C$ (4) $x(\ell nx)^4 + C$

44. $\int \frac{\cos x . \operatorname{cosec}^2 x}{(1 + \sin^5 x)^{4/5}} dx$ is equal to

(1) $-(1 + \sin^5 x)^{1/5} + C$

(2) $-\sin x(1 + \sin^5 x)^{1/5} + C$

(3) $-\frac{(1 + \sin^5 x)^{1/5}}{\sin x} + C$

(4) $-\frac{(1 + \sin^{-5} x)^{1/5}}{\sin x} + C$

45. If $f ¢(x) = \frac{1}{\sqrt{x^2 + 1} - x}$ and $f(0) = -\frac{\sqrt{2} + 1}{2}$, then $f(1)$ is equal to

(1) $1 + \sqrt{2}$ (2) 1

(3) $\ell n(1 + \sqrt{2})$ (4) $\ell n\ (\sqrt{\sqrt{2} + 1})$

46. Let $\vec{a} = f(x)\hat{i} + f'(x)\hat{j}$ and $\vec{b} = f''(x)\hat{i} - f'(x)\hat{j}$ where $f(x)$ is differentiable everywhere with $f ¢(x)$ ¹ 0 and $f(0) = 1, f ¢(0) = 2$, if $\vec{a}.\vec{b} = 0$, then $f(x)$ is

(1) $x^2 + 2x + 1$ (2) $2e^{x-1}$

(3) e^{2x} (4) $4e^{x/2-3}$

47. $\int\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)dx$, where $x\hat{I}\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ is equal to

(1) $-4\cos\left(\frac{x}{2}\right)+C$

(2) $4\sin\left(\frac{x}{2}\right)+C$

(3) $\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)+C$

(4) $-\sin\frac{x}{2}+C$

48. $\int\frac{\operatorname{cosec}^2 x-2017}{\cos^{2017}x}dx$ is equal to

(1) $-\frac{\operatorname{cosec} x}{(\cos x)^{2016}}+C$

(2) $\frac{\cot x}{(\cos x)^{2017}}+C$

(3) $-\frac{\cot x}{(\cos x)^{2016}}$

(4) $\frac{\tan x}{(\cos x)^{2017}}+C$

49. If $xf(x) = 3(f(x))^2 + 2$, then $\int\frac{2x^2-12xf(x)+f(x)}{(6f(x)-x)(x^2-f(x))^2}\,dx$ is equal to

(1) $\frac{1}{x^2-f(x)}+C$ (2) $\frac{1}{x^2+f(x)}+C$

(3) $\frac{1}{x+f(x)}+C$ (4) $\frac{1}{x-f(x)}+C$

50. If $I_{m,n}=\int\cos^m x.\sin nx dx$, then $(7I_{4,3}-4I_{3,2})$ is

(1) $-\cos^2 x+C$

(2) $-\cos 3x.\cos^4 x+C$

(3) constant

(4) $\cos 7x-\cos^4 x+C$

51. $\int\sin(2017x).\sin^{2015}x\,dx$ is equal to

(1) $-\frac{1}{2016}\sin(2016x).(\sin x)^{2016}+C$

(2) $\frac{\sin(2016x)(\sin x)^{2016}}{2016}+C$

(3) $\frac{(\sin x)^{2016}\cos(2016x)}{2016}+C$

(4) $\frac{(\sin x)^{2014}\sin(2017x)}{2014}+C$

52. $\int\frac{(x+1)^2}{x(x^2+1)}dx$ is equal to

(1) $\ell n|x|+C$

(2) $\ell n|x|+2\tan^{-1}x+C$

(3) $\ell n\left(\frac{1}{1+x^2}\right)+C$

(4) $\ell n|x(x^2+1)+C$

53. $\int x^{27}(6x^2+5x+4)(1+x+x^2)^6\,dx$ is equal to

(1) $\frac{x^4}{7}.(1+x+x^2)^7$

(2) $\frac{x^{28}(1+x+x^2)^7}{7}+C$

(3) $\frac{x^{28}(1+x+x^2)^7}{28}+C$

(4) $\frac{x^{28}(1+x+x^2)^6}{6}+C$

54. If $\int\frac{\sin\left(\frac{\pi}{4}-x\right)}{2+\sin 2x}dx=A\tan^{-1}(f(x))+B$, where A and B are constants, then the range of $Af(x)$ is

(1) [0, 1]

(2) [–1, 0]

(3) $\left[-\sqrt{2},\sqrt{2}\right]$

(4) [–1, 1]

55. $\int \frac{e^x(x-2)}{x(x^2+e^x)}dx$ is equal to (where $x > 0$)

(1) $\ell n\left(1+\frac{e^x}{x^2}\right)+C$

(2) $\ell n\left(-\frac{1}{2}+\frac{e^x}{x^2}\right)+C$

(3) $\ell n\left(2+\frac{e^x}{x^2}\right)+C$

(4) $\ell n\left(x+\frac{e^x}{x^2}\right)+C$

56. The value of $\int \frac{\ell n\,|x|}{x\sqrt{1+\ell n\,|x|}}\,dx$ equals:

(1) $\frac{2}{3}\sqrt{1+\ell n\,|x|}$ (*ln*½*x*½ - 2) + *C*

(2) $\frac{2}{3}\sqrt{1+\ell n\,|x|}$ (*ln*½*x*½ + 2) + *C*

(3) $\frac{1}{3}\sqrt{1+\ell n\,|x|}$ (*ln x*½ - 2) + *C*

(4) $2\sqrt{1+\ell n\,|x|}$ (3 *ln*½*x*½ - 2) + *C*

57. $\int \sqrt{\frac{1-\cos x}{\cos\alpha-\cos x}}\,dx$, where 0 < a < *x* < p, is equal to

(1) $2\,ln\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right)+c$

(2) $\sqrt{2}\,ln\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right)+c$

(3) $2\sqrt{2}\,ln\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right)+c$

(4) $-2\sin^{-1}\left(\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}}\right)+c$

58. The value of $\int \frac{\cos^3 x}{\sin^2 x+\sin x}\,dx$ is equal to:

(1) *ln* ½sin *x*½ + sin *x* + *C*

(2) *ln* ½sin *x*½ - sin *x* + *C*

(3) - *ln*½sin *x*½ - sin *x* + *C*

(4) - *ln*½sin *x*½ + sin *x* + *C*

59. The value of $2\int \sin x\,.\,\text{cosec}\,4x\;dx$ is equal to

(1) $\frac{1}{2\sqrt{2}}\,ln\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| - \frac{1}{4}\,ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$

(2) $\frac{1}{2\sqrt{2}}\,ln\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + \frac{1}{4}\,ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$

(3) $\frac{1}{2\sqrt{2}}\,ln\left|\frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}\right| - \frac{1}{4}\,ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$

(4) none of these

60. The value of $\int\{1+2\tan x(\tan x+\sec x)\}^{1/2}\,dx$ is equal to

(1) $ln\,|\sec x\,(\sec x-\tan x)|+C$

(2) $ln\,|\text{cosec}\,x\,(\sec x+\tan x)|+C$

(3) $ln\,|\sec x\,(\sec x+\tan x)|+C$

(4) $ln\,|(\sec x+\tan x)|+C$

61. The value of $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}\,dx$ is:

(1) $\frac{1}{2}\sin 2x+C$ (2) $-\frac{1}{2}\sin 2x+C$

(3) $-\frac{1}{2}\sin x+C$ (4) $-\sin^2 x+C$

62. $\int \frac{x^3-1}{x^3+x}\,dx$ is equal to

(1) $x-ln\,|x|+ln\,(x^2+1)-\tan^{-1}x+C$

(2) $x-ln\,|x|+\frac{1}{2}\,ln\,(x^2+1)-\tan^{-1}x+C$

(3) $x+ln\,|x|+\frac{1}{2}\,ln\,(x^2+1)+\tan^{-1}x+C$

(4) None of these

63. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ is equal to

(1) $\sqrt{x}\,\sqrt{1-x} - 2\sqrt{1-x} + \cos^{-1}\left(\sqrt{x}\right) + c$

(2) $\sqrt{x}\,\sqrt{1-x} + 2\sqrt{1-x} + \cos^{-1}\left(\sqrt{x}\right) + c$

(3) $\sqrt{x}\,\sqrt{1-x} - 2\sqrt{1-x} - \cos^{-1}\left(\sqrt{x}\right) + c$

(4) $\sqrt{x}\,\sqrt{1-x} + 2\sqrt{1-x} - \cos^{-1}\left(\sqrt{x}\right) + c$

64. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}\,dx$ is equal to

(1) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$

(2) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{-1/4} + C$

(3) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$

(4) $\frac{1}{3}\left(\frac{x+1}{x-1}\right)^{1/4} + C$

65. Primitive of $\frac{3x^4-1}{\left(x^4+x+1\right)^2}$ *w.r.t.* x is

(1) $\frac{x}{x^4+x+1} + C$

(2) $\frac{x}{x^4+x^2+1} + C$

(3) $\frac{x^2}{x^4+x+1} + C$

(4) None of these

66. The value of $\int \frac{1+x^4}{(1-x^4)^{3/2}}\,dx$ is equal to

(1) $\frac{2}{\sqrt{\frac{1}{x^2}-x^2}} - C$ (2) $\frac{1}{\sqrt{\frac{1}{x^2}-x^2}} + C$

(3) $\frac{1}{\sqrt{\frac{1}{x^2}+x^2}} + C$ (4) $\frac{1}{\sqrt{\frac{1}{x^2}-x}} + C$

Integer Type Questions (67 to 74)

67. If $f(x) = \frac{(2x+(1+x^2)3x^4)e^{x^3}}{1+2x^2+x^4}$, $g(x) = \frac{x^2e^{x^3}}{x^2+1}$ and $\int f(x)dx = g(x) + f(x)$, $f(0) = 1$, then $\phi(2)$ ______

68. If $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}\,dx = a\cot^{-1}(b\tan^2 x) + C$, then: $a + b =$ ______________

69. If $\int \frac{1}{(x+2)(x^2+1)}\,dx = a\,ln\,(1+x^2) + b\tan^{-1}x + g\,\log_e|x+2| + C$, then $\frac{1}{\alpha+\beta+\gamma} =$ ____________

70. If $\int \frac{1}{\cos^3 x\sqrt{2\sin 2x}}\,dx = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then $5(A + B + C)$ is equal to

71. $\int \frac{1}{5+4\cos x}\,dx = a\tan^{-1}\left(b\tan\frac{x}{2}\right) + C$, then, $a + b =$ _______

72. If $\int x^{-11}(1+x^4)^{-1/2}\,dx = \frac{t^5}{a} - \frac{t^3}{b} - \frac{t}{c} + k$, where $t = \sqrt{1+x^{-4}}$ and k is constant of integration, then $(c - b - a)$ is equal to

73. Let $g(x)$ be the primitive of $\frac{3x+2}{\sqrt{x-9}}$ with respective to x. If $g(13) = 132$, then the value of $g(10)$ is

74. If

$$\int \frac{1}{(1+\sqrt{x})^{2017}}\,dx = 2\left(\frac{1}{\alpha(1+\sqrt{x})^{\alpha}} - \frac{1}{\beta(1+\sqrt{x})^{\beta}}\right) + C$$

, where a, b > 0, then a – b is equal to

CHAPTER 19

DEFINITE INTEGRATION AND APPLICATION OF INTEGRALS

Single Option Correct Type Questions (01 to 60)

1. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$, equals

(1) $\frac{2}{\ell n3}(3^{\sqrt{2}}-1)$ (2) 0

(3) $\frac{2\sqrt{2}}{\ell n3}$ (4) $\frac{3^{\sqrt{2}}}{\sqrt{2}}$

2. $\int_0^{\pi/4} \frac{x \,.\, \sin x}{\cos^3 x}\,dx$ equals to

(1) $\frac{\pi}{4}+\frac{1}{2}$ (2) $\frac{\pi}{4}-\frac{1}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{4}+1$

3. $\int_{\ell n\pi-\ell n2}^{\ell n\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}\,dx$ is equal to

(1) $\sqrt{3}$ (2) $-\sqrt{3}$

(3) $\frac{1}{\sqrt{3}}$ (4) $-\frac{1}{\sqrt{3}}$

4. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ equals

(1) $e\left(\frac{e}{2}-1\right)$ (2) 1

(3) $e(e-1)$ (4) $\frac{e}{2}$

5. If $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^\infty e^{-ax^2}\,dx$ where $a > 0$ is

(1) $\frac{\sqrt{\pi}}{2}$ (2) $\frac{\sqrt{\pi}}{2a}$

(3) $2\frac{\sqrt{\pi}}{a}$ (4) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

6. If $I_1 = \int_e^{e^2} \frac{dx}{\ell n\ x}$ and $I_2 = \int_1^2 \frac{e^x}{x}\,dx$, then

(1) $I_1 = I_2$ (2) $2I_1 = I_2$

(3) $I_1 = 2I_2$ (4) $2I_1 = 3I_2$

7. The value of $\frac{6}{\pi}\int_{\pi/6}^{\pi/3} [2\sin x]\,dx$, where [.] represents greatest integer function is

(1) 1 (2) 2

(3) 3 (4) 0

8. $\int_0^{1.5} [x^2]\,dx$, where [.] denotes the greatest integer function, is equal to

(1) $\sqrt{2}-2$

(2) $2-\sqrt{2}$

(3) $2+\sqrt{2}$

(4) $\frac{1}{\sqrt{2}}$

9. Let $f : R \to R$, $g : R \to R$ be continuous functions. Then the value of integral

$$\int_{\ell n\lambda}^{\ell n 1/\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x)-f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x)+g(-x)]}\,dx \text{ is}$$

(1) depend on λ
(2) a non-zero constant
(3) zero
(4) 1

10. $\displaystyle\int_{2-\log 3}^{3+\log 3} \frac{\log\ (4+x)}{\log(4+x)+\log(9-x)}\,dx$ is equal to

(1) cannot be evaluated (2) $\frac{5}{2}$
(3) $1 + 2\log 3$ (4) $\frac{1}{2} + \log 3$

11. $\displaystyle\int_{-2}^{10} \text{sgn}\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right) dx$ equals ([.] denotes greatest integer function)

(1) 10 (2) 11
(3) 9 (4) 12

12. $\displaystyle\int_{0}^{2n\pi} \left(|\sin x| - \left[\left|\frac{\sin x}{2}\right|\right]\right) dx$ (where [ ] denotes the greatest integer function and $n \in I$) is equal to :

(1) 0 (2) $2n$
(3) $2n\pi$ (4) $4n$

13. $\displaystyle\lim_{h \to 0} \frac{\int_a^{x+h} \ell n^2 t\ dt - \int_a^{x} \ell n^2 t\ dt}{h}$ equals to

(1) 0 (2) $ln^2 x$
(3) $\frac{2\ell n x}{x}$ (4) does not exist

14. The value of the function $f(x) = 1 + x + \int_1^x (ln^2 t + 2\ lnt)\,dt$, where $f'(x)$ vanishes is

(1) e^{-1} (2) 0
(3) $2e^{-1}$ (4) $1+2e^{-1}$

15. The value of $\displaystyle\lim_{x\to 0} \frac{\frac{d}{dx}\int_0^{x^3}\sqrt{\cos t}\ dt}{1-\sqrt{\cos x}}$ is

(1) 0 (2) 11
(3) 10 (4) 12

16. If $I = \displaystyle\int_0^1 \frac{\tan x}{\sqrt{x}}\,dx$ then

(1) $I < \frac{2}{3}$ (2) $I > \frac{2}{3}$
(3) $I < \frac{5}{9}$ (4) $I < \frac{1}{3}$

17. The area bounded by the curve $xy = 4$ and the line $x + y = 5$ is

(1) $\frac{15}{2} + \ell n4$ (2) $\frac{15}{2} + \ell n2$
(3) $\frac{15}{2} - 4\ell n4$ (4) $\frac{15}{2} - \ell n2$

18. The area bounded by the curves $y = \sin x$, $y = \cos x$ and y-axis in I[st] quadrant is

(1) $\sqrt{2}$ (2) $\sqrt{2}+1$
(3) $\sqrt{2}-1$ (4) $\sqrt{2}+2$

19. The area bounded in the first quadrant between the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $3x + 4y = 12$ is

(1) $6(\pi - 1)$ (2) $3(\pi - 2)$
(3) $3(\pi - 1)$ (4) $2(\pi - 2)$

20. The value of the integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$ where $0<\alpha<\frac{\pi}{2}$, is equal to

(1) $\sin\alpha$ (2) $\alpha\sin\alpha$

(3) $\frac{\alpha}{2\sin\alpha}$ (4) $\frac{\alpha}{2}\sin\alpha$

21. The value of $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}\,dx$ is

(1) $\frac{4+\pi}{4\sqrt{2}}$ (2) $\frac{4-\pi}{4\sqrt{2}}$

(3) $\frac{\pi}{2}$ (4) $-\frac{\pi}{2}$

22. The value of $\int_{1/e}^{\tan x} \frac{t}{1+t^2}\,dt + \int_{1/e}^{\cot x} \frac{1}{t(1+t^2)}dt$, where $x \in (\pi/6, \pi/3)$, is equal to

(1) 0

(2) 2

(3) 1

(4) cannot be determined

23. If

$$f(x) = \begin{cases} 0 \;, & where\;\; x=\frac{n}{n+1},\; n=1,\,2,\,3..... \\ 1 \;, & else\;\; where \end{cases}$$

, then the value of $\int_0^2 f(x)\,dx$ is

(1) 1 (2) 0

(3) 2 (4) ∞

24. If $\int_0^{100} f(x)\,dx = a$,

then $\sum_{r=1}^{100}\left(\int_0^1 f\,(r-1+x)\,dx\right) =$

(1) 100 a (2) a

(3) 0 (4) 10 a

25. $\int_0^{\infty}\left[2e^{-x}\right]dx$ where, [.] denotes the greatest integer function, is equal to

(1) 0 (2) $ln\,2$

(3) e^2 (4) $2e^{-1}$

26. $\int_{-\pi/2}^{\pi/2} \frac{|x|\,dx}{8\cos^2 2x+1}$ has the value

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{12}$

(3) $\frac{\pi^2}{24}$ (4) $\frac{\pi}{12}$

27. $\int_0^{\infty} \frac{\ell n(1+x^2)}{1+x^2}\,dx$ equals

(1) $\pi\,ln\,2$ (2) $-\pi\,ln\,2$

(3) $\frac{\pi}{2}\ell n2$ (4) $-\frac{\pi}{2}\ell n2$

28. If $I_1 = \int_0^{\pi} x\;f(\sin^3 x+\cos^2 x)\;dx$ and

$I_2 = \pi\int_0^{\pi/2} f(\sin^3 x+\cos^2 x)\;dx$ then

(1) $I_1 = I_2$ (2) $I_1+I_2=0$

(3) $I_1 = 2I_2$ (4) $2I_1 = I_2$

29. The value of $\int_{-1}^{2}\{2x\}\;dx$ is (where function {.} denotes fractional part function)

(1) 3 (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) $\frac{1}{2}$

30. If $f(x) = \int_{x^2}^{x^3}\frac{dt}{\ell nt}$. $x > 0$ then

(1) $f'(x) = -\frac{1}{6\ell nx}$

(2) f is an increasing function on $(0, \infty)$

(3) f has minimum at $x = 1$

(4) f is a decreasing function on $[0, \infty)$

31. The area of the closed figure bounded by $y = x$, $y = -x$ & the tangent to the curve $y = \sqrt{x^2 - 5}$ at the point (3, 2) is:

(1) 5 (2) $\frac{15}{2}$

(3) 10 (4) $\frac{35}{2}$

32. The area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1)\sin(3b + 4)$, $b \in R$, then $f(x) =$

(1) $(x - 1)\cos(3x + 4)$

(2) $\sin(3x + 4)$

(3) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$

(4) $\cos(3x + 4)$

33. The line $y = mx$ bisects the area enclosed by the curve $y = 1 + 4x - x^2$ & the lines $x = \frac{3}{2}$, $x = 0$ & $y = 0$. Then the value of m is:

(1) $\frac{13}{6}$ (2) $\frac{6}{13}$

(3) $\frac{3}{2}$ (4) 4

34. The area bounded by $x^2 + y^2 - 2x = 0$ & $y = \sin\frac{\pi x}{2}$ in the upper half of the circle is

(1) $\frac{\pi}{2} - \frac{4}{\pi}$ (2) $\frac{\pi}{4} - \frac{2}{\pi}$

(3) $\pi - \frac{8}{\pi}$ (4) $\frac{\pi}{2} + \frac{4}{\pi}$

35. The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is:

(1) $\frac{4 + 3\ \ell n\ 3}{2}$ (2) $\frac{4 - 3\ \ell n\ 3}{2}$

(3) $\frac{3}{2} + \ell n\ 3$ (4) $\frac{1}{2} + \ell n\ 3$

36. Area bounded by $y = x^3 - x$ and $y = x^2 + x$ is

(1) $\frac{37}{24}$ (2) $\frac{37}{12}$

(3) $\frac{11}{24}$ (4) $\frac{37}{23}$

37. The area bounded by the curve $y = 2x^4 - x^2$, x-axis and the two ordinates corresponding to the minima of the function is

(1) $\frac{3}{120}$ (2) $\frac{5}{120}$

(3) $\frac{1}{20}$ (4) $\frac{7}{120}$

38. **STATEMENT-1:** If {.} represents fractional part function, then $\int_0^{5.5} \{x\}\ dx = \frac{21}{8}$

STATEMENT-2: If [.] and {.} represent greatest integer and fractional part functions respectively, then

$$\int_0^t \{x\}\ dx = \frac{[t]}{2} + \frac{\{t\}^2}{2}$$

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(4) Statement-1 is true, statement-2 is false.

39. $\int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2(x + 3\pi)]\, dx$ is equal to

(1) $\left(\frac{\pi^4}{32}\right) + \left(\frac{\pi}{2}\right)$ (2) $\frac{\pi}{2}$

(3) $\left(\frac{\pi}{4}\right) - 1$ (4) $\frac{\pi^4}{32}$

40. Match the column :

	Column- I		Column-II
I	$\int_{-1}^{1} \frac{dx}{1+x^2} =$	P	$\frac{1}{2} ln \left(\frac{2}{3}\right)$
II	$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}} =$	Q	$2 \, ln \left(\frac{2}{3}\right)$
III	$\int_{2}^{3} \frac{dx}{1-x^2} =$	R	$\frac{\pi}{3}$
IV	$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}} =$	S	$\frac{\pi}{2}$

(1) (I)-(S); (II)-(S); (III)-(P); (IV)-(R)
(2) (I)-(R); (II)-(P); (III)-(Q); (IV)-(S)
(3) (I)-(R); (II)-(S); (III)-(Q); (IV)-(S)
(4) (I)-(S); (II)-(R); (III)-(P); (IV)-(Q)

41. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, $f(x) = \int_1^x \frac{\log t}{1+t} dt$.

Then $F(e)$ equals

(1) $\frac{1}{2}$ (2) 0
(3) 1 (4) 2

42. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then, which one of the following is true

(1) $I > \frac{2}{3}$ and > 2
(2) $I < \frac{2}{3}$ and $J < 2$
(3) $I < \frac{2}{3}$ and $J > 2$
(4) $I > \frac{2}{3}$ and $J < 2$

43. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(1) $\frac{5}{3}$ sq unit (2) $\frac{1}{3}$ sq unit
(3) $\frac{2}{3}$ sq unit (4) $\frac{4}{3}$ sq unit

44. $\int_0^{\pi} [\cot x] dx$, where $[\,\cdot\,]$ denotes the greatest integer function, is equal to

(1) 1 (2) – 1
(3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$

45. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(1) $4\sqrt{2}+2$ (2) $4\sqrt{2}-1$
(3) $4\sqrt{2}+1$ (4) $4\sqrt{2}-2$

46. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t} \sin t \, dt$.

Then f has

(1) local maximum at π and 2π.
(2) local minimum at π and 2π
(3) local minimum at π and local maximum at 2π.
(4) local maximum at π and local minimum at 2π.

47. Let [.] denote the greatest integer function then the value of $\int_0^{1.5} x\,[x^2]\,dx$ is

(1) 0 (2) $\frac{3}{2}$
(3) $\frac{3}{4}$ (4) $\frac{5}{4}$

48. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is

(1) $\frac{1}{2}$ square units (2) 1 square units

(3) $\frac{3}{2}$ square units (4) $\frac{5}{2}$ square units

49. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant

(1) 9 (2) 36 (3) 18 (4) $\frac{27}{4}$

50. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\, dx$ equals

(1) $4\sqrt{3}-4$ (2) $4\sqrt{3}-4-\frac{\pi}{3}$

(3) $\pi - 4$ (4) $\frac{2\pi}{3}-4-4\sqrt{3}$

51. The area of the region described by $A = \{(x, y): x^2 + y^2 \le 1$ and $y^2 \le 1 - x\}$ is

(1) $\frac{\pi}{2}-\frac{2}{3}$ (2) $\frac{\pi}{2}+\frac{2}{3}$

(3) $\frac{\pi}{2}+\frac{4}{3}$ (4) $\frac{\pi}{2}-\frac{4}{3}$

52. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$ is equal to

(1) 2 (2) 4 (3) 1 (4) 6

53. The area (in sq. units) of the region described by $\{(x, y); y^2 \le 2x$ and $y \ge 4x - 1\}$ is

(1) $\frac{7}{32}$ (2) $\frac{5}{64}$

(3) $\frac{15}{64}$ (4) $\frac{9}{32}$

54. The area (in sq. units) of the region $\{(x, y) : y^2 \ge 2x$ and $x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is

(1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$

(3) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$

55. The area (in sq. units) of the region $\{(x, y): x \ge 0, x + y \le 3, x^2 \le 4y$ and $y \le 1 + \sqrt{x}\}$ is

(1) $\frac{3}{2}$ (2) $\frac{59}{12}$

(3) $\frac{7}{3}$ (4) $\frac{5}{2}$

56. $\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\, dt}{x^2-\frac{\pi^2}{16}}$ equals

(1) $\frac{8}{\pi} f(2)$ (2) $\frac{2}{\pi} f(2)$

(3) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ (4) $4f(2)$

57. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is (are)

(1) $\frac{22}{7}-\pi$ (2) $\frac{2}{105}$

(3) 0 (4) $\frac{71}{15}-\frac{3\pi}{2}$

58. Let f be a real-valued function defined on the interval $(-1, 1)$ such that

$e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt$, for all

$x \in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

(1) 1 (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{e}$

59. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

(1) $4\left(\sqrt{2}-1\right)$ (2) $2\sqrt{2}\left(\sqrt{2}-1\right)$

(3) $2\left(\sqrt{2}+1\right)$ (4) $2\sqrt{2}\left(\sqrt{2}+1\right)$

60. The following integral $\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17}\, dx$ is equal to

(1) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16}\, du$

(2) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17}\, du$

(3) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17}\, du$

(4) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16}\, du$

Integer Type Questions (61 to 75)

61. If $\alpha = \int_0^1 \left(e^{9x+3\tan^{-1}x}\right)\left(\frac{12+9x^2}{1+x^2}\right)dx$ where $\tan^{-1} x$ takes only principal values, then the value of $\left(\log_e|1+\alpha| - \frac{3\pi}{4}\right)$ is

62. Let $f: R \to R$ be a function defined by $f(x) = \begin{cases} [x], & x \le 2 \\ 0, & x > 2 \end{cases}$ where $[x]$ is the greatest integer less than or equal to x. If $I = \int_{-1}^{2} \frac{xf(x^2)}{2+f(x+1)}\, dx$, then the value of $(4I-1)$ is

63. Area of the region $\left\{(x, y) \in R^2 : y \ge \sqrt{|x+3|}, 5y \le x+9 \le 15\right\}$ is equal to k, then value of $2k$ is

64. $\int_0^{\pi/2} \sqrt{1+\sin 2x}$ dx equals

65. The value of $\int_{-\tan 1}^{0} \left[-\tan^{-1} x\right] dx$, where [.] represents greatest integer function is

66. The value of $\int_{-1}^{3} (|x-2| + [x])\, dx$ is ($[x]$ stands for greatest integer less than or equal to x)

67. The value of $\int_{-1}^{1} \frac{\cot^{-1} x}{\pi}\, dx$ is

68. If $\int_0^{11} \frac{11^x}{11^{[x]}}\, dx = \frac{k}{\log 11}$, (where [·] denotes greatest integer function) then value of k is

69. The value of $\int_0^{10\pi} (|\sin x| + |\cos x|)\, dx$ is

70. The area bounded by curve $y = e^x$, $y = 1$, $y = 3$ and y-axis is $\lambda \ell n 3 + \mu$, $\lambda, \mu \in I$ then $\lambda + \mu$ =

71. If $[x]$ stands for the greatest integer function, the value of $\int_4^{10} \frac{[x^2]}{[x^2 - 28x + 196] + [x^2]}\, dx$ is

72. $\int_{-1}^{1} \frac{e^x + 1}{e^x - 1}\, dx$ equals

73. Area enclosed by the curve $|x - 2| + |y + 1| = 1$ is equal to (in sq. unit)

74. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t\, dt}{x \sin x}$, is equal to

75. The equation of a curve is given by $y = f(x)$, where $f'(x)$ is a continuous function. The tangent at points $(1, f(1))$, $(2, f(2))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$, $\frac{\pi}{3}$ and $\frac{\pi}{4}$ respectively with positive x-axis. Then $\int_2^3 f'(x)f''(x)\, dx + \int_1^3 f''(x)\, dx$, is equal to $-\frac{k}{\sqrt{3}}$, then the value of k is

CHAPTER

20

DIFFERENTIAL EQUATIONS

Single Option Correct Type Questions (01 to 64)

($c, C, d, k, A_1, A_2, A_3$ are constants)

1. If $\frac{dy}{dx} + \frac{2y}{x} = 0$, $y(1) = 1$, then $y(2) =$

(1) $\frac{1}{4}$ (2) 4 (3) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

2. Solution of the differential equation $\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0$ is

(1) $\tan^{-1} y + \sin^{-1} x = c$
(2) $\tan^{-1} x + \sin^{-1} y = c$
(3) $\tan^{-1} y . \sin^{-1} x = c$
(4) $\tan^{-1} x - \sin^{-1} y = c$

3. Solution of $y - \frac{xdy}{dx} = y^2 + \frac{dy}{dx}$ is

(1) $(2x + 1)(1 + y) = c$
(2) $(1 + x)(1 + y) = cx^2$
(3) $(x + 1)(1 - y) = cy$
(4) $x - y + x^2y - 1 = c$

4. If $(x^2 + y^2)\, dy = xydx$ and $y(1) = 1$ and $y(x_0) = e$, then $x_0 =$

(1) $3e$ (2) $\sqrt{2}e$ (3) $\sqrt{3}$ (4) $\sqrt{3}\ e$

5. The solution of $(x + y + 1)\, dy = dx$ is

(1) $x + y + 2 = Ce^y$
(2) $x + y + 4 = C \ln y$
(3) $\ln(x + y + 2) = Cy$
(4) $\ln(x - y + 2) = C + y$

6. If $y(t)$ is solution of $(t + 1)\frac{dy}{dt} - ty = 1$, $y(0) = -1$, then $y(1) =$

(1) $\frac{1}{4}$ (2) -2 (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

7. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx} + f(x)\, y = 0$, then a solution of differential equation $\frac{dy}{dx} + f(x)\, y = r(x)$ is

(1) $\frac{1}{y\ (x)} \int y_1(x)\ dx$

(2) $y_1(x) \int \frac{r(x)}{y_1(x)}\ dx$

(3) $\int r(x) y_1(x)\ dx$

(4) $\int (r(x))^2 y_1(x)\ dx$

8. If $y_1(x)$ and $y_2(x)$ are two solutions of $\frac{dy}{dx} + f(x)\, y = r(x)$ then $y_1(x) + y_2(x)$ is solution of

(1) $\frac{dy}{dx} + f(x)\, y = 0$

(2) $\frac{dy}{dx} + 2f(x)\, y = r(x)$

(3) $\frac{dy}{dx} + f(x)\, y = 2\, r(x)$

(4) $\frac{dy}{dx} + 2f(x)\, y = 2r(x)$

9. Solution of differential equation $f(x)\dfrac{dy}{dx} = f^2(x) + f(x)\,y + f'(x)y$ is

(1) $y = f(x) + ce^x$

(2) $y = -f(x) + ce^x$

(3) $y = -f(x) + ce^x f(x)$

(4) $y = cf(x) + e^x$

10. Solution of $(2x - 10y^3)\dfrac{dy}{dx} + y = 0$ is

(1) $xy^2 = 2y^5 + c$ (2) $x = 10y^3 + cy^2$

(3) $x = 10y^3 + cy$ (4) $xy = 2y^5 + c$

11. Solution of $\sin y \dfrac{dy}{dx} = \cos y\,(1 - x\cos y)$ is

(1) $\sec y = x + 1 + ce^{-x}$

(2) $\sec y = x + 1 + ce^{x}$

(3) $\cos y = x + 1 + ce^{x}$

(4) $\tan y = 1 + ce^{-x}$

12. Solution of $\sec^2 y\,dy + \tan y\,dx = dx$ is

(1) $\cot y = e^x + 2x + c$

(2) $\sec y = x + 1 + ce^x$

(3) $\tan y = e^x$

(4) $\tan y = 1 + ce^{-x}$

13. If $xdy = y(dx + ydy)$, $y(1) = 1$ and $y(x_0) = -3$, then $x_0 =$

(1) $\dfrac{1}{4}$ (2) -15

(3) $-\dfrac{1}{2}$ (4) $\sqrt{3}\ e$

14. The general solution of $(2x^3 - xy^2)\,dx + (2y^3 - x^2y)\,dy = 0$ is

(1) $x^4 + x^2y^2 - y^4 = c$

(2) $x^4 - x^2y^2 + y^4 = c$

(3) $x^4 - x^2y^2 - y^4 = c$

(4) $x^4 + x^2y^2 + y^4 = c$

15. General solution of the differential equation $\dfrac{xdy}{x^2+y^2} + \left(1 - \dfrac{y}{x^2+y^2}\right)dx = 0$ is

(1) $x + \tan^{-1}\left(\dfrac{y}{x}\right) = c$

(2) $x + \tan^{-1}\dfrac{x}{y} = c$

(3) $x - \tan^{-1}\left(\dfrac{y}{x}\right) = c$

(4) $2x - 3\tan^{-1}\left(\dfrac{y}{x}\right) = c$

16. General solution of the differential equation $e^y\,dx + (xe^y - 2y)\,dy = 0$ is

(1) $xe^y - y^2 = c$ (2) $ye^x - x^2 = c$

(3) $ye^y + x = c$ (4) $xe^y - 1 = cy^2$

17. **Statement -1:** The solution of *D.E.* $\dfrac{\frac{xdy}{dx} - y}{\sqrt{x^2 - y^2}} = mx^2$ is given by $\tan^{-1}\dfrac{y}{x} = \dfrac{mx^2}{2} + c$

Statement -2: The solution of differential equation $\dfrac{dy}{dx} + \dfrac{y}{x} = \sin x$ is $x(y + \cos x) = \sin x + c$

(1) Statement-1 and Statement-2 both are True

(2) Statement-1 and Statement-2 both are false

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

18. A curve passes through (2, 0) and slope at point $P(x, y)$ is $\dfrac{(x+1)^2 + (y-3)}{(x+1)}$. The area between curve and x-axis in 4th quadrant is

(1) 2 / 3 (2) 1 / 3

(3) 2 (4) 4 / 3

19. The solution of the equation $\dfrac{d^2y}{dx^2} = e^{-2x}$ is

(1) $\dfrac{e^{-2x}}{4}$ (2) $\dfrac{e^{-2x}}{4} + cx + d$

(3) $\dfrac{1}{4}e^{-2x} + cx^2 + d$ (4) $\dfrac{1}{4}e^{-2x} + c + d$

20. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$, is

(1) $(x - 2) = k e^{\tan^{-1} y}$

(2) $2x e^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

(3) $x\, e^{\tan^{-1} y} = \tan^{-1} y + k$

(4) $x\, e^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

21. The solution of the differential equation $y\, dx + (x + x^2 y)\, dy = 0$ is

(1) $-\frac{1}{xy} = c$

(2) $-\frac{1}{xy} + ln\,|y| = c$

(3) $\frac{1}{xy} + ln\,|y| = c$

(4) $ln\,|y| = cx$

22. If $x\frac{dy}{dx} = y(ln\, y - ln\, x + 1)$, then the solution of the equation is

(1) $ln\left(\frac{x}{y}\right) = cy$ (2) $ln\left(\frac{y}{x}\right) = cx$

(3) $x\, ln\left(\frac{y}{x}\right) = cy$ (4) $y\, ln\left(\frac{x}{y}\right) = cx$

23. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

(1) $y = \log x + x$ (2) $y = x \log x + x^2$

(3) $y = xe^{(x-1)}$ (4) $y = x \log x + x$

24. Solution of the differential equation $\cos x\, dy = y(\sin x - y)\, dx$, $0 < x < \frac{\pi}{2}$ is

(1) $y \sec x = \tan x + c$

(2) $y \tan x = \sec x + c$

(3) $\tan x = (\sec x + c)y$

(4) $\sec x = (\tan x + c)\, y$

25. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(1) $T^2 - \frac{1}{k}$ (2) $I - \frac{kT^2}{2}$

(3) $I - \frac{k(T-t)^2}{2}$ (4) e^{-kT}

26. Consider the differential equation $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If $y(1) = 1$, then x is given by

(1) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$ (2) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

(3) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$ (4) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

27. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\, p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is :

(1) $2\, ln\, 18$ (2) $ln\, 9$

(3) $\frac{1}{2}\, ln\, 18$ (4) $ln\, 18$

28. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P *w.r.t.* additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(1) 2500 (2) 3000

(3) 3500 (4) 4500

29. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals

(1) $600 - 500\, e^{t/2}$ (2) $400 - 300\, e^{-t/2}$
(3) $400 - 300\, e^{t/2}$ (4) $300 - 200\, e^{-t/2}$

30. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\, dx = x dy$, then $f\left(-\frac{1}{2}\right)$ is equal to

(1) $-\frac{4}{5}$ (2) $\frac{2}{5}$ (3) $\frac{4}{5}$ (4) $-\frac{2}{5}$

31. Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0,\pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $-\frac{8}{9}\pi^2$ (2) $-\frac{4}{9}\pi^2$
(3) $\frac{4}{9\sqrt{3}}\pi^2$ (4) $\frac{-8}{9\sqrt{3}}\pi^2$

32. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x)$ is

(1) $\frac{1}{3x} + \frac{2x^2}{3}$ (2) $\frac{-1}{3x} + \frac{4x^2}{3}$
(3) $\frac{-1}{x} + \frac{2}{x^2}$ (4) $\frac{1}{x}$

33. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(1) variable radii and a fixed centre at (0, 1)
(2) variable radii and a fixed centre at (0, –1)
(3) fixed radius 1 and variable centres along the x-axis
(4) fixed radius 1 and variable centres along the y-axis

34. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1-x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

(1) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (2) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
(3) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

35. The order and degree of the differential equation $\frac{d^3y}{dx^3} = \left\{2 + \left(\frac{d^4y}{dx^4}\right)^3\right\}^{5/2}$ are respectively

(1) 4, 1 (2) 3, 3 (3) 4, 15 (4) 4, 3

36. The degree of the differential equation $xy + \frac{d^3y}{dx^3} = \sin\left(\frac{d^4y}{dx^4}\right)$ is

(1) 3 (2) 4
(3) 0 (4) Not defined

37. The general solution of the differential equation $\frac{dy}{dx} = \cot x . \cot y$ is

(1) $\cos x = c \operatorname{cosec} y$ (2) $\sin x = c \sec y$
(3) $\sin x = c \cos y$ (4) $\cos x = c \sin y$

38. The solution of the differential equation $\frac{dy}{dx} = \frac{y^2 - y - 2}{x^2 + 2x - 3}$ is

(1) $\frac{1}{3}\ell n\left|\frac{y-2}{y+1}\right| = \frac{1}{2}\ell n\left|\frac{x+3}{x-1}\right| + c$

(2) $\frac{1}{3}\ell n\left|\frac{y+1}{y-2}\right| = \frac{1}{4}\ell n\left|\frac{x-1}{x+3}\right| + c$

(3) $2\ell n\left|\frac{y+1}{y-2}\right| = 3\ell n\left|\frac{x+3}{x-1}\right| + c$

(4) $4\ell n\left|\frac{y-2}{y+1}\right| = 3\ell n\left|\frac{x-1}{x+3}\right| + c$

39. The solution of differential equation $e^{dy/dx} = x + 1, y(0) = 3$ is

(1) $y = x\ ln\ |x| - x + 2$
(2) $y = (x + 1)\ ln\ |x + 1| - x + 3$
(3) $y = x\ ln\ |x| + x + 3$
(4) $y = -(x + 1)\ ln\ |x + 1| + x + 3$

40. The general solution of the differential equation $\ell n\left(\frac{dy}{dx}\right) = x + y$ is

(1) $e^{-x} + e^{-y} = c$ (2) $e^{x} + e^{-y} = c$
(3) $e^{x} + e^{y} = c$ (4) $e^{-x} + e^{y} = c$

41. The solution of the differential equation $\frac{dy}{dx} = (2x+y)^2$ is

(1) $\frac{1}{2\sqrt{2}}\ell n\left|\frac{2x+y-\sqrt{2}}{2x+y-\sqrt{2}}\right| = x+c$

(2) $\frac{1}{2\sqrt{2}}\ell n\left|\frac{2x+y+\sqrt{2}}{2x+y-\sqrt{2}}\right| = x+c$

(3) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{(2x+y)}{\sqrt{2}} = x+c$

(4) $\tan^{-1}\frac{(2x+y)}{\sqrt{2}} = x+c$

42. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{x+y-3}$ is

(1) $x = ce^{y} - y + 2$ (2) $y = x + ce^{y} - 2$
(3) $x + ce^{-y} - y = 5$ (4) $x + ce^{y} = y + 3$

43. The solution of the differential equation $\left(2\sqrt{xy} - x\right)\ dy + ydx = 0$ is

(1) $\ell ny + \sqrt{x/y} = c$
(2) $e^{y} = \sqrt{x/y} + c$
(3) $\ell ny = \sqrt{x/y} + c$
(4) $e^{y} + \sqrt{x/y} = c$

44. The solution of differential equation $\left(x\sin\frac{y}{x}\right)dy = \left(y\sin\frac{y}{x} - 2x\right)dx$ is

(1) $\sin\left(\frac{y}{x}\right) = 2\ell n\,|x| + c$
(2) $\cos\left(\frac{y}{x}\right) = \ell n\,|x| + c$
(3) $\sin\left(\frac{y}{x}\right) = \ell n\,|x| + c$
(4) $\cos\left(\frac{y}{x}\right) = 2\ell n\,|x| + c$

45. The general solution of $y^2dx + (x^2 - xy + y^2)dy = 0$

(1) $\tan^{-1}\left(\frac{x}{y}\right) + \ell ny + c = 0$
(2) $2\tan^{-1}\left(\frac{x}{y}\right) + \ell ny + c = 0$
(3) $\ell n\left(y + \sqrt{x^2 + y^2}\right) + \ell ny = c$
(4) $\sin^{-1}\left(\frac{x}{y}\right) + \ell ny = c$

46. The solution of differential equation $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$ is

(1) $2xe^{-y} = cx^2 - 1$ (2) $2xe^{y} = x^2 + c$
(3) $2xe^{-y} = cx^2 + 1$ (4) $xe^{-y} = cx^2 + 1$

47. The solution of the differential equation, $\frac{dy}{dx} = \frac{y}{2y\ell ny + y - x}$ is

(1) $xy = y^3 ln\ y + c$ (2) $xy = y^2 ln\ y + c$
(3) $xy = y^5 ln\ y + c$ (4) $xy = y^4 ln\ y + c$

48. The solution of the differential equation $\frac{dy}{dx} = \frac{-(y+y^3)}{1+x+xy^2}$ is

(1) $xy + \frac{1}{2}\ ln\left|\frac{1-y}{1+y}\right| = c$ (2) $xy + \tan^{-1}y = c$
(3) $y = x + \tan^{-1}y + c$ (4) $xy + \sin^{-1}y = c$

49. The solution of the differential equation $\frac{dx}{dy}+\frac{x}{y}=x^3$ is

(1) $2xy^2+cx^2y^2=1$
(2) $2x^2y+cx^2y^2=1$
(3) $xy^2+cx^2y^2=1$
(4) $x^2y+cx^2y^2=1$

50. The solution of the differential equation $\frac{dy}{dx}=\frac{2xy}{x^2-1-2y}$ is

(1) $\frac{x^2}{y}=\frac{1}{y}-2\ln y+c$
(2) $\frac{x^2}{y^2}=\frac{1}{y}-2\ln y+c$
(3) $x^2y=\ln y+c$
(4) $\frac{x^2}{y}=2\ln y+c$

51. The solution of the differential equation $(xy^4+y)dx=xdy$ is

(1) $3x^3y^3+4x^3=cy^3$
(2) $3x^4y^3+4x^3=cy^3$
(3) $4x^4y^3+3x^3=cy^3$
(4) $x^2y^3+4x^3=cy^3$

52. The solution of the differential equation $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=x^2+2y^2+\frac{y^4}{x^2}$ is

(1) $\frac{2y}{x}=x^2+y^2+c$
(2) $\frac{y}{x}=\frac{1}{x^2+y^2}+c$
(3) $\frac{2y}{x}=\frac{1}{x^2+y^2}+c$
(4) $2y=\frac{x^2+y^2}{x}+c$

53. Spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $k>0$, is

(1) $\frac{dr}{dt}+k=0$
(2) $\frac{dr}{dt}-k=0$
(3) $\frac{dr}{dt}=kr$
(4) $\frac{dr}{dt}=2kr$

54. If m, n are order and degree of differential equation $y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)-xy=\cos x$ then

(1) $m<n$
(2) $m=n$
(3) $m>n$
(4) $m-n=3$

55. If $\frac{dy}{dx}=\frac{xy+y}{xy+x}$, then the solution of differential equation is

(1) $y=xe^x+c$
(2) $y=e^x+c$
(3) $y=cxe^{x-y}$
(4) $y=x+c$

56. The solution of $x^2y_1^2+xyy_1-6y^2=0$ are

(1) $y=Cx^2$
(2) $x^3y=C$
(3) $\frac{1}{2}\ln y=C+\ln x$
(4) All of these

57. The solution of the differential equation $\frac{d^2y}{dx^2}=\frac{dy}{dx}$ is

(1) $y=c_1e^x+c_2$
(2) $y=c_1e^{-x}+c_2$
(3) $y=c_1e^{2x}+c_2$
(4) $y=c_1e^{-2x}+c_2$

58. The solution of the differential equation $\frac{d^3y}{dx^3}=8\frac{d^2y}{dx^2}$ satisfying $y(0)=\frac{1}{8}$, $y_1(0)=0$ and $y_2(0)=1$ is

(1) $32y=(e^{8x}-8x)+7$
(2) $64y=(e^{8x}-8x)+7$
(3) $48y=(e^{8x}-8x)-7$
(4) $56y=(e^{8x}+8x)+7$

59. The solution of the differential equation $y_1 y_3 = 3y_2^2$ is

(1) $x = A_1 y^2 + A_2 y + A_3$

(2) $x = A_1 y + A_2$

(3) $x = A_1 y^2 + A_2 y$

(4) $y = A_1 x + A_2$

60. The solution of differential equation $\dfrac{x\ dx - y\ dy}{x\ dy - y\ dx} = \sqrt{\dfrac{1+x^2-y^2}{x^2-y^2}}$ is

(1) $\sqrt{x^2-y^2} + \sqrt{1+x^2-y^2} = \dfrac{c(x+y)}{\sqrt{x^2-y^2}}$

(2) $\sqrt{x^2-y^2} + \sqrt{1+x^2-y^2} = \dfrac{c(x+y)}{x^2+y^2}$

(3) $x^3y + y^5 = 5$

(4) $x^3y - y^5 = c$

61. If gradient of a curve at any point $P(x, y)$ is $\dfrac{x+y+1}{2y+2x+1}$ and it passes through origin, then curve is

(1) $6y + 3x = ln\left|\dfrac{3x+3y+2}{2}\right|$

(2) $6y - 3x = ln\left|\dfrac{3x+3y+2}{2}\right|$

(3) $5y - 3x = ln\left|\dfrac{3x+3y+2}{2}\right|$

(4) $6y - 5x = ln\left|\dfrac{3x+3y+2}{2}\right|$

62. The degree of the differential equation $e^{(d^3y/dx^3)^2} + x\dfrac{d^2y}{dx^2} + y = 0$ is

(1) 1 (2) 2

(3) 3 (4) not defined

63. The equation of curve passing through (3, 4) and satisfying the differential equation $y\left(\dfrac{dy}{dx}\right)^2 + (x-y)\dfrac{dy}{dx} - x = 0$ is

(1) $x - y + 1 = 0$

(2) $x + y + 1 = 0$

(3) $x + y - 1 = 0$

(4) $2x + y - 1 = 0$

64. The general solution of the differential equation $\dfrac{dy}{dx} = 2xe^{x^2-y}$ is

(1) $e^{x^2-y} = c$

(2) $e^{-y} + e^{x^2} = c$

(3) $e^{y} = e^{x^2} + c$

(4) $e^{x^2+y} = c$

Integer Type Questions (65 to 73)

65. If $\dfrac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(ln\ 2)$ is equal to :

66. Let $y(x)$ be the solution of the differential equal $(x \log x)\dfrac{dy}{dx} + y = 2x \log x$, $(x \geq 1)$. Then $y(e)$ is equal to

67. If $(2 + \sin x)\dfrac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\dfrac{\pi}{2}\right)$ is equal to K then $\dfrac{1}{K}$ is

68. If $y(t)$ is a solution of $(1 + t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ is equal to K then $\dfrac{1}{|K|}$ is

69. An right circular cone of height H and radius R is pointed at bottom. It is filled with a volatile liquid completely. If the rate of evaporation is directly proportional to the surface area of the liquid in contact with air (constant of proportionality $k > 0$). The time in which whole liquid evaporates is $\dfrac{nH}{k}$, then n is equal to

70. The solution of the differential equation $\frac{dy}{dx}\sqrt{1+x+y} = x + y - 1$ is $2t + k_1 ln\,|t - 1| + k_2\,ln\,|t + 2| = x + c$,
where $t = \sqrt{x+y+1}$, then $|k_1 + k_2| =$

71. The solution of the differential equation $\frac{dy}{dx} = \frac{x-2y+3}{2x+y+4}$ is $2xy + \frac{y^2}{2} = \frac{x^2}{2} - \lambda_1 y + \lambda_2 x + c$
then $|\lambda_1 + \lambda_2| =$

72. The solution of the differential equation $\frac{dy}{dx} + y\cot x = \sin x$ is $y \sin x = k(2x - \sin 2x) + c$ then k is $\frac{1}{n}$, then n is equal to

73. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1$. If the y-intercept of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to

CHAPTER

21

PROBABILITY

Single Option Correct Type Questions (01 to 62)

1. Two dies are rolled simultaneously. The probability that the sum of the two numbers on the top faces will be at least 10 is:

(1) 1/6 (2) 1/12
(3) 1/18 (4) 1/5

2. In a horse race the odds in favour of three horses are 1:2, 1:3 and 1: 4 . The probability that exactly one of the horse will win the race

(1) $\frac{11}{30}$ (2) $\frac{47}{60}$
(3) $\frac{13}{30}$ (4) $\frac{11}{60}$

3. Let the parameters a, b, c are chosen by throwing a die three times respectively, then the probability that $f(x) = x^3 + 6ax^2 + 2bx + c$, is an increasing function, is

(1) $\frac{1}{36}$ (2) $\frac{1}{6}$
(3) $\frac{1}{256}$ (4) 1

4. The chance that a 13 card combination from a pack of 52 playing cards is dealt to a specified player in a game of bridge, in which 9 cards are of the same suit, is

(1) $\frac{4 \,.\, {}^{13}C_9 \,.\, {}^{39}C_4}{{}^{52}C_{13}}$

(2) $\frac{4! \,.\, {}^{13}C_9 \,.\, {}^{39}C_4}{{}^{52}C_{13}}$

(3) $\frac{{}^{13}C_9 \,.\, {}^{39}C_4}{{}^{52}C_{13}}$

(4) $\frac{{}^{13}C_9}{{}^{52}C_{13}}$

5. 15 coupons are numbered 1, 2, 3,....., 15 respectively. 7 coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is:

(1) $\left(\frac{9}{16}\right)^6$ (2) $\left(\frac{8}{15}\right)^7$
(3) $\left(\frac{3}{5}\right)^7$ (4) $\frac{9^7 - 8^7}{15^7}$

6. $2n$ boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is

(1) $\frac{n}{2n-1}$ (2) $\frac{n-1}{2n-1}$
(3) $\frac{2n-1}{4n^2}$ (4) none of these

7. If 12 tickets numbered 0, 1, 2, ...11 are placed in a bag, and three are drawn out, then the chance that the sum of the numbers on them is equal to 12 is:

(1) $\frac{2}{3}$ (2) $\frac{1}{5}$
(3) $\frac{3}{59}$ (4) $\frac{3}{55}$

8. In drawing of a card from a well shuffled ordinary deck of playing cards the events ‘card drawn is spade’ and ‘card drawn is an ace’ are

(1) mutually exclusive
(2) equally likely
(3) forming an exhaustive system
(4) none of these

9. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for the job. The probability that at least one of the selected persons will be a woman is

(1) 25/39 (2) 14/39
(3) 5/13 (4) 10/13

10. There are three events A, B, C one of which must, and only one can, happen; the odds are 8 to 3 against A, 5 to 2 against B. The odds against C are

(1) 34 : 43 (2) 43 : 34
(3) 53 : 45 (4) 43:53

11. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)$ $p = \alpha\beta$ and $(\beta - 3\gamma)\ p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$

(1) 4 (2) 5
(3) 6 (4) 7

12. A fair die is tossed. If the number is odd, then the probability that it is prime is

(1) $\frac{2}{3}$ (2) $\frac{1}{2}$
(3) 1 (4) $\frac{1}{3}$

13. A pair of dice is thrown. If total of numbers turned up on both the dies is 8, then the probability that the number turned up on the second die is 5' is

(1) $\frac{5}{36}$ (2) $\frac{1}{6}$
(3) $\frac{1}{5}$ (4) $\frac{2}{5}$

14. If odds against solving a question independently by three students are 2 : 1 , 5 : 2 and 5 : 3 respectively, then probability that the question is solved only by one student is

(1) $\frac{31}{56}$ (2) $\frac{24}{56}$
(3) $\frac{25}{56}$ (4) $\frac{29}{56}$

15. Bag A contains 2 white and 3 red marbles and bag B contains 4 white and 5 red marbles. One marble is drawn at random from one of the bags and is found to be red. The probability that it was drawn from the bag B is

(1) $\frac{4}{13}$ (2) $\frac{25}{52}$
(3) $\frac{36}{65}$ (4) $\frac{41}{78}$

16. If M & N are independent events such that $0 < P(M) < 1$ & $0 < P(N) < 1$, then choose the incorrect option:

(1) M & N are mutually exclusive
(2) M & $\bar{N}$ are independent
(3) $\bar{M}$ & $\bar{N}$ are independent
(4) $P\left(M/N\right) + P\left(\bar{M}/N\right) = 1$

17. 2/3rd of the students in a class are boys & the rest girls. It is known that probability of a girl getting a first class is 0.25 & that of a boy is 0.28. The probability that a student chosen at random will get a first class is:

(1) 0.26 (2) 0.265
(3) 0.27 (4) 0.275

18. A basket contains 5 apples and 7 oranges and another basket contains 4 apples and 8 oranges. One fruit is picked out form each basket. Find the probability that both fruits are apples or both are oranges:

(1) $\frac{24}{144}$ (2) $\frac{56}{144}$
(3) $\frac{68}{144}$ (4) $\frac{76}{144}$

19. If E_1 and E_2 are two events such that $P(E_1)=\frac{1}{4}$, $P\left(\frac{E_2}{E_1}\right)=\frac{1}{2}$ and $P\left(\frac{E_1}{E_2}\right)=\frac{1}{4}$ then $P(E_1\cap E_2)$, $P(E_1)$ and $P(E_2)$ are in

(1) A.P.
(2) G.P.
(3) H.P.
(4) Neither G.P. nor A.P. nor H.P.

20. There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} *urn* is $\frac{i^2+1}{34}$ (i = 1, 2, 3, 4). If we randomly select one of the urns & draw a ball, then the probability of ball being white is:

(1) $\frac{569}{1496}$ (2) $\frac{27}{56}$
(3) $\frac{8}{73}$ (4) $\frac{729}{1496}$

21. A natural number P is chosen at random from the first 1000 natural numbers. If [ ] denotes the greatest integer function, then the probability that $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31P}{30}$ (where $x\in R$) is

(1) $\frac{31}{1000}$ (2) $\frac{33}{999}$
(3) $\frac{33}{1000}$ (4) $\frac{67}{1000}$

22. A 9 digit number using the digits 1, 2, 3, 4, 5, 6, 7, 8 & 9 is written randomly without repetition. The probability that the number will be divisible by 9 is:

(1) 1/9 (2) 1/2
(3) 1 (4) $9!/9^9$

23. A bag contains 6 white, 7 red and 5 blue balls. Three balls are drawn at random. The probability of the event 'balls drawn are, one of each colour' is

(1) $\frac{37}{136}$ (2) $\frac{35}{36}$
(3) $\frac{35}{136}$ (4) $\frac{2}{3}$

24. A coin whose faces are marked 3 and 5 is tossed 4 times. what is the probability that the sum of the numbers thrown being less, than 15?

(1) $\frac{5}{16}$
(2) $\frac{1}{2}$
(3) $\frac{1}{5}$
(4) $\frac{1}{3}$

25. Two whole numbers are randomly selected & multiplied. The probability that the unit's place in their product is 0 or 5 is:

(1) 1/3 (2) 16/25
(3) 9/25 (4) 1/5

26. A letter is known to have come either from "KRISHNAGIRI" or "DHARMAPURI". On the post mark only the two consecutive letters "RI" are visible. Then the chance that it came from Krishnagiri is:

(1) 3/5 (2) 2/3
(3) 9/14 (4) none of these

27. A card is drawn from a pack, the card is replaced & the pack shuffled. If this is done 6 times, the probability that the cards drawn are 2 hearts, 2 diamonds & 2 black cards is:

(1) 90/1024
(2) 45/1024
(3) 1/1024
(4) 45/256

28. In an experimental performance of a single throw of a pair of unbiased normal dice, three events E_1, E_2 & E_3 are defined as follows:

E_1: getting a prime numbered face on each dice

E_2: getting the same number on each dice

E_3: getting a sum on two dice equal to 8. Then correct option is

(1) the events E_1, E_2 & E_3 are not mutually exclusive

(2) the events E_1, E_2 & E_3 are not pairwise mutually exclusive

(3) $P(E_3 | E_1) = 2/9$.

(4) All of these

29. In a certain factory, machines A, B and C produce bolts. Of their production, machines A, B and C produce 2%, 1% and 3% defective bolts respectively. Machine A produces 35% of the total output of bolts, machine B produces 25% and machine C produces 40%. A bolt is chosen at random from the factory's production and is found to be defective. The probability it was produced on machine C, is

(1) $\frac{6}{11}$ (2) $\frac{23}{45}$

(3) $\frac{24}{43}$ (4) $\frac{3}{11}$

30. In a purse there are 10 coins, all 5 paise except one which is a rupee. In another purse there are 10 coins all 5 paise. 9 coins are taken out from the former purse & put into the latter & then 9 coins are taken out from the latter & put into the former. Then the chance that the rupee is still in the first purse is:

(1) 9/19 (2) 10/19

(3) 4/9 (4) 10/5

31. A, B, C in order draw a card from a pack of cards, replacing them after each draw, on condition that the first who draws a spade shall win a prize. Their respective chances of winning are

(1) $\frac{16}{37}\frac{12}{37},\frac{9}{37}$ (2) $\frac{16}{37},\frac{9}{37},\frac{11}{37}$

(3) $\frac{1}{2},\frac{1}{3},\frac{1}{6}$ (4) None of these

32. Out of 11 persons sitting at a round table, 3 persons A, B & C are chosen at random, then the probability that no two of these are sitting next to one another is

(1) $\frac{2}{15}$ (2) $\frac{7}{11}$

(3) $\frac{7}{15}$ (4) $\frac{4}{9}$

33. A biased coin with probability p, $0 < p < 1$ of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals

(1) $\frac{5}{16}$ (2) $\frac{1}{2}$

(3) $\frac{1}{5}$ (4) $\frac{1}{3}$

34. **STATEMENT-1:** Since sample space of the experiment 'A coin is tossed if it turns up head, a die is thrown' is $\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), T\}$.

$\therefore$ Prob. of the event $\{(H, 1), (H, 2), (H, 5)\}$ is $\frac{3}{7}$

STATEMENT-2: If all the sample points in the sample space of an experiment are pair wise mutually exclusive, equally likely and exhaustive, then probability of an event E is defined as

$$P(E) = \frac{\text{Number of sample points favourable to the event E}}{\text{Total number of sample points in the sample space}}$$

(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement -1 is false, Statement -2 is true.

(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

35. **STATEMENT-1:** If A and B are two independent events such that $P(A) \neq 0$, $P(B) \neq 0$, then A and B can not be mutually exclusive.

STATEMENT-2: For non-zero independent events A and B, we have $P(A/B) = P(A)$ which is not so for mutually exclusive events.

(1) Statement -1 is true, Statement-2 is true
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is false, Statement -2 is false

36. For an oral test '25' questions are prepared in which 5 questions are good and 20 questions are difficult. If two question is given to two candidates A and B each in that order, the probability that B gets a good question is

(1) $\frac{4}{5}$ (2) $\frac{1}{5}$
(3) $\frac{1}{6}$ (4) $\frac{5}{6}$

37. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial, is

(1) $\frac{1}{25}$ (2) $\frac{24}{25}$
(3) $\frac{2}{25}$ (4) $\frac{3}{5}$

38. The probability of India winning a test match against West-Indies is 1/2 assuming independence from match to match. The probability that in a match series India's second win occurs at the third test is

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$
(3) $\frac{1}{2}$ (4) $\frac{2}{3}$

39. In a combat, A targets B, and both B and C target A, The probabilities of A, B, C hitting their targets are 2/3, 1/2 and 1/3 respectively. They shoot simultaneously and A is hit. The probability that B hits his target whereas C does not is

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$
(3) $\frac{1}{2}$ (4) $\frac{1}{6}$

40. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse, is

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$
(3) $\frac{1}{5}$ (4) $\frac{2}{5}$

41. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. The set of possible values of x are in the interval:

(1) $\left[\frac{1}{3}, \frac{1}{2}\right]$
(2) $\left[\frac{1}{3}, \frac{2}{3}\right]$
(3) $\left[\frac{1}{3}, \frac{13}{3}\right]$
(4) $[0, 1]$

42. The probability that A speaks truth is 4/5 while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact, is

(1) 3/20 (2) 1/5
(3) 7/20 (4) 4/5

43. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are
(1) mutually exclusive and independent
(2) independent but not equally likely
(3) equally likely but not independent
(4) equally likely and mutually exclusive

44. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others, The probability that all the three apply for the same house, is :
(1) 7/9 (2) 8/9
(3) 1/9 (4) 2/9

45. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$ then $P\left(\frac{A \cap B}{\bar{A} \cup \bar{B}}\right)$ is equal to:
(1) $\frac{1}{10}$ (2) $\frac{7}{10}$
(3) $\frac{1}{7}$ (4) Zero

46. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$. Then, $P(B)$ is:
(1) $\frac{1}{6}$ (2) $\frac{1}{3}$
(3) $\frac{2}{3}$ (4) $\frac{1}{2}$

47. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is
(1) $\frac{3}{5}$ (2) 0
(3) 1 (4) $\frac{2}{5}$

48. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equal :
(1) $\frac{1}{7}$ (2) $\frac{5}{14}$
(3) $\frac{1}{50}$ (4) $\frac{1}{14}$

49. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, ...,20\}$.
Statement -1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$
Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common
difference is $\{\pm1, \pm2, \pm3, \pm4, \pm5\}$
(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

50. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is:
(1) $\frac{2}{7}$
(2) $\frac{1}{21}$
(3) $\frac{2}{23}$
(4) $\frac{1}{3}$

51. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is

(1) $\frac{3}{8}$ (2) $\frac{1}{5}$

(3) $\frac{1}{4}$ (4) $\frac{2}{5}$

52. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$

(3) $\frac{3}{5}$ (4) $\frac{2}{3}$

53. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT True?

(1) E_2 and E_3 are independent

(2) E_1 and E_3 are independent

(3) E_1, E_2 and E_3 are independent

(4) E_1 and E_2 are independent

54. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs)

= P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P (All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is

(1) $\frac{7}{32}$ (2) $\frac{7}{16}$

(3) $\frac{7}{64}$ (4) $\frac{3}{16}$

55. It two different numbers are taken from the set {0, 1, 2, 3,....., 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is

(1) $\frac{6}{55}$ (2) $\frac{12}{55}$

(3) $\frac{14}{45}$ (4) $\frac{7}{55}$

56. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \bar{C}) = \frac{1}{3}$ and $P\left(\bar{A} \cap B \cap \bar{C}\right) = \frac{1}{3}$, then $P(B \cap C)$ is:

(1) 1/12 (2) 1/6

(3) 1/15 (4) 1/9

57. Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(1) $\frac{4}{25}$ (2) $\frac{4}{35}$

(3) $\frac{4}{55}$ (4) $\frac{4}{1155}$

58. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{2}{5}$ (4) $\frac{1}{5}$

59. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is

(1) $\frac{1}{2}$ (2) $\frac{36}{55}$

(3) $\frac{6}{11}$ (4) $\frac{5}{11}$

60. If two events A and B are such that $P(A^c) = 0.3$ and $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, then the value of $P[B/(A \cup B^c)]$, is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$

61. Three persons A, B and C are to speak at a function along with five others. If they all speak in random order, then probability that A speaks before B and B speaks before C is

(1) $\frac{3}{8}$ (2) $\frac{1}{6}$

(3) $\frac{3}{5}$ (4) $\frac{5}{6}$

62. The numbers 'a and b' are randomly selected from the set of natural numbers. Probability that the number $3^a + 2^b$ has a digit equal to 1 at the units place, is $\frac{p}{q}$ then $p + q$ is: (Where p & q are co-prime natural numbers)

(1) 13 (2) 19

(3) 5 (4) 20

Integer Type Questions (63 to 72)

63. A number is chosen at random among the set of first 120 natural numbers the probability of the number chosen from the set being a multiple of 5 or 15, is K then 5 K, is equal to:

64. If A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{2}{3}$, If $P(\bar{A} \cap B)$ is $\frac{K}{12}$, then K is equal to

65. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more that 0.99?

66. Two number b and c are chosen one by one at random (with replacement) from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9. The probability that $x^2 + bx + c > 0$ for all real x is $\frac{p}{q}$ (p and q are co-prime natural numbers) then $|p - q|$ is

67. A bag contains $(n + 1)$ coins. It is known that one of these coins has a head on both sides, whereas the other coins are normal. One of these coins is selected at random & tossed. If the probability that the toss results in head, is 7/12, then the value of n is.

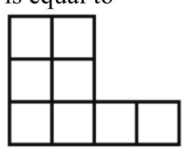
68. Two balls are drawn in succession from a box containing 4 red, 3 white and 5 blue balls. The probability of the event 'one ball is red and other ball is white' is K then 11 K is equal to:

69. Two whole numbers are randomly selected & multiplied. If the probability that the units place in their product is "Even" is p & the probability that the units place in their product is "Odd" is q, then p/q is:

70. An urn contains 6 red and 4 blue balls. Two balls are drawn without replacement. If the probability of the event that 'the second ball drawn is red' is $\frac{p}{q}$, where p and q are coprime, then the value of $p + q$ is

71. If the probability that units digit in square of an even integer is 4 is p, then the value of $5p$ is

72. If alphabets of word RAKESH are written in given boxes, then probability that no row remains empty is $\frac{a}{b}$ (where a & b are coprime) then $a + b$, is equal to



CHAPTER 22

STATISTICS

Single Option Correct Type Questions (01 to 66)

1. Find the A.M. of the series 1, 2, 4, 8, 16, ..., 2^n

(1) $\frac{2^{n+1}-1}{n+1}$ (2) $\frac{2^{n+2}-1}{n}$

(3) $\frac{2^n-1}{n+1}$ (4) $\frac{2^n-1}{n}$

2. N observations on a variable x are $x_i = A + iB$ for i = 1, 2, 3, ..., n where A, B are real constants. The mean of the observation is

(1) $A + B\,\frac{(n+1)}{2}$

(2) $nA + B\,\frac{n+1}{2}$

(3) $A + Bn\frac{n+1}{2}$

(4) $A + B\left(\frac{n}{2}\right)$

3. $M(x_1, x_2, x_3 \text{ .... } x_n)$ defines a measure of central tendency based on n values $x_1, x_2, x_3, \text{ ....} x_n$. Consider the following measured of central tendency

(1) Arithmetic mean
(2) Median
(3) Geometric mean

which of the above measure satisfy/ satisfies the property

$$\frac{M\ x_1, x_2, x_3 \text{............} x_n}{M\ y_1, y_2, y_3 \text{............} y_n}$$

$$= M.\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, \text{.......}, \frac{x_n}{y_n}\right)?$$

select the correct answer using the code below

(1) 1 only (2) 2 only

(3) 3 only (4) 1 and 3

4. If values a, b, c,...... j, p occur with frequencies ${}^{10}C_0, {}^{10}C_1, {}^{10}C_2, ..., {}^{10}C_{10}$ then mode is

(1) a (2) e

(3) f (4) k

5. The mean of 21 observations (all different) is 40. If each observation greater than the median are increased by 21, then mean of observations will become

(1) 50 (2) 50.5

(3) 30 (4) 45

6. The scores of a batsman in ten innings are 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. Find the mean deviation about median.

(1) $\frac{43}{5}$ (2) $\frac{44}{5}$

(3) $\frac{41}{5}$ (4) $\frac{42}{5}$

7. The mean deviation about median of variates 13, 14, 15, ... , 99, 100 is

(1) 1936 (2) 21.5

(3) 23.5 (4) 22

8. The mean deviation of an ungrouped data is 50. If each observation is increased by 2%, then the new mean deviation is

(1) 50 (2) 51

(3) 49 (4) 50.5

9. The mean deviation of an ungrouped data is 80. If each observation is decreased by 5%, then the new mean deviation is

(1) 76 (2) 77
(3) 78 (4) 79

10. The mean deviation from mean of the observations a, $a + d$, $a + 2d$, ... $a + 2nd$ is

(1) $\frac{n(n+1)d^2}{3}$

(2) $\frac{n(n+1)}{2}d^2$

(3) $a+\frac{n(n+1)d^2}{2}$

(4) $\frac{n(n+1)|d|}{(2n+1)}$

11. If $\bar{X}$ is the mean of x_1, x_2, x_3 ...... x_n. Then the algebraic sum of the deviations about mean $\bar{X}$ is

(1) 0 (2) $\frac{\bar{X}}{n}$
(3) $n\bar{X}$ (4) $(n-1)\ \bar{X}$

12. If the algebraic sum of deviations of 20 observation from 30 is 20, then the mean of the observation is

(1) 30 (2) 30.1
(3) 29 (4) 31

13. Variance of first 20 natural number is

(1) $\frac{133}{4}$ (2) $\frac{379}{12}$
(3) $\frac{133}{2}$ (4) $\frac{399}{4}$

14. The mean & variance of 7 observations are 8, 16 respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the LCM of remaining two observations is

(1) 16 (2) 24
(3) 20 (4) 14

15. If $n = 10$, $\bar{x} = \sqrt{12}$, $\sum x^2 = 1560$, then standard deviation σ is

(1) 12 (2) 13
(3) $\sqrt{166}$ (4) $\sqrt{12}$

16. The mean of distribution is 4 if coefficient of variation is 58%. Then standard deviation of distribution is

(1) 2.23 (2) 3.23
(3) 2.32 (4) 2.75

17. The sum of squares of deviations for 10 observations taken from mean 50 is 250. The co-efficient of variation is

(1) 50% (2) 10%
(3) 40% (4) 30%

18. If the standard deviation of x_1, x_2, ... x_n is 3.5, then the standard deviation of $-2x_1 - 3, -2x_2 - 3, \ldots -2x_n - 3$ is

(1) – 7 (2) – 4
(3) 7 (4) 1.75

19. The marks of some students were listed out of a maximum 100. The standard deviation of marks was found to be 9. Subsequently the marks raised to a maximum of 150 and standard deviation of new marks was calculeted. The new standard deviation

(1) 9 (2) 13.5
(3) –13.5 (4) –9

20. Find the arithmetic mean of ${}^{2n+1}C_0$, ${}^{2n+1}C_1$, ...${}^{2n+1}C_n$.

(1) $\frac{2^{2n+2}}{n}$ (2) $\frac{2^{2n}}{n+1}$
(3) $\frac{2^{n+1}}{n+1}$ (4) None of these

21. A variable takes the values of 0, 1, 2, ... , n with frequencies proportional to the binomial coefficients ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$, then mean of the distribution is

(1) $\frac{n\ \ n+1}{4}$ (2) $\frac{n}{2}$
(3) $\frac{n\ \ n-1}{2}$ (4) $\frac{n\ \ n+1}{2}$

22. Following is the record of goals scored by team A in football session

Numbers of goals scored	0	1	2	3	4
Numbers of matches	1	9	7	5	3

For team 'B' mean number of goals scored per match is was 2 goals with variance 1.25. The team which is more consistent

(1) A (2) B

(3) A or B (4) Not A, B

23. The mean of two samples of sizes 200 and 300 were found to be 25, 10 respectively. Their standard deviations were 3 and 4 respectively. The variance of combined sample of size 500 is

(1) 70 (2) 60

(3) 67.2 (4) 80

24. The first of the two samples has 100 items with mean 15 and S.D. 3. If the whole group has 250 items with mean 15.6 and S.D. = $\sqrt{13.44}$ then S.D. of the second group is

(1) 5 (2) 4

(3) 6 (4) 3.52

25. The average marks of 10 students in a class was 60 with standard deviation 4. While the average marks of other 10 students was 40 with a standard deviation 6. If all the 20 students are taken together, their standard deviation will be

(1) 5.0 (2) 7.5

(3) 9.8 (4) 11.2

26. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. From these observations three are 1, 2 and 6 and $\lambda = |x_1 - x_2| + 8$ where x_1 & x_2 are remaining observations. Then number of solution of equation $10 - x^2 - 2x = \lambda$ are

(1) 1 (2) 2

(3) 3 (4) 4

27. The mean and variance of 10 numbers were calculated as 11.3 and 3.3 respectively. It was subsequently found that one of the numbers was misread as 10 instead of 12. How does the variance change?

(1) variance decreases

(2) variance increases

(3) nothing can be said about variance

(4) variance remains unchanged.

28. **STATEMENT-1:** If $\sum_{i=1}^{9}(x_i - 8) = 9$ and $\sum_{i=1}^{9}(x_i - 8)^2 = 45$ then S.D. of $x_1, x_2, ..., x_9$ is 2.

STATEMENT-2: S.D. is independent of change of origin.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

29. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

(1) 20.5 (2) 22.0

(3) 24.0 (4) 25.5

30. Suppose a population A has 100 observations 101, 102, ..., 200 and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations respectively, then V_A/V_B is

(1) 1 (2) 9/4

(3) 4/9 (4) 2/3

31. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

(1) 40% (2) 20%

(3) 80% (4) 60%

32. The mean of the number a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the folowing gives possible values of a and b?

(1) $a = 3, b = 4$ (2) $a = 0, b = 7$

(3) $a = 5, b = 2$ (4) $a = 1, b = 6$

33. **Statement-1:** The variance of first n even natural numbers is $\frac{n^2-1}{4}$

Statement-2: The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

34. If the mean deviation of numbers 1, 1 + d, 1 + 2d, ...,1 + 100d from their mean is 255, then the value of $|d|$ is equal to

(1) 10.0 (2) 20.0

(3) 10.1 (4) 20.2

35. For two data sets, each of size 5, the variance are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) 5.5 (2) 6

(3) 7 (4) 8.8

36. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively:

(1) 32, 2 (2) 32,4

(3) 28, 2 (4) 28, 4

37. Let x_1, x_2 , ..., x_n be n observations, and let $\overline{x}$ be their arithmetic mean and σ^2 be the variance

Statement-1: Variance of $2x_1$, $2x_2$, ..., $2x_n$ is $4\sigma^2$.

Statement-2: Arithmetic mean $2x_1$, $2x_2$, ..., $2x_n$ is $4\overline{x}$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, statement-2 is true

(3) Statement-1 is false, statement-2 is false

(4) Statement-1 is true, statement-2 is false.

38. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?

(1) mean (2) median

(3) mode (4) variance

39. The variance of first 50 even natural number is

(1) $\frac{833}{4}$ (2) 833

(3) 437 (4) $\frac{437}{4}$

40. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is

(1) 16.8 (2) 16.0

(3) 15.8 (4) 14.0

41. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(1) $3a^2 - 32a + 84 = 0$

(2) $3a^2 - 34a + 91 = 0$

(3) $3a^2 - 23a + 44 = 0$

(4) $3a^2 - 26a + 55 = 0$

42. The mean weight of 150 person in a group is 60 kg. The mean weight of men in the group is 70 kg and that of women is 55kg. The number of men is

(1) 50 (2) 75

(3) 100 (4) 25

43. Then mean of 11 observations is 25. If each observation is decreased by 5, the new mean will be

(1) 25 (2) 30
(3) 20 (4) 15

44. The marks of some students were listed out of a maximum 60. The standard deviation of marks was found to be 5. Subsequently the marks raised to a maximum of 100 and variance of new marks was calculated. The new variance is

(1) $\frac{25}{3}$ (2) $\frac{625}{3}$
(3) $\frac{625}{9}$ (4) $\frac{15}{9}$

45. Variance of first 10 natural numbers is

(1) $\frac{133}{4}$ (2) $\frac{33}{4}$
(3) 33 (4) $\frac{33}{2}$

46. Find the median of values 12, 17, 19, 8, 4, 23, 27

(1) 27 (2) 23
(3) 17 (4) 18

47. Find the mode of the data 3, 1, 1, 2, 3, 0, –3, 4, 1, 2, 3, 3, 5

(1) 1 (2) 2
(3) 0 (4) 3

48. If difference between mean and mode is 3, the difference between mean and median is

(1) 3 (2) 1
(3) 4 (4) 2

49. Mode of the data is

x_i (variate)	frequency
$^4C_1$	$^8C_0$
$^4C_2$	$^8C_2$
$^4C_3$	$^8C_4$
$^4C_4$	$^8C_6$

(1) $^4C_2$ (2) $^8C_2$
(3) $^8C_4$ (4) $^4C_3$

50. Mean of 1, 4, 7, 10, 13...........n terms is

(1) $(3n–1)n$
(2) $(3n–1)\frac{n}{2}$
(3) $\frac{3n-1}{2}$
(4) $(3n–1)$

51. If var(x_i) = λ then var($2x_i$ + 3) is

(1) 2λ + 3 (2) $2\lambda^2$
(3) 4λ (4) 4λ + 9

52. Consider the following statement and choose correct option

(i) variance can not be negative
(ii) S.D can not be negative
(III) Median is influenced by extreme value in set of numbers.

(1) TTT (2) FTT
(3) FTF (4) TTF

53. The mean and variance of 7 observations are 7 and $\frac{100}{7}$. If 5 of the observation are 2, 4, 7, 11, 10, Then the remaining 2 observations are

(1) 3, 6
(2) 3, 12
(3) 4, 11
(4) 5, 10

54. If standard deviation of 1, 2, 3, 4, 5 is $\sqrt{2}$ then which of the following is correct?

(1) standard deviation of 1, 4, 9, 16, 25, is 2
(2) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2000}$
(3) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2}$
(4) standard deviation of 1, 8, 27, 16, 25 is $\sqrt{2}$

55. The mean and variance of 100 numbers were calculated as 11 and 2 respectively. Later it was found that one of the number was misread 5 instead of 9. How does the variance change.
(1) Variance doesn't change
(2) Variance Increases
(3) Variance decreases
(4) Can't comment

56. If variance of x_1, x_2, x_3, x_4, x_5 is σ^2, then variance of $3x_1+4$, $3x_2+4$, $3x_3+4$, $3x_4+4$, $3x_5+4$ is
(1) $4\sigma^2+3$ (2) $4\sigma^2+9$
(3) $9\sigma^2$ (4) $4\sigma^2-3$

57. The mean deviation about median of 34, 38, 42, 55, 63, 46, 54, 44, 70,48
(1) 8.2 (2) 8.4
(3) 8.6 (4) 8.8

58. Variance of first n natural numbers is
(1) $\frac{n^2-1}{24}$ (2) $\frac{n^2-1}{12}$
(3) $\frac{n^2-1}{6}$ (4) $\frac{n^2-1}{3}$

59. In a batch of 20 students 8 have failed. The marks of the successful candidates are 23, 27, 29, 18, 17, 19, 21, 27, 20, 24, 26, 28 . The median marks are
(1) 22 (2) 18
(3) 18.5 (4) can't determine

60. The coefficient of variation of two series are 60% and 70% if their standard deviation are 21 and 14, then find ratio of their AMs
(1) $\frac{6}{7}$ (2) $\frac{2}{3}$
(3) $\frac{4}{7}$ (4) $\frac{7}{4}$

61. The mean and median of some data is 14 and 12. Later it was discovered that every data element should be increased by 2 units then new mean and median will be
(1) 16, 12 (2) 16,14
(3) 14, 12 (4) 10, 8

62. Rohan worked for a firm as given below?

No. of weeks	Days each week he worked
2 weeks	1 day each week
14 weeks	2 day each week
8 weeks	5 day each week
32 weeks	7 day each week

What is the mean number of days rohan works per week
(1) 5 (2) 6
(3) 5.5 (4) 5.25

63. If a variate X is expressed as a linear function of two variates U and V in the form $X = a\,U + b\,V$, then mean $\bar{X}$ of X is
(1) $a\bar{U} + b\bar{V}$ (2) $\bar{U}+\bar{V}$
(3) $a\bar{U} + a\bar{U}$ (4) None of these

64. The AM of n numbers of a series is $\bar{X}$. If the sum of first $(n - 1)$ terms is k, then the nth number is
(1) $\bar{X} - k$ (2) $n\bar{X} - k$
(3) $\bar{X} - nk$ (4) $\frac{\bar{X}}{3}$

65. If $\bar{X}_1$ and $\bar{X}_2$ are the means of two distributions such that $\bar{X}_1 < \bar{X}_2$ and $\bar{X}$ is the mean of the combined distribution, then
(1) $\bar{X} < \bar{X}_1$
(2) $\bar{X} > \bar{X}_2$
(3) $\bar{X} = \frac{\bar{X}_1+\bar{X}_2}{2}$
(4) $\bar{X}_1 < \bar{X} < \bar{X}_2$

66. Coefficient of variation of two distribution are 50% and 60% and their arithmetic means are 30 and 25 respectively. Difference of their standard deviations is

(1) 0 (2) 1

(3) 1.5 (4) 2 .5

Integer Type Questions (67 to 75)

67. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the S.D. of the observations is 2, then $|a|$ equals

68. If the mean deviation about the median of the numbers $a, 2a, \ldots, 50a$ is 50, then $|a|$ equals :

69. If $\sum_{i=1}^{11} x_i - 4 \; = 11$ and $\sum_{i=1}^{11} x_i - 4 \;^{2} = 44$ then the variance of $x_1, x_2, x_3 \ldots x_{11}$ is

70. If S. D. of $x_1, x_2, x_3 \ldots x_4$ is 3 then S. D of $-4x_1, -4x_2 \ldots -4x_n$ is

71. The mean of distribution is 6, If coefficient of variation is 50%, then standard deviation of distribution is

72. The mean deviation about median of variation 53, 54, 55......100 is

73. The mean of two samples of sizes 20 and 10 were found to be 11, 8 respectively. Their variance were 4 and 34 respectively. The variance of combined sample of size 30 is

74. The variance of first 5 even natural numbers is

75. The mean of 2 samples of sizes 50 & 40 were found to be 63 and 54. Their variance were 81 & 36. The variance of combined sample of size 90 is

01. SETS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 57)

1.	(4)	9.	(2)	17.	(2)	25.	(4)	33.	(1)	41.	(2)	49.	(3)	57.	(1)
2.	(2)	10.	(2)	18.	(3)	26.	(2)	34.	(1)	42.	(2)	50.	(2)		
3.	(1)	11.	(2)	19.	(1)	27.	(3)	35.	(1)	43.	(1)	51.	(3)		
4.	(1)	12.	(2)	20.	(4)	28	(1)	36.	(1)	44.	(3)	52.	(3)		
5.	(4)	13.	(1)	21.	(2)	29.	(2)	37.	(2)	45.	(4)	53.	(2)		
6.	(4)	14.	(4)	22.	(3)	30.	(3)	38.	(2)	46.	(3)	54.	(3)		
7.	(3)	15.	(2)	23.	(3)	31.	(3)	39.	(2)	47.	(4)	55.	(1)		
8.	(2)	16.	(1)	24.	(2)	32.	(3)	40.	(2)	48.	(4)	56.	(1)		

INTEGER TYPE QUESTIONS (58 TO 67)

58.	(32)	60.	(300)	62.	(15)	64.	(18)	66.	(30)
59.	(6)	61.	(60)	63.	(7)	65.	(266)	67.	(18)

02. RELATIONS & FUNCTIONS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 63)

1.	(4)	9.	(1)	17.	(4)	25.	(1)	33.	(2)	41.	(1)	49.	(4)	57.	(3)
2.	(4)	10.	(4)	18.	(4)	26.	(1)	34.	(1)	42.	(4)	50.	(2)	58.	(3)
3.	(4)	11.	(4)	19.	(1)	27.	(2)	35.	(2)	43.	(4)	51.	(1)	59.	(1)
4.	(4)	12.	(4)	20.	(3)	28.	(2)	36.	(1)	44.	(2)	52.	(2)	60.	(1)
5.	(1)	13.	(1)	21.	(3)	29.	(2)	37.	(1)	45.	(3)	53.	(1)	61.	(2)
6.	(1)	14.	(4)	22.	(4)	30.	(3)	38.	(3)	46.	(4)	54.	(3)	62.	(1)
7.	(3)	15.	(2)	23.	(2)	31.	(4)	39.	(2)	47.	(4)	55.	(4)	63.	(4)
8.	(4)	16.	(3)	24.	(1)	32.	(4)	40.	(3)	48.	(2)	56.	(1)		

INTEGER TYPE QUESTIONS (64 TO 73)

64.	(2)	66.	(22)	68.	(14)	70.	(0)	72.	(24)
65.	(0)	67.	(0)	69.	(4)	71.	(12)	73.	(6)

03. TRIGONOMETRY (TRI)

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 65)

1.	(3)	10.	(2)	19.	(2)	28.	(2)	37.	(2)	46.	(2)	55.	(2)	64.	(2)
2.	(2)	11.	(1)	20.	(4)	29.	(2)	38.	(1)	47.	(D)	56.	(2)	65.	(3)
3.	(2)	12.	(3)	21.	(4)	30.	(1)	39.	(2)	48.	(3)	57.	(2)		
4.	(4)	13.	(3)	22.	(4)	31.	(2)	40.	(4)	49.	(2)	58.	(1)		
5.	(2)	14.	(2)	23.	(4)	32.	(3)	41.	(1)	50.	(3)	59.	(1)		
6.	(3)	15.	(2)	24.	(1)	33.	(1)	42.	(1)	51.	(4)	60.	(2)		
7.	(1)	16.	(2)	25.	(1)	34.	(4)	43.	(2)	52.	(3)	61.	(1)		
8.	(3)	17.	(2)	26.	(2)	35.	(1)	44.	(2)	53.	(3)	62.	(1)		
9.	(4)	18.	(1)	27.	(4)	36.	(1)	45.	(4)	54.	(1)	63.	(2)		

INTEGER TYPE QUESTIONS (66 TO 75)

66.	(25)	68.	(1)	70.	(0)	72.	(3)	74.	(0)
67.	(1)	69.	(4)	71.	(9)	73.	(2)	75.	(7)

04. QUADRATIC EQUATIONS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 65)

1.	(3)	10.	(2)	19.	(3)	28.	(3)	37.	(4)	46.	(1)	55.	(3)	64.	(4)
2.	(1)	11.	(3)	20.	(1)	29.	(2)	38.	(3)	47.	(3)	56.	(2)	65.	(1)
3.	(3)	12.	(4)	21.	(2)	30.	(4)	39.	(3)	48.	(3)	57.	(1)		
4.	(1)	13.	(1)	22.	(2)	31.	(4)	40.	(1)	49.	(3)	58.	(1)		
5.	(2)	14.	(3)	23.	(2)	32.	(3)	41.	(2)	50.	(2)	59.	(4)		
6.	(1)	15.	(4)	24.	(1)	33.	(1)	42.	(2)	51.	(3)	60.	(1)		
7.	(2)	16.	(1)	25.	(1)	34.	(4)	43.	(2)	52.	(3)	61.	(4)		
8.	(2)	17.	(4)	26.	(3)	35.	(2)	44.	(2)	53.	(4)	62.	(2)		
9.	(3)	18.	(3)	27.	(2)	36.	(2)	45.	(4)	54.	(1)	63.	(2)		

INTEGER TYPE QUESTIONS (66 TO 75)

66.	(3)	68.	(3)	70.	(2)	72.	(3)	74.	(0)
67.	(1)	69.	(41)	71.	(18)	73.	(2)	75.	(0)

05. COMPLEX NUMBERS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 62)

1.	(4)	9.	(1)	17.	(3)	25.	(1)	33.	(1)	41.	(4)	49.	(2)	57.	(4)
2.	(3)	10.	(1)	18.	(2)	26.	(2)	34.	(3)	42.	(1)	50.	(4)	58.	(1)
3.	(2)	11.	(4)	19.	(1)	27.	(1)	35.	(4)	43.	(3)	51.	(3)	59.	(3)
4.	(1)	12.	(3)	20.	(1)	28.	(1)	36.	(4)	44.	(4)	52.	(1)	60.	(4)
5.	(1)	13.	(1)	21.	(4)	29.	(1)	37.	(3)	45.	(2)	53.	(3)	61.	(4)
6.	(1)	14.	(1)	22.	(1)	30.	(1)	38.	(3)	46.	(2)	54.	(1)	62.	(4)
7.	(1)	15.	(3)	23.	(3)	31.	(4)	39.	(4)	47.	(2)	55.	(1)		
8.	(1)	16.	(3)	24.	(2)	32.	(4)	40.	(3)	48.	(3)	56.	(3)		

INTEGER TYPE QUESTIONS (63 TO 73)

63.	(0)	65.	(1)	67.	(54)	69.	(1)	71.	(1)	73.	(20)
64.	(5)	66.	(4)	68.	(6)	70.	(1)	72.	(0)		

06. BINOMIAL THEOREM

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 60)

1.	(2)	9.	(4)	17.	(1)	25.	(2)	33.	(3)	41.	(4)	49.	(2)	57.	(2)
2.	(3)	10.	(3)	18.	(4)	26.	(3)	34.	(1)	42.	(3)	50.	(2)	58.	(1)
3.	(3)	11.	(2)	19.	(4)	27.	(3)	35.	(1)	43.	(3)	51.	(1)	59.	(4)
4.	(3)	12.	(4)	20.	(3)	28.	(1)	36.	(4)	44.	(4)	52.	(3)	60.	(4)
5.	(4)	13.	(1)	21.	(3)	29.	(1)	37.	(2)	45.	(1)	53.	(2)		
6.	(3)	14.	(3)	22.	(2)	30.	(2)	38.	(4)	46.	(2)	54.	(1)		
7.	(3)	15.	(2)	23.	(1)	31.	(3)	39.	(4)	47.	(2)	55.	(1)		
8.	(4)	16.	(3)	24.	(2)	32.	(1)	40.	(2)	48.	(3)	56.	(2)		

INTEGER TYPE QUESTIONS (61 TO 74)

61.	(2)	63.	(49)	65.	(55)	67.	(101)	69.	(15)	71.	(7)	73.	(8)
62.	(2)	64.	(5)	66.	(9)	68.	(35)	70.	(9)	72.	(1)	74.	(29)

07. PERMUTATIONS AND COMBINATIONS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 60)

1.	(2)	9.	(2)	17.	(3)	25.	(4)	33.	(1)	41.	(2)	49.	(3)	57.	(4)
2.	(2)	10.	(1)	18.	(4)	26.	(1)	34.	(4)	42.	(3)	50.	(3)	58.	(2)
3.	(4)	11.	(1)	19.	(1)	27.	(2)	35.	(2)	43.	(1)	51.	(2)	59.	(3)
4.	(2)	12.	(3)	20.	(1)	28.	(4)	36.	(4)	44.	(1)	52.	(3)	60.	(1)
5.	(2)	13.	(2)	21.	(1)	29.	(1)	37.	(1)	45.	(1)	53.	(4)		
6.	(2)	14.	(2)	22.	(4)	30.	(1)	38.	(2)	46.	(4)	54.	(4)		
7.	(1)	15.	(1)	23.	(1)	31.	(4)	39.	(4)	47.	(3)	55.	(1)		
8.	(2)	16.	(1)	24.	(2)	32.	(2)	40.	(2)	48.	(4)	56.	(4)		

INTEGER TYPE QUESTIONS (61 TO 75)

61.	(20)	63.	(21)	65.	(196)	67.	(108)	69.	(485)	71.	(420)	73.	(150)	75.	(540)
62.	(18)	64.	(7)	66.	(21)	68.	(192)	70.	(40)	72.	(77)	74.	(456)		

08. SEQUENCE AND SERIES

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 60)

1.	(4)	9.	(2)	17.	(1)	25.	(3)	33.	(2)	41.	(4)	49.	(3)	57.	(2)
2.	(1)	10.	(2)	18.	(1)	26.	(3)	34.	(1)	42.	(2)	50.	(3)	58.	(3)
3.	(3)	11.	(1)	19.	(3)	27.	(4)	35.	(2)	43.	(4)	51.	(2)	59.	(1)
4.	(2)	12.	(2)	20.	(1)	28.	(4)	36.	(1)	44.	(2)	52.	(4)	60.	(3)
5.	(2)	13.	(2)	21.	(1)	29.	(2)	37.	(4)	45.	(2)	53.	(3)		
6.	(3)	14.	(2)	22.	(4)	30.	(4)	38.	(1)	46.	(1)	54.	(4)		
7.	(2)	15.	(4)	23.	(2)	31.	(3)	39.	(3)	47.	(3)	55.	(3)		
8.	(1)	16.	(2)	24.	(1)	32.	(2)	40.	(3)	48.	(3)	56.	(4)		

INTEGER TYPE QUESTIONS (61 TO 75)

61.	(8)	63.	(3)	65.	(1)	67.	(0)	69.	(1)	71.	(3)	73.	(0)	75.	(11)
62.	(3)	64.	(3)	66.	(34)	68.	(35)	70.	(4)	72.	(12)	74.	(0)		

09. STRAIGHT LINES

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 60)

1.	(2)	9.	(3)	17.	(3)	25.	(1)	33.	(2)	41.	(2)	49.	(2)	57.	(2)
2.	(1)	10.	(1)	18.	(4)	26.	(4)	34.	(1)	42.	(2)	50.	(2)	58.	(1)
3.	(2)	11.	(3)	19.	(3)	27.	(2)	35.	(2)	43.	(3)	51.	(4)	59.	(1)
4.	(2)	12.	(2)	20.	(1)	28.	(4)	36.	(1)	44.	(1)	52.	(1)	60.	(4)
5.	(2)	13.	(1)	21.	(2)	29.	(2)	37.	(4)	45.	(1)	53.	(1)		
6.	(2)	14.	(3)	22.	(2)	30.	(3)	38.	(4)	46.	(1)	54.	(3)		
7.	(2)	15.	(3)	23.	(1)	31.	(1)	39.	(2)	47.	(3)	55.	(2)		
8.	(3)	16.	(1)	24.	(3)	32.	(1)	40.	(2)	48.	(2)	56.	(1)		

INTEGER TYPE QUESTIONS (61 TO 70)

61.	(8)	63.	(2)	65.	(3)	67.	(6)	69.	(780)
62.	(3)	64.	(2)	66.	(4)	68.	(2)	70.	(2)

10. CIRCLES

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 60)

1.	(3)	9.	(1)	17.	(2)	25.	(2)	33.	(3)	41.	(3)	49.	(2)	57.	(2)
2.	(4)	10.	(2)	18.	(3)	26.	(1)	34.	(3)	42.	(2)	50.	(3)	58.	(1)
3.	(2)	11.	(1)	19.	(2)	27.	(2)	35.	(3)	43.	(1)	51.	(1)	59.	(3)
4.	(1)	12.	(1)	20.	(2)	28.	(1)	36.	(1)	44.	(1)	52.	(3)	60.	(1)
5.	(1)	13.	(3)	21.	(2)	29.	(1)	37.	(2)	45.	(2)	53.	(2)		
6.	(2)	14.	(2)	22.	(1)	30.	(1)	38.	(1)	46.	(4)	54.	(2)		
7.	(1)	15.	(1)	23.	(4)	31.	(1)	39.	(2)	47.	(2)	55.	(3)		
8.	(3)	16.	(2)	24.	(1)	32.	(3)	40.	(1)	48.	(3)	56.	(1)		

INTEGER TYPE QUESTIONS (61 TO 70)

61.	(3)	63.	(8)	65.	(2)	67.	(6)	69.	(28)
62.	(727)	64.	(2)	66.	(3)	68.	(6)	70.	(816)

11. CONIC SECTIONS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 63)

1.	(4)	9.	(1)	17.	(2)	25.	(1)	33.	(1)	41.	(3)	49.	(4)	57.	(3)
2.	(2)	10.	(3)	18.	(2)	26.	(4)	34.	(4)	42.	(1)	50.	(4)	58.	(1)
3.	(1)	11.	(4)	19.	(3)	27.	(4)	35.	(1)	43.	(3)	51.	(4)	59.	(2)
4.	(1)	12.	(3)	20.	(3)	28.	(3)	36.	(3)	44.	(1)	52.	(2)	60.	(3)
5.	(3)	13.	(4)	21.	(1)	29.	(4)	37.	(2)	45.	(4)	53.	(2)	61.	(3)
6.	(1)	14.	(4)	22.	(3)	30.	(4)	38.	(4)	46.	(4)	54.	(4)	62.	(4)
7.	(4)	15.	(3)	23.	(2)	31.	(1)	39.	(3)	47.	(4)	55.	(1)	63.	(4)
8.	(2)	16.	(2)	24.	(2)	32.	(3)	40.	(1)	48.	(1)	56.	(4)		

INTEGER TYPE QUESTIONS (64 TO 74)

64.	(8)	66.	(8)	68.	(1)	70.	(9)	72.	(6)	74.	(27)
65.	(5)	67.	(12)	69.	(4)	71.	(2)	73.	(4)		

12. VECTORS & 3-D GEOMETRY

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 57)

1.	(1)	9.	(1)	17.	(1)	25.	(4)	33.	(3)	41.	(3)	49.	(3)	57.	(2)
2.	(3)	10.	(3)	18.	(3)	26.	(3)	34.	(4)	42.	(4)	50.	(3)		
3.	(3)	11.	(4)	19.	(4)	27.	(4)	35.	(4)	43.	(4)	51.	(2)		
4.	(4)	12.	(1)	20.	(2)	28.	(1)	36.	(4)	44.	(2)	52.	(3)		
5.	(2)	13.	(3)	21.	(2)	29.	(3)	37.	(4)	45.	(4)	53.	(3)		
6.	(4)	14.	(4)	22.	(1)	30.	(2)	38.	(4)	46.	(3)	54.	(3)		
7.	(3)	15.	(3)	23.	(1)	31.	(1)	39.	(3)	47.	(2)	55.	(2)		
8.	(4)	16.	(3)	24.	(1)	32.	(2)	40.	(1)	48.	(3)	56.	(3)		

INTEGER TYPE QUESTIONS (58 TO 65)

58.	(3)	59.	(5)	60.	(5)	61.	(7)	62.	(2)	63.	(16)	64.	(2)	65.	(4)

13. LIMITS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 65)

1.	(4)	10.	(3)	19.	(1)	28.	(3)	37.	(3)	46.	(1)	55.	(1)	64.	(3)
2.	(4)	11.	(4)	20.	(2)	29.	(3)	38.	(2)	47.	(2)	56.	(4)	65.	(2)
3.	(3)	12.	(3)	21.	(3)	30.	(4)	39.	(1)	48.	(2)	57.	(2)		
4.	(3)	13.	(2)	22.	(4)	31.	(4)	40.	(4)	49.	(3)	58.	(2)		
5.	(4)	14.	(1)	23.	(2)	32.	(2)	41.	(1)	50.	(4)	59.	(2)		
6.	(2)	15.	(3)	24.	(2)	33.	(2)	42.	(4)	51.	(3)	60.	(1)		
7.	(4)	16.	(1)	25.	(2)	34.	(2)	43.	(2)	52.	(3)	61.	(3)		
8.	(2)	17.	(1)	26.	(1)	35.	(2)	44.	(4)	53.	(4)	62.	(3)		
9.	(2)	18.	(3)	27.	(1)	36.	(1)	45.	(2)	54.	(4)	63.	(4)		

INTEGER TYPE QUESTIONS (66 TO 75)

66.	(0)	68.	(1)	70.	(1)	72.	(0)	74.	(1)
67.	(3)	69.	(0)	71.	(2)	73.	(32)	75.	(2)

14. CONTINUITY, DIFFERENTIABILITY, MOD

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 50)

1.	(2)	8.	(3)	15.	(3)	22.	(2)	29.	(4)	36.	(4)	43.	(3)	50.	(2)
2.	(2)	9.	(2)	16.	(1)	23.	(2)	30.	(4)	37.	(1)	44.	(3)		
3.	(3)	10.	(1)	17.	(3)	24.	(3)	31.	(1)	38.	(2)	45.	(1)		
4.	(3)	11.	(1)	18.	(1)	25.	(1)	32.	(3)	39.	(3)	46.	(1)		
5.	(3)	12.	(3)	19.	(3)	26.	(1)	33.	(3)	40.	(2)	47.	(4)		
6.	(2)	13.	(2)	20.	(2)	27.	(2)	34.	(3)	41.	(3)	48.	(3)		
7.	(2)	14.	(3)	21.	(3)	28.	(2)	35.	(3)	42.	(3)	49.	(3)		

INTEGER TYPE QUESTIONS (51 TO 58)

51.	(2)	52.	(2)	53.	(0)	54.	(5)	55.	(1)	56.	(1)	57.	(2)	58.	(2)

15. INVERSE TRIGONOMETRIC FUNCTIONS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 60)

1.	(4)	9.	(4)	17.	(3)	25.	(1)	33.	(2)	41.	(3)	49.	(3)	57.	(3)
2.	(4)	10.	(2)	18.	(4)	26.	(4)	34.	(4)	42.	(3)	50.	(1)	58.	(2)
3.	(3)	11.	(3)	19.	(2)	27.	(3)	35.	(2)	43.	(4)	51.	(2)	59.	(1)
4.	(3)	12.	(3)	20.	(2)	28.	(4)	36.	(2)	44.	(2)	52.	(3)	60.	(1)
5.	(2)	13.	(2)	21.	(1)	29.	(3)	37.	(2)	45.	(1)	53.	(4)		
6.	(4)	14.	(3)	22.	(4)	30.	(1)	38.	(1)	46.	(3)	54.	(2)		
7.	(3)	15.	(1)	23.	(4)	31.	(3)	39.	(4)	47.	(3)	55.	(1)		
8.	(2)	16.	(2)	24.	(2)	32.	(2)	40.	(4)	48.	(2)	56.	(1)		

INTEGER TYPE QUESTIONS (61 TO 71)

61.	(0)	63.	(3)	65.	(1)	67.	(1)	69.	(5)	71.	(1)
62.	(20)	64.	(3)	66.	(15)	68.	(3)	70.	(3)		

16. MATRICES & DETERMINANTS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 57)

1.	(1)	9.	(4)	17.	(3)	25.	(2)	33.	(1)	41.	(3)	49.	(3)	57.	(2)
2.	(2)	10.	(4)	18.	(2)	26.	(1)	34.	(4)	42.	(1)	50.	(2)		
3.	(4)	11.	(1)	19.	(3)	27.	(2)	35.	(1)	43.	(3)	51.	(4)		
4.	(1)	12.	(3)	20.	(1)	28.	(4)	36.	(3)	44.	(3)	52.	(1)		
5.	(1)	13.	(1)	21.	(3)	29.	(3)	37.	(4)	45.	(4)	53.	(4)		
6.	(3)	14.	(4)	22.	(3)	30.	(4)	38.	(2)	46.	(2)	54.	(3)		
7.	(1)	15.	(4)	23.	(4)	31.	(4)	39.	(4)	47.	(3)	55.	(1)		
8.	(1)	16.	(3)	24.	(3)	32.	(4)	40.	(2)	48.	(4)	56.	(4)		

INTEGER TYPE QUESTIONS (58 TO 67)

58.	(8)	60.	(1)	62.	(2)	64.	(1)	66.	(2)
59.	(5)	61.	(3)	63.	(8)	65.	(4)	67.	(11)

17. APPLICATION OF DERIVATIVES

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 63)

1.	(2)	9.	(2)	17.	(4)	25.	(2)	33.	(4)	41.	(2)	49.	(3)	57.	(2)
2.	(3)	10.	(1)	18.	(4)	26.	(3)	34.	(1)	42.	(4)	50.	(4)	58.	(1)
3.	(3)	11.	(3)	19.	(2)	27.	(4)	35.	(2)	43.	(2)	51.	(2)	59.	(3)
4.	(4)	12.	(1)	20.	(4)	28.	(4)	36.	(1)	44.	(2)	52.	(3)	60.	(1)
5.	(4)	13.	(4)	21.	(1)	29.	(4)	37.	(2)	45.	(1)	53.	(4)	61.	(3)
6.	(4)	14.	(2)	22.	(4)	30.	(3)	38.	(3)	46.	(2)	54.	(4)	62.	(1)
7.	(2)	15.	(3)	23.	(1)	31.	(3)	39.	(3)	47.	(1)	55.	(1)	63.	(4)
8.	(1)	16.	(3)	24.	(4)	32.	(1)	40.	(4)	48.	(2)	56.	(2)		

INTEGER TYPE QUESTIONS (64 TO 73)

64.	(4)	66.	(3)	68.	(1)	70.	(25)	72.	(2)
65.	(2)	67.	(2)	69.	(2)	71.	(2)	73.	(4)

18. INDEFINITE INTEGRATION

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 66)

1.	(2)	10.	(3)	19.	(1)	28.	(3)	37.	(2)	46.	(3)	55.	(1)	64.	(1)
2.	(1)	11.	(3)	20.	(1)	29.	(4)	38.	(4)	47.	(2)	56.	(1)	65.	(4)
3.	(2)	12.	(1)	21.	(2)	30.	(1)	39.	(3)	48.	(1)	57.	(4)	66.	(2)
4.	(3)	13.	(2)	22.	(4)	31.	(4)	40.	(3)	49.	(1)	58.	(2)		
5.	(3)	14.	(2)	23.	(1)	32.	(3)	41.	(3)	50.	(2)	59.	(1)		
6.	(1)	15.	(4)	24.	(4)	33.	(3)	42.	(4)	51.	(2)	60.	(3)		
7.	(2)	16.	(3)	25.	(3)	34.	(1)	43.	(3)	52.	(2)	61.	(2)		
8.	(3)	17.	(2)	26.	(4)	35.	(3)	44.	(3)	53.	(2)	62.	(2)		
9.	(2)	18.	(3)	27.	(2)	36.	(1)	45.	(4)	54.	(4)	63.	(1)		

INTEGER TYPE QUESTIONS (67 TO 74)

67.	(1)	68.	(0)	69.	(2)	70.	(16)	71.	(1)	72.	(15)	73.	(60)	74.	(1)

19. DEFINITE INTEGRATION AND ITS APPLICATION

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 60)

1.	(1)	9.	(3)	17.	(3)	25.	(2)	33.	(1)	41.	(1)	49.	(1)	57.	(1)
2.	(2)	10.	(4)	18.	(3)	26.	(2)	34.	(1)	42.	(2)	50.	(2)	58.	(2)
3.	(1)	11.	(4)	19.	(2)	27.	(1)	35.	(2)	43.	(4)	51.	(3)	59.	(2)
4.	(1)	12.	(4)	20.	(3)	28.	(1)	36.	(2)	44.	(3)	52.	(3)	60.	(1)
5.	(4)	13.	(2)	21.	(2)	29.	(2)	37.	(4)	45.	(4)	53.	(4)		
6.	(1)	14.	(4)	22.	(3)	30.	(2)	38.	(2)	46.	(4)	54.	(1)		
7.	(1)	15.	(4)	23.	(3)	31.	(1)	39.	(2)	47.	(3)	55.	(4)		
8.	(2)	16.	(2)	24.	(2)	32.	(3)	40.	(1)	48.	(3)	56.	(1)		

INTEGER TYPE QUESTIONS (61 TO 75)

61.	(9)	63.	(3)	65.	(0)	67.	(1)	69.	(40)	71.	(3)	73.	(2)	75.	(1)
62.	(0)	64.	(2)	66.	(7)	68.	(110)	70.	(1)	72.	(0)	74.	(1)		

20. DIFFERENTIAL EQUATIONS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 64)

1.	(1)	9.	(3)	17.	(4)	25.	(2)	33.	(3)	41.	(3)	49.	(2)	57.	(1)
2.	(1)	10.	(1)	18.	(4)	26.	(3)	34.	(2)	42.	(1)	50.	(1)	58.	(2)
3.	(3)	11.	(2)	19.	(2)	27.	(1)	35.	(3)	43.	(1)	51.	(2)	59.	(1)
4.	(4)	12.	(4)	20.	(2)	28.	(3)	36.	(4)	44.	(4)	52.	(3)	60.	(1)
5.	(1)	13.	(2)	21.	(2)	29.	(3)	37.	(2)	45.	(1)	53.	(1)	61.	(2)
6.	(3)	14.	(2)	22.	(2)	30.	(3)	38.	(4)	46.	(3)	54.	(3)	62.	(4)
7.	(2)	15.	(1)	23.	(4)	31.	(1)	39.	(2)	47.	(2)	55.	(3)	63.	(1)
8.	(3)	16.	(1)	24.	(4)	32.	(1)	40.	(2)	48.	(2)	56.	(4)	64.	(3)

INTEGER TYPE QUESTIONS (65 TO 73)

65.	(7)	67.	(3)	69.	(1)	71.	(7)	73.	(9)
66.	(2)	68.	(2)	70.	(2)	72.	(4)		

21. PROBABILITY

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 62)

1.	(1)	9.	(1)	17.	(3)	25.	(3)	33.	(4)	41.	(1)	49.	(2)	57.	(4)
2.	(3)	10.	(2)	18.	(4)	26.	(3)	34.	(3)	42.	(3)	50.	(1)	58.	(3)
3.	(1)	11.	(3)	19.	(2)	27.	(1)	35.	(1)	43.	(2)	51.	(2)	59.	(3)
4.	(1)	12.	(1)	20.	(1)	28.	(4)	36.	(2)	44.	(3)	52.	(2)	60.	(3)
5.	(4)	13.	(3)	21.	(3)	29.	(3)	37.	(2)	45.	(4)	53.	(3)	61.	(2)
6.	(1)	14.	(3)	22.	(3)	30.	(2)	38.	(2)	46.	(2)	54.	(2)	62.	(2)
7.	(4)	15.	(2)	23.	(3)	31.	(1)	39.	(3)	47.	(3)	55.	(1)		
8.	(4)	16.	(1)	24.	(1)	32.	(3)	40.	(4)	48.	(4)	56.	(1)		

INTEGER TYPE QUESTIONS (63 TO 72)

63.	(1)	65.	(4)	67.	(5)	69.	(3)	71.	(2)
64.	(5)	66.	(49)	68.	(2)	70.	(8)	72.	(27)

22. STATISTICS

SINGLE OPTION CORRECT TYPE QUESTIONS (01 TO 66)

1.	(1)	10.	(4)	19.	(2)	28.	(1)	37.	(4)	46.	(3)	55.	(3)	64.	(2)
2.	(1)	11.	(1)	20.	(2)	29.	(3)	38.	(4)	47.	(4)	56.	(3)	65.	(4)
3.	(3)	12.	(4)	21.	(2)	30.	(1)	39.	(2)	48.	(2)	57.	(3)	66.	(1)
4.	(3)	13.	(1)	22.	(1)	31.	(3)	40.	(4)	49.	(4)	58.	(2)		
5.	(1)	14.	(2)	23.	(3)	32.	(1)	41.	(1)	50.	(3)	59.	(3)		
6.	(1)	15.	(1)	24.	(2)	33.	(4)	42.	(1)	51.	(3)	60.	(4)		
7.	(4)	16.	(3)	25.	(4)	34.	(3)	43.	(3)	52.	(4)	61.	(2)		
8.	(2)	17.	(2)	26.	(1)	35.	(1)	44.	(3)	53.	(2)	62.	(4)		
9.	(1)	18.	(3)	27.	(1)	36.	(1)	45.	(2)	54.	(3)	63.	(1)		

INTEGER TYPE QUESTIONS (67 TO 75)

67.	(2)	69.	(3)	71.	(3)	73.	(16)	75.	(81)
68.	(4)	70.	(12)	72.	(12)	74.	(8)		

Hints & Solutions

SETS

Single Option Correct Type Questions (01 to 57)

1. **(4)**

Sol. Since, intelligency is not defined for students in a class so set of intelligent students in a class is not well defined collection.

2. **(2)**

Sol. (i) $x^2 - 1 = 0$ $\quad x = \pm 1$

(ii) $x^2 + 1 = 0$ $\quad x = \pm i\ x \in \phi$

(iii) $x^2 - 9 = 0$ $\quad x = \pm 3$

(iv) $x^2 - x - 2 = 0, \quad x = 2, -1$

3. **(1)**

Sol. $x^2 = 16$

$\Rightarrow \quad x = \pm 4$

$2x = 6 \quad x = 3$

No common value of x

4. **(1)**

Sol. Obvious

5. **(4)**

Sol. $P(A) = \{\phi, \{7\}, \{10\}, \{11\}, \{7, 10\}, \{7, 11\}, \{10, 11\}, \{7, 10, 11\}\}$

Number of subsets $= 2^n = 2^8 = 256$

6. **(4)**

Sol. Collection of all intelligent women in Jalandhar is not a set as it is not a well defined collection. It is not possible to decide logically which woman is to be included in the collection and which is not to be included.

7. **(3)**

Sol. 2,3,5 and 7 are the only positive primes less than 10.

8. **(2)**

Sol. Between any two real numbers there lie infinitely many real numbers.

9. **(2)**

Sol. $P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\} = \{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

10. **(2)**

Sol. $A = \{1, 2, 3\}$

$B = \{3, 4\}$

$C = \{4, 5, 6\}$

$B \cap C = \{4\}$

$A \cup (B \cap C) = \{1, 2, 3, 4\}$

11. **(2)**

Sol. $A = [x : x \in R, -1 < x < 1]$

$B = [x : x \in R : x \leq 0 \text{ or } x \geq 2]$

$\therefore \quad A \cup B = R - D,$

where $D = [x : x \in R, 1 \leq x < 2]$

12. **(2)**

Sol. Obvious

13. **(1)**

Sol. $A \cap B = \{3, 4, 10\}$

$A \cap C = \{4\}$

$(A \cap B) \cup (A \cap C) = \{3, 4, 10\}$

14. **(4)**

Sol. Obviously $(A - (B \cup C)$

15. **(2)**

Sol. $B' = U - B = \{1, 2, 3, 4, 5, 8, 9, 10\}$

$A \cap B' = \{1,2,5\} = A$

16. **(1)**

Sol. $A = \{5, 9, 13, 17, 21\}$ and $B = \{3, 6, 9, 12, 15, 18, 21, 24\}$

$A - B = \{5, 13, 17\}$

$A - (A - B) = \{9, 21\}$

17. (2)

Sol. Let $A \cup B = A \cap B$

Now, $x \in A$

$\Rightarrow x \in A \cup B \quad (\because A \subseteq A \cup B)$

$\Rightarrow x \in A \cap B \quad (\because A \cup B = A \cap B)$

$\Rightarrow x \in B$

Similarly, $x \in B$ implies $x \in A \quad \therefore A = B$

Conversly, let $A = B$

$\therefore A \cup B = A \cup A = A = A \cap A = A \cap B$

$\therefore A \cup B = A \cap B$

18. (3)

Sol. $bN \cap cN$

(+*ve* integral multiple of b) $\cap$ (+ve integral multiple of c)

since b & c are relatively primes:

$= b\,c\,N \quad \therefore d = bc$

19. (1)

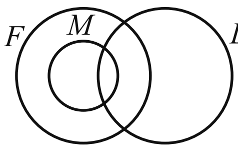
Sol. 1. $(N \cup B) \cap Z = (N \cap Z) \cup (B \cap Z)$

$= N \cup (B \cap Z)$

2. $A = \{3, 6, 9, 12, 15, 18, 21, 24\}$

20. (4)

Sol. $M \equiv$ Mother; $F \equiv$ Female ; D = Doctor



21. (2)

Sol. (i) $A \cup B \geq A \cap B$

(ii) $A \cap B \leq A \cup B$

(iii) $A \cap B = A \cup B$ not always

22. (3)

Sol. Let number of newspapers is x.

As every newspaper is read by 60 students

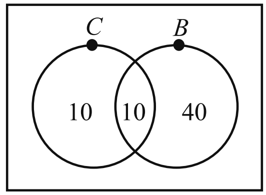
Since, every students reads 5 newspapers

$\therefore 60x = 300(5)$

$\Rightarrow x = 25.$

23. (3)

Sol.



$P = 10 + 10 + 40 = 60\ \%$

24. (2)

Sol. $n(A) = 40\%$ of $10{,}000 = 4{,}000$

$n(B) = 20\%$ of $10{,}000 = 2{,}000$

$n(C) = 10\%$ of $10{,}000 = 1{,}000$

$n(A \cap B) = 5\%$ of $10{,}000 = 500$

$n(B \cap C) = 3\%$ of $10{,}000 = 300$

$n(C \cap A) = 4\%$ of $10{,}000 = 400$

$n(A \cap B \cap C) = 2\%$ of $10{,}000 = 200$

$n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$

$= n(A) - n[A \cap (B \cup C)]$

$= n(A) - n\,[(A \cap B) \cup (A \cap C)]$

$= n\,(A) - [n\,(A \cap B) + n\,(A \cap C) - n\,(A \cap B \cap C)]$

$= 4000 - [500 + 400 - 200]$

$= 4000 - 700 = 3300.$

25. (4)

Sol. Now n(only hindi) $= n(H) - n(H \cap B)$

$= 750 - 150 = 600$

26. (2)

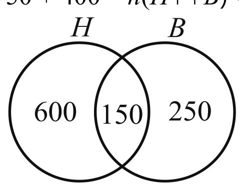
Sol. n(only bengali) $= n(B) - n(H \cap B)$

$400 - 150 = 250$

27. (3)

Sol. $n(H \cup B) = n(H) + n(B) - n(H \cap B)$

$1000 = 750 + 400 - n(H \cap B) = 150$



28 (1)

Sol. $A_1 \cup A_2 \cup A_3$ is the smallest element containing subset of all we set A_1, A_2 and A_3

29. **(2)**

Sol. 1. $((A \cap B) \cup C)' \cap B')'$
$= (A \cap B) \cup C) \cup B$
$= (A \cap B) \cup B \cup C$
$= B \cup C \neq B \cap C$

2. $(A' \cap B') \cap (A \cup B \cup C')$
$= (A \cup B)' \cap ((A \cup B) \cup C')$
$= \phi \cup ((A \cup B)' \cap C')$
$= ((A \cup B) \cup C)'$
$= (A \cup (B \cup C))'$

30. **(3)**

Sol. $n(A \cup B) = 280$
$n(A' \cup B') = 2009 - n(A \cup B)$
$= 2009 - 280 = 1729 = 12^3 + 1^3$
$= 10^3 + 9^3$
$n(A - B) = 1681 - 1075 = 606$
$= 4 + 2 \times 301 = 4 + 2 \times 7 \times 43$
$= (2)^2 + 2 \times 7 \times 43$

31. **(3)**

Sol. $x^3 + (x-1)^3 = 1$
$x^3 + x^3 - 3x^2 + 3x - 1 = 1$
$2x^3 - 3x^2 + 3x - 2 = 0$
$(x-1)(2x^2 - x + 2) = 0$
$x = 1$
$y = 0\,(1, 0)$
Statement 1 is True
Statement 2:
$x^3 + (1 - x)^3 = 1 \Rightarrow x^3 + 1 - 3x + 3x^2 - x^3 = 1$
$\Rightarrow x^2 - x = 0$
$x = 0, 1$ $(0, 1)\ (1, 0)$

32. **(3)**

Sol. $n(M) = 23, n(P) = 24, n(C) = 19$
$n(M \cap P) = 12, n(M \cap C) = 9, n(P \cap C) = 7$
$n(M \cap P \cap C) = 4$
$n\,(M \cap P' \cap C') = n[M \cap (P \cup C)']$
$= n(M) - n(M \cap (P \cup C))$
$= n(M) - n[(M \cap P) \cup (M \cap C)]$
$= n(M) - n(M \cap P) - n(M \cap C)$
$+ n(M \cap P \cap C)$
$= 23 - 12 - 9 + 4 = 27 - 21 = 6$

$n(P \cap M' \cap C') = n[P \cap (M \cup C)']$
$= n(P) - n[P \cap (M \cup C)$
$= n(P) - n\ [P \cap M) \cup (P \cap C)]$
$= n(P) - n(P \cap M) - n(P \cap C)$
$+ n(P \cap M \cap C)$
$= 24 - 12 - 7 + 4 = 9$
$n(C \cap M' \cap P') = n(C) - n(C \cap P) - n(C \cap M)$
$+ n(C \cap P \cap M)$
$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

33. **(1)**

Sol. For any $x \in R$, we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number
$\Rightarrow$ xRx for all x. So, R is reflexive.
R is not symmetric, because $\sqrt{2}R1$ but $1 \not R \sqrt{2}$,R is not transitive also because $\sqrt{2}\,R1$ and $1R2\sqrt{2}$ but $\sqrt{2} \not R\ 2\sqrt{2}$

34. **(1)**

Sol. $X \cap (Y \cup X)' = X \cap (Y' \cap X')$
$= X \cap X' \cap Y' = \phi$
$\Rightarrow$ Statement – 1 true.
$X \Delta Y = (X \sim Y) \cup (Y \sim X) = (X \cup Y) \sim (X \cap Y)$
$\Rightarrow$ number of element in $X \Delta Y = m - n$.
$\Rightarrow$ Statement-2 is true

35. **(1)**

Sol. (1) The set $\{3^{2n} - 8n - 1 : n \in N\}$ contains 0 and every element of this set is a multiple of 64.
(2) $2^{3n} - 1$ is always divisible by 7.
(3) $3^{2n} - 1$ is always divisible by 8.
(4) $2^{2n} - 7n - 1$ is always divisible by 49 and $2^{3n} - 7n - 1 = 0$ for $n = 1$.

36. **(1)**

Sol. $n(A \cup B)$ is minimum when $n(A \cap B)$ is maximum i.e. 3.
$\therefore$ minimum $n(A \cup B) = 6$
$n(A \cup B)$ is maximum when $n(A \cap B)$ is minimum i.e. 0
$\therefore$ maximum $n(A \cup B) = 9$

37. **(2)**

Sol. $n(A) = 21, n(B) = 26, n(C) = 29$

$n(A \cap B) = 14,\ n(A \cap C) = 12,\ m(B \cap C) = 13,$

$n(A \cap B \cap C) = 8$

$n(C \cap A' \cap B') = n(C \cap \overline{A \cup B})$

$= n(C) - n((C \cap A) \cup (C \cap B))$

$n(C) - [n(C \cap A) + n(C \cap B) - n(A \cap B \cap C)]$

$29 - [12 + 13 - 8] = 12$

$n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$

$= 14 - 8 = 6$

38. **(2)**

Sol. We have, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C$

$[\because (A \cup C) \cap C = C]$

$\Rightarrow (A \cap B) \cup (B \cap C) = C$ ...(i)

$[\because A \cap C = A \cap B]$

Again, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$

$\Rightarrow B = (A \cap B) \cup (C \cap B)$

$\Rightarrow (A \cap B) \cup (C \cap B) = B$

$\Rightarrow (A \cap B) \cup (B \cap C) = B$ ...(ii)

From (i) and (ii), we get $B = C$

39. **(2)**

Sol. Every element has 3 options. Either set Y or set Z or none

So, number of ordered pairs = 3^5

40. **(2)**

Sol. $X = \{0, 9, ... 4^n - 3n - 1\}$

$Y = \{0, 9, ..., 9(n-1)\}$

Now $4^n - 3n - 1 = (3+1)^n - 3n - 1$

$= 3^n + n.3^{n-1} + ... + {}^nC_2 .9.$

is a multiple of 9.

Also Y consists of all multiples of '9' from 0, 9,.......

Hence all values of X are subset of values of Y.

Thus $X \cup Y = Y$

41. **(2)**

Sol. Number of subsets = 2^m

also number of subsets = 2^n

$2^m - 2^n = 56$

$m = 6$ $n = 3$

$(m, n) = (6, 3)$

42. **(2)**

Sol. Obvious

43. **(1)**

Sol. Obvious

44. **(3)**

Sol. Obvious

45. **(4)**

Sol. Obvious

46. **(3)**

Sol. Obvious

47. **(4)**

Sol. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$0 \le n(A \cap B) \le 5$

$7 \le n(A \cup B) \le 12$

48. **(4)**

Sol. $A = \{2, 3, 5, 7, 11, 13, 17, 19, 13\}$

$B = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$n(A \cup B) = 19,\ n(A \cap B) = 0$

$n(A \cap B') = 9$

49. **(3)**

Sol. $A = \phi$

$P(A) = \{\phi\} \Rightarrow n(P(A)) = 1$

$n(P(P(\phi))) = 2^1$

$n(P(P(P(\phi)))) = 2^2 = 4$

$n(P(P(P(P(\phi))))) = 2^4 = 16$

50. **(2)**

Sol. $A' \cup \{(A \cup B) \cap B'\}$

$A' \cup \{(A \cap B') \cup (B \cap)')\}$

$A' \cup (A \cap B') = (A' \cup A) \cap (A' \cup B')$

$= A' \cup B' = (A \cap B)'$

51. **(3)**

Sol. $X \cap (X \cup Y)' = X \cap (X' \cap Y') = (X \cap X') \cap Y'$

$= \phi \cap Y' = \phi$

52. **(3)**

Sol. $N_3 \cap N_5 = n_{15}$

[$\because$ 3 and 5 are relatively prime numbers]

53. **(2)**

Sol. $\therefore\ x^2 + 4y^2 = 45$

We can see that $x = \pm 3, y = \pm 3$

(3,3) (3,–3), (–3,–3), (–3,3) are 4 elements

54. **(3)**

Sol. $2^m + 2^n = 144$

$2^n\{2^{m-n} + 1\} = 2^4 \times 3^2$

$n = 4, m - n = 3$

$n = 4$, $m = 7$

55. **(1)**

Sol. Every element of X have 3 options
Either in Y or in Z or more
So, number of ordered pairs = 3^4

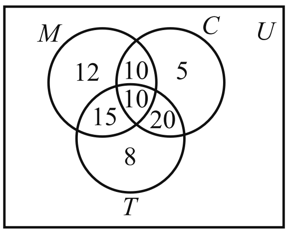
56. **(1)**

Sol. $n(U) = 100$

$n(M \cap C \cap T) = 10; n(M \cap C) = 20$

$n(C \cap T) = 30; n(M \cap T) + 25$

$n(M$ only) = 12; $n($ only $C)$ = 5; n (only$T$) = 8



$\therefore\ n(M \cup C \cup T)$

12 + 10 + 5 + 15 + 10 + 20 + 8 =80

$\therefore\ n(M \cap C \cup T)' = 100 - 80 = 20$

57. **(1)**

Sol. Subsets = $2^9 - 1 - 1 = 510$

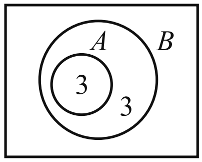
Integer Type Questions (58 to 67)

58. **(32)**

Sol. $A = \{-2, -1, 0, 1, 2\}$
No. of subsets = $2^n = 2^5 = 32$

59. **(6)**

Sol.



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow$ minimum value of $n(A \cup B)$

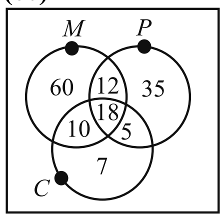
= 3 + 6 – 3 = 6

60. **(300)**

Sol. $n(A^c \cap B^c) = n[(\{A \cup B)^c] = n(U) - n(A \cup B)$
$= n(U) - [n(A) + n(B) - n(A \cap B)]$
= 700 – [200 + 300 – 100] = 300.

61. **(60)**

Sol.



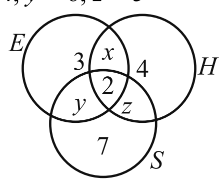
$n(M) = 100$
$n(P) = 70$
$n(C) = 40$
$n(M \cap P) = 30$
$n(M \cap C) = 28$
$n(P \cap C) = 23$
$n(M \cap P \cap C) = 18$

62. **(15)**

Sol. $x + y = 10$; $x + z = 9$; $y + z = 11$

$\Rightarrow$ $x + y + z = 15$

$x = 4, y = 6, z = 5$



63. **(7)**

Sol.

$70 + 72 - t_1 = 100$

$t_1 = 42\%$

$\Rightarrow$ min. in $P \cap C = 42\%$

$t_2 = 85\% - 20\% = 65\%$

$\Rightarrow$ min. $M \cap E = 65\%$

$t = 42 - 35 = 7\%$

min. in $((P \cap C) \cap (M \cap E)) = 7\%$

64. (18)

Sol. $a = 18$, $a + d = 23$, $c + d = 8$

$c + f = 8$, $a + b + c + d = 26$

$c + d + f + g = 48$

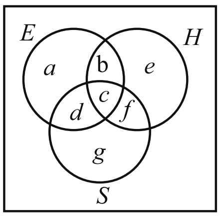
$a + b + c + d + e + f + g = 100 - 24 = 76$

$a = 18$, $d = 5$, $c = 3$, $f = 5$, $b = 0$

$g = 48 - (3 + 5 + 5) = 35$

$e = 76 - (18 + 0 + 3 + 5 + 5 + 35) = 10$

Now $b + c + e + f = 0 + 3 + 10 + 5 = 18$



65. (266)

Sol. $n(2) = \left[\frac{1000}{2}\right] = 500$

$n(3) = \left[\frac{1000}{3}\right] = 333$

$n(5) = \left[\frac{1000}{5}\right] = 200$

$n(2 \cap 3) = 166, n(3 \cap 5) = 66$

$n(5 \cap 2) = 100,\ n(2 \cap 3 \cap 5)$

$= 33, n(2 \cup 3 \cup 5) = 734$

$n(2' \cap 3' \cap 5') = 1000 - 734 = 266$

66. (30)

Sol. $80 = 40 + 50 + 60 - 2$

$(n(A \cap B) + n(B \cap C) + n(C \cap A)) + 30$

$\Rightarrow n(A \cap B) + n(B \cap C) + (C \cap A) = 50$

Required number of members

$T = n(A \cap B) + n(B \cap C) + n(C \cap A)$

$-2n(A \cap B \cap C)$

$= 50 - 2 \times 10 = 30$

67. (18)

Sol. $0 \le n(A \cap B) \le \text{min. } \{n(A), n(B)\}$

$0 \le n(A \cap B) \le 12$

$n(A' \cap B) = n(B) - n(A \cap B)$

$3 \le n(A' \cap B) \le 15$

$\Rightarrow x = 3,\ y = 15$

RELATIONS & FUNCTIONS

Single Option Correct Type Questions (01 to 63)

1. **(4)**

Sol: We have $(a, b)\ R\ (a, b)$ for all $(a, b) \in N \times N$
Since $a + b = b + a$. Hence, R is reflexive
R is symmetric for all $(a, b), (c, d) \in N \times N$ we have $(a, b)\ R\ (c, d)$

$\Rightarrow a + b = b + c$

$\Rightarrow c + b = d + a$

$\Rightarrow (c, d)\ R\ (a, b)$

$(a, b)\ R\ (c, d)$ and $(c, d) R (e, f)$

$a + d + c + d = b + c + d + e$

$\Rightarrow a + f = b + e$

$\Rightarrow (a, b)\ R\ (e, f) \Rightarrow R$ is transitive

Thus, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow (a, b)\ R\ (e, f)$

2. **(4)**

Sol: Reflexive Relation

$A \cdot A \neq 0$ for $\forall A \in S$ so Relation is not Reflexive Relation

Symmetric Relation

$A. \cdot B = 0 \Rightarrow BA = 0$ Not true $\forall\ A, B \in S$

For example

$$A = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow AB = 0$$

but $BA \neq 0$

Transitive Relation:

$AB = 0,\ BC = 0 \Rightarrow AC = 0$ Not True, $\forall\ A, B, C \in S$

For example

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$\Rightarrow AB = 0, BC = 0$

but $AC \neq 0$

3. **(4)**

Sol: $f(x) = \dfrac{2x^2 - x + 5}{7x^2 + 2x + 10}$, Domain $x \in R$

$$\Rightarrow f'(x) = \frac{(4x-1)(7x^2+2x+10) - (14x+2)(2x^2-x+5)}{(7x^2+2x+10)^2}$$

$$f'(x) = \frac{11x^2 - 30x - 20}{(7x^2+2x+10)^2} > 0$$

$$\Rightarrow x \in (-\infty, 0) \cup \left(\frac{30}{11}, \infty\right)$$

$$f'(x) < 0 \quad \Rightarrow \quad x \in \left(0, \frac{30}{11}\right)$$

$$\Rightarrow \quad f'(x) = 0 \quad \Rightarrow x = 0, \frac{30}{11}$$

Function is increasing and decreasing in different intervals, so non monotonic

$\therefore$ Many one function

4. **(4)**

Sol: $f(x) = \sin\left(\sqrt{[a]}x\right) \quad \Rightarrow$ Period $= \dfrac{2\pi}{\sqrt{[a]}} = \pi$

$[a] = 4$

$\Rightarrow \quad a \in [4, 5)$

5. **(1)**

Sol: For any $x \in R$, we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number

$\Rightarrow xRx$ for all x, So, R is reflexive.

R is not symmetric, because $\sqrt{2}\,R1$ but 1 R $\sqrt{2}, R$ is not transitive also because $\sqrt{2}R1$ and 1R2 $\sqrt{2}$ but $\sqrt{2}\,R\,2\sqrt{2}$

6. **(1)**

Sol: $((m, n), (p, q)) \in S$

$\Rightarrow m+q=n+p$

$((p,q),(r,s)) \in S \quad \Rightarrow p+s=r+q$

$\Rightarrow ((r,s),(m,n)) \in s$

As $r+n = m+s$

Now if we add above equation

$((m,n),(p,q)) \in s \ \Rightarrow m+q=n+p$

$\Rightarrow \ (n+p)=m+q$ &

Hence $((p, q), (m, n))$

7. **(3)**

Sol: $R_1 : m + 4n = 5n + (m - n)$

$R_2 : m + 9n = 10n + (m - n)$

If $5n + (m - n)$ is divisible by 5 then $10\,n + (m - n)$ is also divisible by 5 and vice versa.

Hence $R_1 = R_2$

Also R_1 & R_2 is symmetric relation on Z.

8. **(4)**

Sol: $(x,y) \in X \quad \Rightarrow x<y$ and $y = x + 5$

$\Rightarrow \ (x,x) \notin X$, Hence not reflective

If $(x,y) \in X \quad \Rightarrow \ x<y$ and $y=x+5$

$(y,x) \in X \quad \Rightarrow X$ is not symmetric

Let $(x,y) \in X$ and

$(y,z) \in X \quad \Rightarrow x<y; y=x+5$

and y < z; z = y + 5

$\Rightarrow x<z$ and $z = x + 10 \quad \Rightarrow \ (x,z) \notin X$

Not transssitive

9. **(1)**

Sol: $\left[x-\frac{1}{3}\right]=\left[x+\frac{2}{3}\right]-1$

$\therefore$ the equation becomes

$[x]+\left[x+\frac{1}{2}\right]+\left[x+\frac{2}{3}\right]=9$

Let $n \le x < n+1$, then

$[x]=n, \left[x+\frac{1}{2}\right]=n$ or $n+1, \left[x+\frac{2}{3}\right]$

$=n$ or $n+1$

$\therefore \quad [x]+\left[x+\frac{1}{2}\right]+\left[x+\frac{2}{3}\right]=3n, 3n+1, 3n+2$

$\therefore$ the only possible case is $3n = 9$ *i.e.* $n = 3$

$\therefore \ 3 \le x < 4, \ \ 3 \le x+\frac{1}{2} < 4$

and $3 \le x+\frac{2}{3} < 4$ *i.e.* $3 \le x < \frac{10}{3}$

$a=3, b=\frac{10}{3}$

10. **(4)**

Sol: $f(x)=\{x\}\times 100$

$f(\sqrt{3})=\{\sqrt{3}\}\times 100$

$[f(\sqrt{3})]=[73.2----]=73$

11. **(4)**

Sol: $x^2-10x+25\,\text{sgn}(x^2+4x-32) \le 0$

$$\begin{cases} x^2-10x+25 \le 0 & \text{when} \quad (x+8)(x-4)>0 \quad \text{or} \quad x\in(-\infty,-\infty)\cup(4,\infty) \\ x^2-10x-25 \le 0 & \text{when} \quad (x+8)(x-4)<0 \quad \text{or} \quad x\in(-8,4) \\ x^2-10x \le 0 & \text{when} \quad (x+8)(x-4)=0 \quad \text{or} \quad x\in\{4,-8\} \end{cases}$$

Total solutions = –2, –1,0, 1, 2, 3, 4, 5

12. **(4)**

Sol: $[x + [2x]] < 3$

$[x] + [2x] < 3$

True for $(-\infty, 1)$

13. **(1)**

Sol: (i) $-x^2+5x-6 \ge 0 \Rightarrow \qquad x\in[2,3]$

And

(ii) $2\{x\}<1 \quad \Rightarrow \quad \{x\}<\frac{1}{2}$

$\Rightarrow \quad x \in \left[2, \frac{5}{2}\right) \cup \{3\}$

14. **(4)**

Sol: $f(x)=\log_{1/2}\left(-\log_2\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$

$\Rightarrow -\log_2\left(1+\frac{1}{x^{1/4}}\right)-1>0$

$\Rightarrow \quad -\infty<\log_2\left(1+\frac{1}{x^{1/4}}\right)<-1$

$\Rightarrow 0<1+\frac{1}{x^{1/4}}<\frac{1}{2}\Rightarrow -1<\frac{1}{x^{1/4}}<-\frac{1}{2}$

$\Rightarrow x\in\phi$ (null set) $\Rightarrow x\in\phi$

15. **(2)**

Sol: $q^2-4pr=0, p>0$

$\Rightarrow f(x)=\log(px^3+(p+q)x^2+(q+r)x+r)$

Let

$g(x)=px^3+(p+q)x^2+(q+r)x+r$

$\Rightarrow \quad g(x)=(x+1)\,(px^2+qx+r)$

Discriminant of px^2+qx+r is $q^2-4pr=0$

Domain $(x+1)(px^2+qx+r)>0$

$\Rightarrow \quad p(x+1)\left(x+\frac{q}{2p}\right)^2>0$

$\Rightarrow \quad x\neq-\frac{q}{2p}$ and $x>-1$

$\therefore \ x\in R-[(-\infty,-1]\cup\left\{-\frac{q}{2p}\right\}$

16. **(3)**

Sol: $f(x)=\frac{x-[x]}{1+x-[x]}=\frac{\{x\}}{1+\{x\}}=1-\frac{1}{1+\{x\}}$

$\because \ \{x\}\in[0,1) \quad \Rightarrow f(x)\in\left[0,\frac{1}{2}\right)$

17. **(4)**

Sol: $f(x)=\frac{e^x-e^{-|x|}}{e^x+e^{|x|}}$

If $x\geq 0,\quad f(x)=\frac{e^x-e^{-x}}{2e^x}=\frac{1}{2}-\frac{1}{2(e^x)^2}=\frac{1}{2}$

$\left(1-\frac{1}{(e^x)^2}\right); f(x)\in\left[0,\frac{1}{2}\right) \quad$ ….(i)

$f(x)\in\left[0,\frac{1}{2}\right)$

If $x<0,\ f(x)=\frac{e^x-e^x}{e^x+e^{-x}}=0 \qquad$ ….(ii)

$\therefore$ range of $f(x)$ is (i) $\cup$(ii) $=\left[0,\frac{1}{2}\right)$

18. **(4)**

Sol: Here

$(2-\log_2(16\sin^2 x+1)>0$

$\Rightarrow \ 0<16\sin^2 x+1<4$

$\Rightarrow \ \frac{-1}{16}\leq\sin^2 x<\frac{3}{16}$

$\Rightarrow \ 1\leq 16\sin^2 x+1\leq 4$

$\Rightarrow 0\leq\log_2(16\sin^2 x+1)<2$

$\Rightarrow \ 2\geq 2-\log_2(16\sin^2 x+1)>0$

$\Rightarrow \quad \log_{\sqrt{2}}2\geq\log_{\sqrt{2}}(2-\log_2(16\sin^2 x+1))>-\infty$

$\Rightarrow 2\geq y>-\infty$

Hence range is $y\in(-\infty,2]$

19. **(1)**

Sol: $f(6\{x\}^2-5\{x\}+1)$

$\Rightarrow \ f((3\{x\}-1)\,(2\{x\}-1))$

$\Rightarrow \ (3\{x\}-1)\,(2\{x\}-1)\leq 0$

Or $\{x\}\in\left[\frac{1}{3},\frac{1}{2}\right] \qquad \therefore x\in\bigcup_{n\in I}\left[n+\frac{1}{3},n+\frac{1}{2}\right]$

20. **(3)**

Sol: $f:(e,\infty)\to R \qquad \Rightarrow \ f(x)=\ell n(\ell n(\ell nx))$

$D:\ell n(\ell nx)>0 \qquad$ or $\qquad \ell nx>1 \quad$ or $x>$ e

$R:(-\infty,\infty) \qquad \Rightarrow$ one-one and onto function

21. **(3)**

Sol: $f(x)=2[x]+\cos x$;

$f(x)=\cos x \qquad x\in[0,1)$

$= 2+\cos x \qquad x\in[1,2)$

$= 4 + \cos x \quad x \in [2,3)$

$= 6 + \cos x \qquad x \in [3,4)$

For $x \in [0,1) \qquad f'(x) = -ve$

$x \in [1,2) \qquad f'(x) = -ve$

$x \in [2,3) \qquad f'(x) = -ve$

$x \in [3,4) \qquad f'(x) = +ve$

$\Rightarrow$ function is not one-one

If $x \in [0,1)$ range : [1, cos 1)

$x \in [1,2)$ range : [2 + cos 1, 2 + cos 2]

Not onto function

22. (4)

Sol: $f(2) = f(3^{1/4}) = 0$ many one

$f(x) \neq -\sqrt{3} \qquad \therefore$ Into

23. (2)

Sol: $f:[1,\infty) \to [1,\infty)$

$f(x) = 2^{x(x-1)}$

$x(x-1)$ is strictly increasing in domain

$f(x) = 2^{x(x-1)}$ is one one & onto function so inverse is defined

$2^{x(x-1)} = y \quad \Rightarrow x^2 - x = \log_2 y$

$\Rightarrow x^2 - x - \log_2 y = 0$

$$x = \left(\frac{1 \pm \sqrt{1+4\log_2 y}}{2}\right)$$

– ve sign rejected

$$f^{-1}(x) = \left(\frac{1 \pm \sqrt{1+4\log_2 x}}{2}\right)$$

24. (1)

Sol: $f: N \to N$

$f(x) = x + (-1)^{x-1}$

$$\Rightarrow f(x) = \begin{cases} x-1, & x \to \text{Even natural} \\ x+1, & x \to \text{Odd natural} \end{cases}$$

$$\Rightarrow f^{-1}(x) = \begin{cases} x+1, & x \to \text{Odd natural} \\ x-1, & x \to \text{Even natural} \end{cases}$$

$\therefore f^{-1}(x) = x + (-1)^{x-1}$

25. (1)

Sol: Clearly (II) also satisfied (i), (ii), (iii) but not (iv) but (I) satisfies all the condign

26. (1)

Sol: $\cos 4\theta = \frac{1}{3} \Rightarrow 2\cos^2 2\theta - 1 = \frac{1}{3}$

$\Rightarrow \cos^2 2\theta = \frac{2}{3} = \quad \Rightarrow \cos 2\theta = \pm\sqrt{\frac{2}{3}}$

Now

$$f(\cos 4\theta) = \frac{2}{2-\sec^2\theta} = \frac{1+\cos 2\theta}{\cos 2\theta} = 1 + \frac{1}{\cos 2\theta}$$

$$\Rightarrow f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$$

27. (2)

Sol: $(x,x) \in R$ for $w = 1$

$\therefore$ R is reflexive

If $x \neq 0$, then $(0,x) \in R$ for $w = 0$ but $(x,0) \in R$ for any w

$\therefore$ R is not symmetric $\Rightarrow$ R is not equivalence relation

$\left(\frac{m}{n}, \frac{p}{q}\right) \in S \Rightarrow qm = pm \Rightarrow \frac{m}{n} = \frac{p}{q}$

(i) $\frac{m}{n} = \frac{m}{n} \Rightarrow \left(\frac{m}{n}, \frac{m}{n}\right) \in S \Rightarrow$ Reflexive

(ii) $\frac{m}{n} = \frac{p}{q} \Rightarrow \frac{p}{q} = \frac{m}{n} \Rightarrow$ Symmetric

(iii) $\frac{m}{n} = \frac{p}{q}$ and $\frac{p}{q} = \frac{x}{y} \Rightarrow \frac{m}{n} = \frac{x}{y}$

$\Rightarrow$ transitive $\Rightarrow$ S is equivalence relation

28. (2)

Sol: For reflexive $(A,A) \in R \Rightarrow A = P^{-1}AP$

Which is true for P = I

$\therefore$ reflexive

For symmetric: As $(A, B) \in R$ for matrix P

$A = P^{-1} BP$

$\Rightarrow \quad PA = PP^{-1}BP \quad \Rightarrow \quad PAP^{1} = IBPP^{-1}$

$\Rightarrow PAP^{-1} = |B| \quad \Rightarrow \quad PAP^{-1} = B$

$\Rightarrow B = PAP^{-1}$

$\therefore \quad (B, A) \in R$ for matrix P–1

$\therefore \quad R$ is symmetric

For transitivity

$A = P^{-1}BP$ and

$\Rightarrow A = (P^{-1})^2 CP^2$

$\Rightarrow A = (P^2)^{-1} C(P^2)$

$\therefore \quad (A, C) \in R$ fro matrix P^2

$\therefore \quad R$ is transitive

So R is equivalence

29. **(2)**

Sol: $K = \{4, 8, 12, 16, 20\}$

$F(k)$ can take values from set $\{3, 6, 9, 12, 15, 18\}$ this can be done is ${}^6C_5 \times 5!$ Ways. = 6! Ways and options for remaining 15 elements of A + 15!

30. **(3)**

Sol: $y = 2[x] + 3$ and $y = 3[x–2]$

$2[x] + 3 = 3[x] – 6$

$\Rightarrow \quad [x] = 9$

$\Rightarrow \quad x \in [9, 10)$

$\therefore \quad y = 21$

$\therefore \quad [x + y] = 30$

31. **(4)**

Sol: (1) $\sqrt{1 + \sin x} = \left|\sin\frac{x}{2} + \cos\frac{x}{2}\right|$

$\sin\frac{x}{2} + \cos\frac{x}{2}$ Non-identical function

(2) $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1} x, x \in [-1, 1]$ only

$2\tan^{-1} x, x \in R$ Non-identical function

(3) $\sqrt{x^2} = |x|, x \in R$

$(\sqrt{x})^2 = x, x \in R^+ \cup \{0\}$

Non-identical function

(4) $\ell nx^3 + \ell nx^2 = 5\ell nx, x > 0$

$5\ell nx, x > 0$ Identical function

32. **(4)**

Sol: $f(x) = \dfrac{a^x - 1}{x^n(a^x + 1)}$

$\because \quad f(x) = f(-x)$

$\Rightarrow \quad \dfrac{1 - a^x}{(-x)^n(1 + a^x)} = \dfrac{a^x - 1}{x^n(a^x + 1)}$

$\Rightarrow (-x)^n = -x^n$

33. **(2)**

Sol: $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1 = \dfrac{x}{2}\left(\dfrac{e^x + 1}{e^x - 1}\right) + 1$

$f(-x) = -\dfrac{x}{2}\left(\dfrac{e^{-x} + 1}{e^{-x} - 1}\right) + 1 = \dfrac{x}{2}\left(\dfrac{e^x + 1}{e^x - 1}\right) + 1$,

$= f(x)$ even function

34. **(1)**

Sol: $f : R \to [-1, 1]$

$$f(x) = \sin\left(\frac{\pi}{2}[x]\right) = \begin{cases} -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ 1, & 1 \le x \le 2 \end{cases}$$

Many-one function

Into function

Also

$f(x + 4) = \sin\left(\frac{\pi}{2}[x + 4]\right)$

$= \sin\left(2\pi + \frac{\pi}{2}[x]\right) = \sin\left(\frac{\pi}{2}[x]\right)$

35. **(2)**

Sol: $f : R \to R, \quad f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$D \le 0$ or $4a^2 - 12b \le 0$

Or $a^2 \le 3b$

36. **(1)**

Sol: $x_1 = x_2$

As f is one-one function

So $f(x_1) = f(x_2)$

As g is one-one function

So g{$f(x_1)$} = g{$f(x_2)$}
gof(x_1) = gof(x_2)

37. (1)

Sol: $f\left(x+\frac{1}{3}\right)$

$=\left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right]+[x+1]-3\left(x+\frac{1}{3}\right)+15$

$=\left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right]+[x]-3x+15=f(x)$

$\therefore$ Fundamental period is 1/3

38. (3)

Sol: $f(x)=[\sin 3x]-|\cos 6x|$

period $\Rightarrow \frac{2\pi}{3},\frac{\pi}{3}$

period of $f(x)$ = L.C.M $\left(\frac{2\pi}{3},\frac{\pi}{3}\right)=\frac{2\pi}{3}$

Fundamental period $=\frac{2\pi}{3}$

39. (2)

Sol: $f(x)=|x-1|,\ \ f:R^+\to R$

$g(x)=e^x,\ \ \ g:[-1,\infty)\to R$

$fog(x)=f[g(x)]=|e^x-1|$

$D:[-1,\infty)$

$R:[0,\infty)$

40. (3)

Sol: $x\in(2,4)\quad\Rightarrow\left[\frac{x}{2}\right]=1$

So $f(x)=x-1\quad\Rightarrow\quad y=x-1$

$\Rightarrow\ x=y+1\quad\Rightarrow\ f^{-1}(x)=x+1$

41. (1)

Sol: $f(g)(x_1))=f(g(x_2))$

$\Rightarrow g(x_1)=g(x_2)$

As f is one-one function

$\Rightarrow\ x_1=x_2$

As g is one-one function
Hence $f(g(x_1))=f(g(x_2))$

$\Rightarrow\ x_1=x_2$

$\Rightarrow$ $f(x)$ is one-one function

42. (4)

Sol: If $f(x)$ is increasing continuous function in $[\alpha,\beta]$, then its range is $[f(\alpha),f(\beta)]$ but for discontinuous function the statement is not true.

43. (4)

Sol: $y=f(x)$ and $y=f^{-1}(x)$ can intersect at points other than $y=x$

e.g. $y=-x+c$ or $y=\sqrt{1-x^2}$

44. (2)

Sol: $\cos\sqrt{x+T}=\cos\sqrt{x}$

$\sqrt{x+T}=2n\pi\pm\sqrt{x}$ On squaring both sides

$x+T=4n^2\pi^2+x\pm 4n\pi\sqrt{x}$

$\Rightarrow\ T=4n^2\pi^2\pm 4n\pi\sqrt{x}$

T is not independent of x. Hence function is non periodic

45. (3)

Sol: $f:N\to I$

$$f(n)=\begin{cases}\frac{n-1}{2}, & n\to\text{odd}\\ -\frac{n}{2} & n\to\text{even}\end{cases}$$

For $n\to$ odd numbers

$f(n)=0,1,2,3,.......$

For $f(n)\to -1,-2,-3,....$

$\therefore$ $f(n)$ is one-one

Range $\in I$ $\therefore$ onto function

46. (4)

Sol: $f(x+y)=f(x)+f(y)$
Function should be $y=mx$
$f(1)=7$

$\therefore$ $m=7$

$\Rightarrow$ $f(x)=7x$

$\sum_{r=1}^{n}f(r)=7\sum_{r=1}^{n}=\frac{7n(n+1)}{2}$

47. (4)

Sol: $4-x^2\neq 0,\ x^3-x>0$

$x \neq \pm 2$ and $-1 < x < 0$ or $1 < x < \infty$

$\therefore \quad D = (-1, 0) \cup (1, \infty) - \{2\}$

or $D = (-1, 0) \cup (1, 2) \cup (2, \infty)$

48. **(2)**

Sol: $f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$

$f(-x) = \log\left(-x + \sqrt{x^2 + 1}\right)$

$= \log\left(\frac{-x^2 + x^2 + 1}{x + \sqrt{x^2 + 1}}\right)$

$= -\log\left(x + \sqrt{x^2 + 1}\right) = -f(x)$

$\Rightarrow \quad f(x)$ is an odd function

49. **(4)**

Sol: We know that

$-\sqrt{a^2 + b^2} \leq a \sin\theta + b \cos\theta \leq \sqrt{a^2 + b^2}$

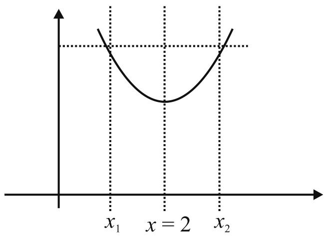
$\therefore \quad -2 \leq \sin x - \sqrt{3} \cos x \leq 2$

$\Rightarrow \quad -1 \leq \sin x - \sqrt{3} \cos x + 1 \leq 3$

$\therefore \; f(x) \in [-1, 3]$

50. **(2)**

Sol: Let us consider a graph symmetric w.r.t. line $x = 2$ as shown in figure



From figure $f(x_1) = f(x_2)$

Where $x_1 = 2 - x$ & $x_2 = 2 + x$

$\therefore \quad f(2 - x) = f(2 + x)$

51. **(1)**

Sol: $f(x) = (x-1)^2 + 1, x \geq 1$

$f : [1, \infty) \to [1, \infty)$ is a bijective function

$\Rightarrow \quad y = (x-1)^2 + 1 \Rightarrow (x-1)^2 = y - 1$

$\Rightarrow \; x = 1 \pm \sqrt{y-1} \Rightarrow f^{-1}(y) = 1 \pm \sqrt{y-1}$

$\Rightarrow \quad f^{-1}(x) = 1 + \sqrt{x-1} \quad \{\because x \geq 1\}$

So statement-2 is correct

Now

$f(x) = f^{-1}(x) \Rightarrow f(x) = x$

$\Rightarrow \; (x-1)^2 + 1 = x$

$\Rightarrow \; x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$

So statement-1 is correct

52. **(2)**

Sol: $f(x) + 2f\left(\frac{1}{x}\right) = 3x$

$S : f(x) = f(-x)$

$f(x) + 2f\left(\frac{1}{x}\right) = 3x \qquad \ldots\ldots(1)$

$x \to \qquad f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \qquad \ldots\ldots.(2)$

(1) $-2\times(2), -3f(x) = 3x - \frac{6}{x}, \; f(x) = \frac{2}{x} - x$

Now $\qquad f(x) = f(-x)$

$\therefore \; \frac{2}{x} - x = \frac{2}{x} + x, \; \frac{4}{x} = 2x$

$\frac{2}{x} = x \quad \Rightarrow x = \pm\sqrt{2}$

Exactly two elements

53. **(1)**

Sol: $f(x) = |\ell n 2 - \sin x|$

$f(f(x)) = |\ell n 2 - \sin|\ell n 2 - \sin x|$

$g(x) = \ell n 2 - \sin(\ell n 2 - \sin x)$

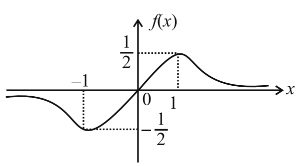
Hence $g(x)$ is differentiable at $x = 0$ as it is sum and composite of differentiable function

$g'(x) = \cos(\ell n 2 - \sin x).\cos x$

$g'(0) = \cos(\ell n 2)$

54. **(3)**

Sol:

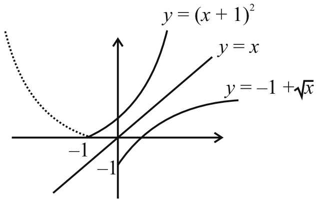


$f(x)=\frac{x}{1+x^2}; f:R\to\left[-\frac{1}{2},\frac{1}{2}\right]$

From the graph of $f(x)$ we can observe that function is many one and onto

55. **(4)**

Sol: $f(x)=y=(x+1)^2, x\ge -1$ and $y\ge 0$



$\Rightarrow \quad x=-1\pm\sqrt{y}$

$\therefore \quad f^{-1}(x)=-1+\sqrt{x}, \quad x>0 \Rightarrow \quad y\ge -1$

56. **(1)**

Sol: $f(x)=2x+\sin x$

$f'(x)=2+\cos x>0$

Continuous increasing function. One-One function

$\lim_{x\to\infty}(2x+\sin x)=+\infty$

$\lim_{x\to-\infty}(2x+\sin x)=-\infty$

$\therefore$ Range = R (onto function)

57. **(3)**

Sol: $y=\frac{x^2+x+2}{x^2+x+1}, x\in R$

$\Rightarrow x^2(x-1)+x(y-1)+(y-2)=0$

$D\ge 0 \Rightarrow (y-1)^2-4(y-1)(y-2)\ge 0$

$\Rightarrow (y-1)(3y-7)\le 0 \Rightarrow 1\le y\le \frac{7}{3}$

But for y = 1 quadratic vanishes to

Put y= 1, $1=\frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow 1=2$ not possible

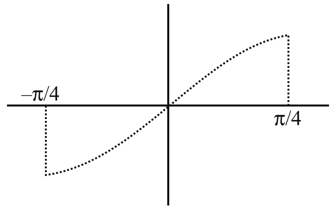
$\Rightarrow$ Not real values of x.

$\therefore$ y = 1 is not in the range

58. **(3)**

Sol: $g(f(x))=(\sin x+\cos x)^2-1$

$=1+\sin 2x-1=\sin 2x \Rightarrow \frac{-\pi}{2}\le 2x\le \frac{\pi}{2}$



$\therefore$ gof(x) is invertible in $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$

59. **(1)**

Sol: $y=(f-g)x=\begin{cases}-x, & x\in Q\\ x, & x\notin Q\end{cases}$

Which is one-one and onto function

60. **(1)**

Sol: $f(x)=x^2; g(x)=\sin x$

$\Rightarrow gof(x)=\sin x^2$

$\Rightarrow gogof(x)=\sin(\sin x^2)$

$\Rightarrow (fogogof)(x)=(\sin(\sin x^2))^2=\sin^2(\sin x^2)$

Now

$\sin^2(\sin x^2)=\sin(\sin x^2)$

$\Rightarrow \sin(\sin x^2)=0,1$

$\Rightarrow \sin x^2=n\pi,(4n+1)\frac{\pi}{2}, n\in I$

$\Rightarrow \sin x^2=0$

$\Rightarrow x^2=n\pi \quad \Rightarrow gogof(x)=\sin(\sin x^2)$

61. **(2)**

Sol: $f:[0,3]\to[1,29]$

$f(x)=2x^3-15x^2+36x+1$

$f'(x)=6x^2-30x+36$

$=6(x^2-5x+6)=6(x-2)(x-3)$

In given domain function has local maxima, it is many-one

Now at x = 0, $f(0)$ = 1

$x=2, \ f(2)=16-60+72+1=29$

x = 3, $f(3)$ = 54–135+108+1=163–135=28

Has range = [1, 29]

Hence given function is onto

62. **(1)**

Sol: (i) $f(-x)=-f(x)$ so it is odd function

(ii) $f'(x)=3(\log(\sec x+\tan x))^2 \dfrac{1}{(\sec x+\tan x)}$

$(\sec x\tan x+\sec^2 x)>0$

(ii) Range of $f(x)$ is R

as $f\left(-\dfrac{\pi}{2}\right) \Rightarrow -\infty$

$f\left(\dfrac{\pi}{2}\right) \Rightarrow \infty$

63. **(4)**

Sol: Let $y=x^2+1 \Rightarrow x=\pm\sqrt{y-1}$

$\Rightarrow f^{-1}(y)=\pm\sqrt{y-1}$

$\Rightarrow f^{-1}(x)=\pm\sqrt{x-1}$

$\Rightarrow f^{-1}(17)=\pm\sqrt{17-1}=\pm 4$

and $f^{-1}(-3)=\pm\sqrt{-3-1}=\pm\sqrt{-4}$,

which is not possible

Integer Type Questions (64 to 73)

64. **(2)**

Sol: f & g are 2 distinct functions $[-1,1] \to [0,2]$ onto functions

So f & g are either $-x+1$ or $x+1$

Case-I: $f(x)=-x+1$; $g(x)=x+1$

$$h(x)=\frac{f(x)}{g(x)}=\frac{1-x}{1+x} \quad ; \quad h(h(x))=\frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}=x$$

$$h(1/x)=\frac{x-1}{x+1} \quad ; \quad h(h(1/x)]=\frac{1-\frac{x-1}{x+1}}{1+\frac{x-1}{x+1}}=\frac{1}{x}$$

$$\left|h(h(x))+h\left(h\left(\frac{1}{x}\right)\right)\right|=|x+1/x|>0$$

as domain does not contain point $x=\pm 1$

Case-II: $f(x)=1+x$; $g(x)=1-x$

$$h(x)=\frac{1+x}{1-x} \quad ;h(h(x))=\frac{1+\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}}=-\frac{1}{x}$$

$$h(h(1/x))=-x \quad ;\left|h(h(x))+h\left(h\left(\frac{1}{x}\right)\right)\right|$$

$$=|-x-1/x|=(x+1/x)>2$$

65. **(0)**

Sol: $f(-x)=-[ax^7+bx^3+cx]-5$

; $f(-x)=-[f(x)+5]-5$

$f(-x)=-f(x)=-10$ put $x=7$

. $f(7)=-17$.

so $f(7)+17\cos x=-17(cox-1)\in[-34,0]$

66. **(22)**

Sol: 21 x

22 y

23 z

	Case-I	Case-II	Case-II
$f(21)=x$	T	F	F
$f(22)\neq x$	F	T	F
$f(23)\neq y$	F	F	T

Case-I $f(21)=x,\ f(23)=y$

Then $f(21)=x$ is not true

Case-II $f(23)=y,\ f(22)=z, f(21)=x$

Not possible

Case-III $f(22)=x, f(23)=z, f(21)=y$

$\therefore\ f^{-1}(x)=22$

67. **(0)**

Sol: Given $f(y)=\log y \quad \Rightarrow f(1/y)=\log(1/y)$

then $f(y)+f\left(\dfrac{1}{y}\right)=\log y+\log(1/y)=\log 1=0$

68. **(14)**

Sol: Number of surjection from A to B

$= \sum_{r=1}^{2} (-1)^{2-r}\ {}^2C_r (r)^4$

$= (-1)^{2-1}\ {}^2C_1 (1)^4 + (-1)^{2-2}\ {}^2C_2 (2)^4$

$= -2 + 16 = 14$

Therefore, number of surjection from A to $B = 14$.

69. **(4)**

Sol: Case-I $x \le \frac{1}{2}$

$-2x + 1 = 3[x] + 2\{x\}$

$\Rightarrow x = \frac{1}{4}$

Case-II $x > \frac{1}{2}$

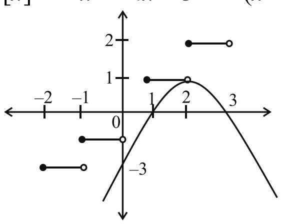
$\Rightarrow 2x - 1 = 3[x] + 2\{x\}$

$\Rightarrow 2[x] + 2\{x\} - 1 = 3[x] + 2\{x\}$

$[x] = -1 \Rightarrow x \in \phi$

70. **(0)**

Sol: $[x] = -x^2 + 4x - 3 = -(x-1)(x-3)$



No solution

71. **(12)**

Sol: $\therefore \sin x$ has period $= 2\pi$

$\Rightarrow \sin\frac{\pi x}{2}$ has period $= \frac{2\pi}{\frac{\pi}{2}} = 4$

$\because \cos x$ has period $= 2\pi \Rightarrow \cos\frac{\pi x}{3}$ has period

has period $= \frac{2\pi}{\frac{\pi}{3}} = 6 \Rightarrow 2\cos\frac{\pi x}{3}$

has period = 6

$\because \tan x$ has period $= \pi \Rightarrow \tan\frac{\pi x}{4}$ has period

$= \frac{\pi}{\frac{\pi}{4}} = 4$.

L.C.M. of 4, 6 and 4 = 12, period of $f(x) = 12$

72. **(24)**

Sol: $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$

$\frac{x}{9} - \left\{\frac{x}{9}\right\} = \frac{x}{11} - \left\{\frac{x}{11}\right\}$

$\frac{2x}{99} = \left\{\frac{x}{9}\right\} - \left\{\frac{x}{11}\right\} \in (-1, 1)$

$x \in \left(-\frac{99}{2}, \frac{99}{2}\right) \Rightarrow x \in \left(0, \frac{99}{2}\right)$

Here $x \in \{1, 2, 3, \ldots\ldots\ldots\ldots 8\}$

$\cup \{11, 12, 13, 14, 15, \ldots\ldots 17\}$

$\cup \{22, 23, \ldots\ldots, 26\} \cup \{33, 34, 35\} \cup \{44\}$

Total positive integers = 24

73. **(6)**

Sol: $f(x) = \left[\frac{x}{15}\right]\left[\frac{-15}{x}\right]$

If $x \ge 15$, then $\left[-\frac{15}{x}\right] = -1$

$\therefore f(x) = -\left[\frac{x}{15}\right]$

so $x \in [15, 90)$, $\frac{x}{15} \in [1, 6)$

$\therefore f(x) = -1, -2, -3, -4, -5$ and if

$x \in (0, 15)$, then $\left[\frac{x}{15}\right] = 0$

$\therefore f(x) = 0$

Hence $f(x) \in \{-5, -4, -3, -2, -1, 0\}$

TRIGONOMETRY (TRI)

Single Option Correct Type Questions (01 to 65

1. **(3)**

Sol: $\text{cosec}^2 A - \cot^2 A = 1$ ......(i)

$\text{cosec } A + \cot A = \frac{11}{2}$ ......(ii)

dividing, we get $\text{cosec } A - \cot A = \frac{2}{11}$

subtracting (ii) from (i), $2 \cot A = \frac{117}{22}$

$\Rightarrow \tan A = \frac{44}{117}$

2. **(2)**

Sol: $\tan\alpha + \cot\alpha = a$

$\Rightarrow \tan^2\alpha + \cot^2\alpha + 2 = a^2$

$\Rightarrow \tan^4\alpha + \cot^4\alpha = (a^2 - 2)^2 - 2 = a^4 - 4a^2 + 2$

3. **(2)**

Sol: $\tan\theta = -\frac{5}{12} \because \frac{3\pi}{2} < \theta < 2\pi \Rightarrow \sin\theta = -\frac{5}{13}$

and $\cot\theta = -\frac{12}{5}$

$\frac{-\sin\theta - \cot\theta}{-\text{cosec}\theta - \text{cosec}\theta} = \frac{\sin\theta + \cot\theta}{2\ \text{cosec}\theta}$

$= \frac{-\frac{5}{13} - \frac{12}{5}}{-2 \times \frac{13}{5}} = \frac{181}{338}$

4. **(4)**

Sol: $\frac{(-\cot x)\sin x + \cos^3 x}{\sin x(-\cot x)} = \frac{-\cos x}{-\cos x}(1 - \cos^2 x) = \sin^2 x$

5. **(2)**

Sol: $3\{\cos^4\alpha + \sin^4\alpha\} - 2\{\cos^6\alpha + \sin^6\alpha\}$

$= 3\{1 - 2\sin^2\alpha\cos^2\alpha\} - 2\{1 \times (\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha)\}$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2\{1 - 3\sin^2\alpha\cos^2\alpha\}$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha = 1$

6. **(3)**

Sol: Given $= \sin^2 5^\circ + \sin^2 10^\circ + ... + \sin^2 40^\circ + \sin^2 45^\circ + \sin^2 50^\circ + ... + \sin^2 85^\circ + 1$

$= \sin^2 5^\circ + \sin^2 10^\circ + ... + \sin^2 40^\circ + \frac{1}{2} + \cos^2 40^\circ + ... + \cos^2 5^\circ + 1 = 8 + \frac{1}{2} + 1 = \frac{19}{2}$

7. **(1)**

Sol: $\frac{\sin 24^\circ \cos 6^\circ - \sin 6^\circ \cos 24^\circ}{\sin 21^\circ \cos 39^\circ - \sin 39^\circ \cos 21^\circ}$

$= \frac{\sin(24^\circ - 6^\circ)}{\sin(21^\circ - 39^\circ)} = \frac{\sin 18^\circ}{\sin(-18^\circ)} = -1$

8. **(3)**

Sol: $\tan A - \tan B = x$

$\cot B - \cot A = y$

$\frac{\tan A - \tan B}{\tan A \tan B} = y$

$\Rightarrow \tan A \tan B = \frac{x}{y}$

Now $\cot(A - B) = \frac{1}{\tan(A - B)} = \frac{1 + \tan A \tan B}{\tan A - \tan B}$

$= \frac{1 + \frac{x}{y}}{x} = \frac{1}{x} + \frac{1}{y}$

9. **(4)**

Sol: $\because 3\sin\alpha = 5\sin\beta \Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{5}{3}$

$\Rightarrow \frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta} = \frac{8}{2} \Rightarrow \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$

10. **(2)**

Sol: $\frac{(\cos 6x+\cos 4x)+5(\cos 4x+\cos 2x)+10(\cos 2x+1)}{\cos 5x+5\cos 3x+10\cos x}$

$= \frac{2\cos 5x\cos x+5\times 2\cos 3x\cos x+10\times 2\cos^2 x}{\cos 5x+5\cos 3x+10\cos x}$

$= 2\cos x\,\frac{[\cos 5x+5\cos 3x+10\cos x]}{[\cos 5x+5\cos 3x+10\cos x]}$

$= 2\cos x$

11. **(1)**

Sol: $\frac{\tan(180°-25°)-\tan(90°+25°)}{1+(\tan(180°-25°)\tan(90°+25°)}$

$= \frac{-\tan 25°+\frac{1}{\tan 25°}}{2} = \frac{1-x^2}{2x}$

12. **(3)**

Sol: given $= 2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + 2\cos^2(\alpha+\beta) - 1$

$= 2\sin^2\beta + 2\cos(\alpha+\beta)\{2\sin\alpha\sin\beta + \cos(\alpha+\beta)\} - 1$

$= 2\sin^2\beta + 2\cos(\alpha+\beta)\cos(\alpha-\beta) - 1$

$= 2\sin^2\beta + 2\cos^2\alpha - 2\sin^2\beta - 1 = \cos 2\alpha$

13. **(3)**

Sol: $\cos A = \frac{3}{4}$

$16\cos^2\frac{A}{2} - 32\sin\frac{A}{2}\sin\frac{5A}{2}$

$= \frac{16(1+\cos A)}{2} - 16(\cos 2A - \cos 3A)$

$= \frac{16(1+\cos A)}{2} - 16\{(2\cos^2 A - 1) - (4\cos^3 A - 3\cos A)\}$

$= 8\left(1+\frac{3}{4}\right) - 16\left\{2\times\frac{9}{16} - 1 - 4\times\frac{27}{64} + 3\times\frac{3}{4}\right\} = 3$

14. **(2)**

Sol: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta = \cos\theta(4\cos^2\theta - 3)$

$= \frac{1}{2}\left(a+\frac{1}{a}\right)\left\{4\times\frac{1}{4}\left(a+\frac{1}{a}\right)^2 - 3\right\}$

$= \frac{1}{2}\left(a+\frac{1}{a}\right)\left\{a^2+\frac{1}{a^2}-1\right\}$

$= \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$

15. **(2)**

Sol: $\sin t + \cos t = \frac{1}{5}$

$\Rightarrow \frac{2\tan\frac{t}{2}+1-\tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}} = \frac{1}{5}$

$\Rightarrow 10\tan\frac{t}{2} + 5 - 5\tan^2\frac{t}{2} = 1 + \tan^2\frac{t}{2}$

$\Rightarrow 6\tan^2\frac{t}{2} - 10\tan\frac{t}{2} - 4 = 0$

$\Rightarrow 3\tan^2\frac{t}{2} - 6\tan\frac{t}{2} + \tan\frac{t}{2} - 2 = 0$

$\Rightarrow 3\tan\frac{t}{2}\left(\tan\frac{t}{2}-2\right) + 1\left(\tan\frac{t}{2}-2\right) = 0$

$\Rightarrow \tan\frac{t}{2} = 2,\ \tan\frac{t}{2} = -\frac{1}{3}$

16. **(2)**

Sol: Given $= \left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$

$= \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$

$= \sin^2\frac{\pi}{10}\,.\,\sin^2\frac{3\pi}{10} = \left(\frac{\sqrt{5}-1}{4}\cdot\frac{\sqrt{5}+1}{4}\right)^2$

$= \left(\frac{4}{16}\right)^2 = \frac{1}{16}$

17. (2)

Sol: $\dfrac{\cos 20° + 8\sin 70° \sin 50° \sin 10°}{\sin^2 80°}$

$= \dfrac{\cos 20° + 8\sin 10° \sin(60° - 10°)\sin(60° + 10°)}{\sin^2 80°}$

$= \dfrac{\cos 20° + 8.\frac{1}{4}\sin 30°}{\cos^2 10°} = \dfrac{2(\cos 20° + 1)}{1 + \cos 20°} = 2$

18. (1)

Sol: $A = \tan 6° \tan 42°$

$B = \cot 66° \cot 78°$

$\dfrac{A}{B} = \tan 6° \tan 42° \tan 66° \tan 78°$

$\Rightarrow \dfrac{A}{B} = \dfrac{\tan 6° \tan(60° - 6°)\tan(60° + 6°)}{\tan 54°}.$

$\tan 78° \tan 42°$

$\Rightarrow \dfrac{A}{B} = \dfrac{\tan 18° . \tan(60° - 18°)\tan(60° + 18°)}{\tan 54°}$

$= \dfrac{\tan 54°}{\tan 54°}$

$\Rightarrow \dfrac{A}{B} = 1$

19. (2)

Sol: Add & subtract $\cot\alpha$.

$(\tan\alpha - \cot\alpha) + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha + \cot\alpha$

$= -2\cot 2\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha + \cot\alpha$

$= -4\cot 4\alpha + 4\tan 4\alpha + 8\cot 8\alpha + \cot\alpha$

$= -8\cot 8\alpha + 8\cot 8\alpha + \cot\alpha = \cot\alpha$

20. (4)

Sol: $A + B + C = \dfrac{3\pi}{2}$

$\cos 2A + \cos 2B + \cos 2C = 2\cos(A + B).\cos(A - B) + 1 - 2\sin^2 C$

$= 2\cos\left(\dfrac{3\pi}{2} - C\right).\cos(A - B) - 2\sin^2 C + 1$

$(\because A + B + C = \dfrac{3\pi}{2})$

$= -2\sin C\{\cos(A - B) + \sin C\} + 1$

$= -2\sin C\{\cos(A - B) + \sin(\dfrac{3\pi}{2} - (A + B))\} + 1$

$= -2\sin C\{\cos(A - B) - \cos(A + B)\} + 1$

$= 1 - 4\sin A \sin B \sin C.$

21. (4)

Sol: $1 + \cos\dfrac{\pi}{9} + \cos\dfrac{2\pi}{9} + \cos\dfrac{3\pi}{9} + \cos\dfrac{4\pi}{9} - \cos\dfrac{4\pi}{9} - \cos\dfrac{3\pi}{9} - \cos\dfrac{2\pi}{9} - \cos\dfrac{\pi}{9} = 1$

22. (4)

Sol: $\sin\theta + \sin(\theta + \phi) + \sin(\theta + 2\phi) + \ldots\ldots\ldots\ldots + \sin(\theta + (n-1)\phi)$

$= \dfrac{\sin\left(\frac{n\phi}{2}\right)}{\sin\frac{\phi}{2}} . \sin\left(\dfrac{2\theta + (n-1)\phi}{2}\right)$

$\because \phi = \dfrac{2\pi}{n}$ (External angle of regular polygon)

So $\dfrac{\sin\left(n\left(\frac{\pi}{n}\right)\right)}{\sin\left(\frac{\pi}{n}\right)} \sin\left(\dfrac{2\theta + \frac{(n-1)2\pi}{n}}{2}\right) = 0$

23. (4)

Sol: $\cos\dfrac{\pi}{11} \cos\dfrac{2\pi}{11} \cos\dfrac{3\pi}{11} . \cos\dfrac{4\pi}{11} . \cos\dfrac{5\pi}{11} = \cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{4\pi}{11}\cos\dfrac{8\pi}{11}\cos\dfrac{16\pi}{11}$

$= \dfrac{\sin\frac{32\pi}{11}}{2^5.\sin\frac{\pi}{11}} = \dfrac{\sin\left(3\pi - \frac{\pi}{11}\right)}{32.\sin\frac{\pi}{11}} = \dfrac{1}{32}$

24. **(1)**

Sol: $y = 10\cos^2 x - 6\sin\cos x + 2\sin^2 x$

$= 5(1 + \cos 2x) - 3\sin 2x + 1 - \cos 2x$

$= 4\cos 2x - 3\sin 2x + 6$

$\because -\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

$y_{max} = 5 + 6 = 11$

$y_{min} = -5 + 6 = 1$

25. **(1)**

Sol: $y = 1 + 2\sin x + 3\cos^2 x \Rightarrow y = 1 + 2\sin x + 3 - 3\sin^2 x$

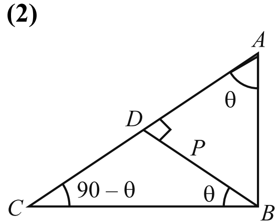
$y = 1 - (3\sin^2 x - 2\sin x - 3) \Rightarrow y = 1 - 3(\sin^2 x - \sin x + \frac{1}{9} - \frac{1}{9} - 1)$

$y = 1 - 3\left[\left(\sin x - \frac{1}{3}\right)^2 - \frac{10}{9}\right]$

$= -3\left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3}$

$y_{max} = \frac{13}{3}$, $y_{min} = -3\left(\frac{16}{9}\right) + \frac{13}{3} = -1$

26. **(2)**

Sol: 

$AC = 2\sqrt{2}\ P$

$\frac{P}{AD} = \frac{DC}{P} = \tan\theta$

$\because\ AD + DC = 2\sqrt{2}\ P$

$\Rightarrow \frac{P}{\tan\theta} + P\tan\theta = 2\sqrt{2}P$

$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{2\sin\theta\ \cos\theta} = \sqrt{2} \quad \Rightarrow \sin\ 2\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{8}$

So $\phi = \pi - \left(\frac{\pi}{2} + \frac{\pi}{8}\right) \Rightarrow \phi = \frac{3\pi}{8}$

27. **(4)**

Sol: Clearly, $\sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1$

$\therefore\ \cos\theta_1 = \cos\theta_2 = \cos\theta_3 = 0$

$\therefore\ \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

28. **(2)**

Sol: $\because\ \tan A < 0$ and $A + B + C = 180°$

$\Rightarrow A > 90°$

$\Rightarrow B + C < 90°$

$\Rightarrow \tan(B + C) > 0$

$\Rightarrow \frac{\tan\ B + \tan C}{1 - \tan B\ \ \tan C} > 0$

$\Rightarrow 1 - \tan B\tan C > 0$

$\Rightarrow \tan B\tan C < 1$

29. **(2)**

Sol: $LHS = \alpha(4\cos^3\theta - 3\cos\theta)^2 + \beta\cos^4\theta$

$= 16\alpha\cos^6\theta - 24\alpha\cos^4\theta + 9\alpha\cos^2\theta + \beta\cos^4\theta$

$= 16\alpha\cos^6\theta + (\beta - 24\alpha)\cos^4\theta + 9\alpha\cos^2\theta$

$RHS = 16\cos^6\theta + 9\cos^2\theta$

comparing LHS and RHS, we get $\alpha = 1$, $\beta = 24$

30. **(1)**

Sol: $A + B = \frac{\pi}{3}$

$\therefore \cos(A + B) = \frac{1}{2}$

$\cos^2 A + \cos^2 B - \cos A\cos B$

$= \cos^2 A - \sin^2 B + 1 - \cos A\cos B$

$= \cos(A + B)\cos(A - B) + 1 - \cos A\cos B$

$= \frac{1}{2}(\cos A\cos B + \sin A\sin B) + 1 - \cos A\cos B$

$= 1 - \frac{1}{2}(\cos A\cos B - \sin A\sin B)$

$= 1 - \frac{1}{2}\cos(A + B) = \frac{3}{4}$

31. **(2)**

Sol: $\dfrac{1}{\cos(270^\circ+20^\circ)}+\dfrac{1}{\sqrt{3}\sin(270^\circ-20^\circ)}$

$=\dfrac{1}{\sin 20^\circ}-\dfrac{1}{\sqrt{3}\cos 20^\circ}$

$=\dfrac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ\right)}{2\frac{\sqrt{3}}{2}\sin 20^\circ\ \cos 20^\circ}$

$=\dfrac{4\sin(60^\circ-20^\circ)}{\sqrt{3}\sin 40^\circ}=\dfrac{4}{\sqrt{3}}\dfrac{\sin 40^\circ}{\sin 40^\circ}=\dfrac{4\sqrt{3}}{3}$.

32. **(3)**

Sol: $a\sec\theta = 1 - b\tan\theta$ ... (1)

$a^2\sec^2\theta = 5 + b^2\tan^2\theta$ ... (2)

$(1)^2$ is $a^2\sec^2\theta = 1 + b^2\tan^2\theta - 2b\tan\theta$

From equation (2)

$5 + b^2\tan^2\theta = 1 + b^2\tan^2\theta - 2b\tan\theta$

$\tan\theta = -\dfrac{2}{b} \Rightarrow \sec^2\theta = \dfrac{b^2+4}{b^2}$

so $a^2\dfrac{(b^2+4)}{b^2} = 5 + 4 \Rightarrow a^2b^2 + 4a^2 = 9b^2$

33. **(1)**

Sol: $\sin x + \sin y = a$ .....(1)

$\cos x + \cos y = b$ .....(2)

$(1)^2 + (2)^2$

$\Rightarrow 1 + 1 + 2\cos(x-y) = a^2 + b^2$

$\cos(x-y) = \dfrac{a^2+b^2-2}{2}$

$\because \tan^2\left(\dfrac{x-y}{2}\right) = \dfrac{1-\cos(x-y)}{1+\cos(x-y)} \Rightarrow \tan^2$

$\left(\dfrac{x-y}{2}\right) = \dfrac{1-\left(\frac{a^2+b^2-2}{2}\right)}{1+\frac{a^2+b^2-2}{2}}$

$\Rightarrow \tan\left(\dfrac{x-y}{2}\right) = \pm\sqrt{\dfrac{4-a^2-b^2}{a^2+b^2}}$

34. **(4)**

Sol: $y = 3\cos\left(\theta+\dfrac{\pi}{3}\right) + 5\cos\theta + 3$

$y = 3\cos\theta.\dfrac{1}{2} - 3\dfrac{\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3$

$y = \dfrac{3}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3$

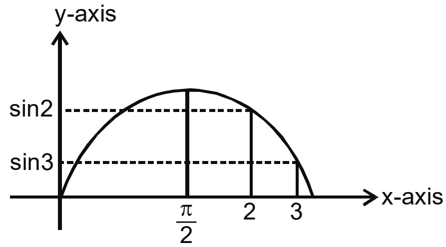
$y = \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 3$

$y_{max} = \sqrt{\dfrac{169}{4}+\dfrac{27}{4}} + 3 = 7 + 3 = 10$

$y_{min} = -\sqrt{\dfrac{169}{4}+\dfrac{27}{4}} + 3 = -7 + 3 = -4$

35. **(1)**

Sol:



36. **(1)**

Sol: $y = \dfrac{\tan\theta}{\tan 3\theta} = \dfrac{\tan\theta}{\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}}$

$\Rightarrow y = \dfrac{1-3\tan^2\theta}{3-\tan^2\theta} \qquad \Rightarrow \tan^2\theta = \dfrac{3y-1}{y-3}$

> 0

$\Rightarrow y \in \left(-\infty, \dfrac{1}{3}\right) \cup (3, \infty)$

$\therefore$ statement-1 and statement-2 both are true and statement-2 explains statement-1

37. **(2)**

Sol: $\because \alpha$ is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$.

$\therefore 25\cos^2\alpha + 5\cos\alpha - 12 = 0$

$\Rightarrow \cos\alpha = -\dfrac{4}{5}, \dfrac{3}{5}$ But $\dfrac{\pi}{2} < \alpha < \pi$ (IInd quadrant)

$\therefore\ \cos\alpha = \frac{-4}{5}$ and $\sin\alpha = \frac{3}{5}$

$\Rightarrow \sin 2\alpha = 2\sin\alpha\cos\alpha = -\frac{24}{25}$

38. **(1)**

Sol: $\sin(\alpha+\beta) = 1 \Rightarrow \alpha+\beta = \frac{\pi}{2}$ ..... (i)

$\sin(\alpha-\beta) = \frac{1}{2} \Rightarrow \alpha-\beta = \frac{\pi}{6}$ ..... (ii)

on solving (i) & (ii)

$\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{6}$

$\therefore\ \tan(\alpha+2\beta).\tan(2\alpha+\beta)$

$= \tan\left(\frac{2\pi}{3}\right)\ \tan\left(\frac{5\pi}{6}\right)$

39. **(2)**

Sol: $\because \sec^2\theta \geq 1$

$\therefore\ \frac{4xy}{(x+y)^2} \geq 1 \Rightarrow x^2+y^2+2xy-4xy \leq 0$

$\Rightarrow (x-y)^2 \leq 0$

which is true only if $x = y$

40. **(4)**

Sol: $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta}$

$+ \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow u^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2$

$\theta + 2\sqrt{a^2\cos^2\theta + b^2\sin^2\theta}$

$\times \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow u^2 = (a^2+b^2) + 2$

$\sqrt{\{a^2+(b^2-a^2)\sin^2\theta\}\times\{a^2+(b^2-a^2)\cos^2\theta\}}$

$\Rightarrow u^2 = (a^2+b^2) + 2$

$\sqrt{a^4 + a^2(b^2-a^2) + (b^2-a^2)^2\sin^2\theta\cos^2\theta}$

$\Rightarrow u^2 = (a^2+b^2) + 2$

$\sqrt{a^2b^2 + \left(\frac{b^2-a^2}{2}\right)^2\sin^2 2\theta}$.

$\therefore\ \min(u^2) = a^2+b^2+2ab = (a+b)^2$

and $\max(u^2) = a^2+b^2+(a^2+b^2) = 2(a^2+b^2)$

Now, $\max(u^2) - \min(u^2) = (a-b)^2$

41. **(1)**

Sol: $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$

squaring and adding, we get

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha.\cos\beta$

$= \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$

$\Rightarrow 2 + 2\cos(\alpha-\beta) = \frac{1170}{4225}$

$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1170}{4\times4225} = \frac{9}{130}$

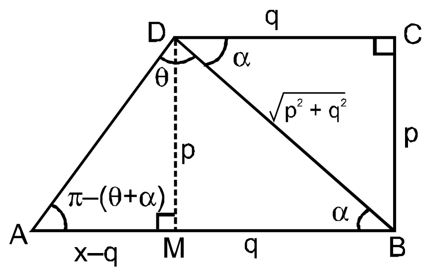
$\Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$ $(\because \pi < \alpha-\beta < 3\pi$

$\Rightarrow \frac{\pi}{2} < \left(\frac{\alpha-\beta}{2}\right) < \frac{3\pi}{2})$

42. **(1)**

Sol: Let $AB = x$

$\tan(\pi-\theta-\alpha) = \frac{p}{x-q} \Rightarrow \tan(\theta+\alpha) = \frac{p}{q-x}$



$\Rightarrow q - x = p\cot(\theta+\alpha)$

$\Rightarrow x = q - p\cot(\theta+\alpha)$

$= q - p\left(\frac{\cot\theta\ \cot\alpha - 1}{\cot\alpha + \cot\theta}\right)$

$= q - p \left(\dfrac{\frac{q}{p}\cot\theta - 1}{\frac{q}{p} + \cot\theta}\right) = q - p$

$\left(\dfrac{q\cot\theta - p}{q + p\cot\theta}\right) = q - p\left(\dfrac{q\cos\theta - p\sin\theta}{q\sin\theta + p\cos\theta}\right)$

$\Rightarrow x = \dfrac{q^2\sin\theta + pq\cos\theta - pq\cos\theta + p^2\sin\theta}{p\cos\theta + q\sin\theta}$

$\Rightarrow AB = \dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$.

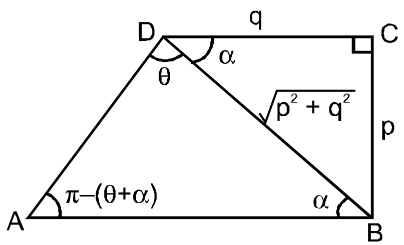
Alternative

From Sine Rule

$\dfrac{AB}{\sin\theta} = \dfrac{\sqrt{p^2+q^2}}{\sin(\pi - (\theta+\alpha))}$

$AB = \dfrac{\sqrt{p^2+q^2}\ \sin\theta}{\sin\theta\ \cos\alpha + \cos\theta\ \sin\alpha}$

$= \dfrac{(p^2+q^2)\ \sin\theta}{q\sin\ \theta + p\cos\theta}\left(\because \cos\alpha = \dfrac{q}{\sqrt{p^2+q^2}}\right)$



$= \dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$.

43. (2)

Sol: Given expression

$= \dfrac{\sin A}{\cos A} \times \dfrac{\sin A}{\sin A - \cos A} + \dfrac{\cos A}{\sin A} \times \dfrac{\cos A}{\cos A - \sin A}$

$= \dfrac{1}{\sin A - \cos A}\left\{\dfrac{\sin^3 A - \cos^3 A}{\cos A\ \sin A}\right\} =$

$\dfrac{\sin^2 A + \sin A\cos A + \cos^2 A}{\sin A\ \cos A} = 1 + \sec A \text{ cosec } A$

44. (2)

Sol: $f_k(x) = \dfrac{1}{k}\ (\sin^k x + \cos^k x)$

$f_4 - f_6 = \dfrac{1}{4}(\sin^4 x + \cos^4 x) - \dfrac{1}{6}(\sin^6 x + \cos^6 x)$

$= \dfrac{1}{4}\ (1 - 2\sin^2 x\cos^2 x) - \dfrac{1}{6}\ (1 - 3\sin^2 x\cos^2 x)$

$\dfrac{1}{4} - \dfrac{1}{6} = \dfrac{1}{12}$

45. (4)

Sol: $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$5\left(\tan^2 x - \dfrac{1}{1+\tan^2 x}\right) = 2\left(\dfrac{1-\tan^2 x}{1+\tan^2 x}\right) + 9$

$5(\tan^4 x + \tan^2 x - 1) = 2 - 2\tan^2 x + 9 + 9\tan^2 x$

$5\tan^4 x - 2\tan^2 x - 16 = 0$

$5\tan^4 x - 10\tan^2 x + 8\tan^2 x - 16 = 0$

$5\tan^2 x\ (\tan^2 x - 2) + 8\ (\tan^2 x - 2) = 0$

$(5\tan^2 x + 8)\ (\tan^2 x - 2) = 0$

$\tan^2 x = 2$

$\cos 2x = \dfrac{1-2}{1+2} = -\dfrac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = -\dfrac{7}{9}$

46. (2)

Sol: $\theta \in \left(0, \dfrac{\pi}{4}\right)$

$\because$ $\tan\theta \uparrow$ in $\theta \in \left(0, \dfrac{\pi}{4}\right)$ and $0 < \tan\theta < 1$

$\cot\theta \downarrow$ in $\theta \in \left(0, \dfrac{\pi}{4}\right)$ and $\cot\theta > 1$

Let $\tan\theta = 1 - \lambda_1$ and $\cot\theta = 1 + \lambda_2$ where λ_1 and λ_2 are very small and positive, then

$t_1 = (1-\lambda_1)^{1-\lambda_1}$, $t_2 = (1-\lambda_1)^{1+\lambda_2}$, $t_3 = (1+\lambda_2)^{1-\lambda_1}$, $t_4 = (1+\lambda_2)^{(1+\lambda_2)}$

$\therefore$ $t_4 > t_3 > t_1 > t_2$

OR

$\because \tan\theta \uparrow$ in $\theta \in \left(0, \dfrac{\pi}{4}\right)$ and $0 < \tan\theta < 1$

$\cot\theta \downarrow$ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and $\cot\theta > 1$ think only above and conculude result

47. (D)

Sol: $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta = (\sqrt{2}+1)\cos\theta \Rightarrow \tan\theta = \sqrt{2}+1 \Rightarrow \theta = n\pi + \frac{3\pi}{8}$; $n \in I$

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\therefore \cos\theta = (\sqrt{2}-1)\sin\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1 \Rightarrow \theta = n\pi + \frac{3\pi}{8}$; $n \in I$

$\therefore P = Q$

48. (3)

Sol: $$\sum_{k=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right]}{\sin\frac{\pi}{6}\left(\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right)}$$

$$= 2\sum_{k=1}^{13}\left(\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right)$$

$$=2\left(\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right)$$

$$= 2\left(1-\cot\left(\frac{29\pi}{12}\right)\right) = 2\left(1-\cot\left(\frac{5\pi}{12}\right)\right) = 2$$

$(1-(2-\sqrt{3})) = 2(-1+\sqrt{3})$

$= 2(\sqrt{3}-1)$

49. (2)

Sol: $\sin\theta + \cos\theta = m$ ....(1)

$\sin\theta\cos\theta = \frac{m}{n}$ ...(2)

squaring (1)

$1 + 2\sin\theta\cos\theta = m^2$

$\sin\theta\cos\theta = \frac{m^2-1}{2}$ ...(3)

by (2) & (3) $n(m^2-1) = 2m$

50. (3)

Sol: $E = \frac{13}{4} - (\cos x + \frac{1}{2})^2 \Rightarrow$ maximum value $= \frac{13}{4}$

minimum value $= 1$

51. (4)

Sol: $\tan(A+B) =$

$$\frac{\frac{a}{a+1}+\frac{1}{2a+1}}{1-\frac{a}{(a+1)(2a+1)}} = \frac{2a^2+a+a+1}{2a^2+3a+1-a}$$

$$= \frac{2a^2+2a+1}{2a^2+2a+1} \Rightarrow A+B = \frac{\pi}{4}$$

52. (3)

Sol: Checking option $A = 60°$, $B = 30°$, $C = 0°$

53. (3)

Sol: $$\frac{3\cos\theta+4\cos^3\theta-3\cos\theta}{3\sin\theta-(3\sin\theta-4\sin^3\theta)}$$

$$= \frac{4\cos^3\theta}{4\sin^3\theta} = \cot^3\theta$$

54. (1)

Sol: $A = \sin^2 x + \cos^4 x$

$= \sin^2 x + (1-\sin^2 x)^2$

$= \sin^4 x - \sin^2 x + 1$

$= \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$= \frac{3}{4} \le A \le 1$

55. (2)

Sol: Let $e^{\sin x} = t$

$\Rightarrow t^2 - 4t - 1 = 0 \Rightarrow t = \frac{4\pm\sqrt{16+4}}{2}$

$\Rightarrow t = e^{\sin x} = 2 \pm \sqrt{5} \Rightarrow e^{\sin x} = 2-\sqrt{5}$,

$e^{\sin x} = 2+\sqrt{5}$

$e^{\sin x} = 2-\sqrt{5} < 0, \Rightarrow \sin x = ln(2+\sqrt{5}) > 1$

so rejected hence no solution

56. (2)

Sol: $\cos x + \sin x = \frac{1}{2}$

$\therefore \frac{1-\tan^2 x/2}{1+\tan^2 x/2} + \frac{2\tan x/2}{1+\tan^2 x/2} = \frac{1}{2}$, Let $\tan \frac{x}{2} = t$

$\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2} \Rightarrow 3t^2 - 4t - 1 = 0$

$\therefore t = \frac{2 \pm \sqrt{7}}{3}$

as $0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$

$\therefore \tan \frac{x}{2}$ is positive

$\therefore t = \tan \frac{x}{2} = \frac{2+\sqrt{7}}{3}$

Now $\tan x = \frac{2\tan x/2}{1-\tan^2 x/2} = \frac{2t}{1-t^2}$

$\Rightarrow \tan x = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2} = -\left(\frac{4+\sqrt{7}}{3}\right)$

57. (2)

Sol: $2\{\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta)\} + 3 = 0$

$(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

$\sum \cos\alpha = 0 = \sum \sin\alpha$

58. (1)

Sol: $\tan 2\alpha = \tan((\alpha+\beta) + (\alpha-\beta))$

$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}$

$= \frac{(9+5)4}{48-15} = \frac{14\times 4}{33} = \frac{56}{33}$

Hence correct option is (1)

59. (1)

Sol: sum of roots $= \frac{1+\tan^2\frac{\theta}{2}}{\tan\frac{\theta}{2}} = \frac{2}{\sin\theta} = 6$

$x^2 - 6x + 1 = 0$

60. (2)

Sol: $\frac{\sin\frac{2^5\pi}{33}}{2^5\sin\frac{\pi}{33}} = \frac{\sin\frac{32\pi}{33}}{32\sin\frac{\pi}{33}} = \frac{\sin\left(\pi-\frac{\pi}{33}\right)}{32\sin\frac{\pi}{33}}$

$= \frac{\sin\frac{\pi}{33}}{32\sin\frac{\pi}{33}} = \frac{1}{32}$

61. (1)

Sol: $-13 \le 12\sin\frac{11\theta}{2} + 5\cos\frac{11\theta}{2} \le 13$

$0 \le 13 + 12\sin\frac{11\theta}{2} + 5\cos\frac{11\theta}{2} \le 26$

range = [0,26]

62. (1)

Sol: $\frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2} = \pi$

$\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2}\right) = 0$

63. (2)

Sol: $\sin^6 x + \cos^6 x = a^2$

$\Rightarrow (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x) = a^2$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x = a^2 \Rightarrow 1 - 3\sin^2 x\cos^2 x = a^2$

$\Rightarrow 1 - \frac{3}{4}\sin^2 2x = a^2 \Rightarrow \frac{4(1-a^2)}{3} = \sin^2 2x$

$\Rightarrow 0 \le \frac{4}{3}(1-a^2) \le 1$

$1 - a^2 \ge 0$ and $4 - 4a^2 \le 3$

$a^2 \le 1$ and $\frac{1}{4} \le a^2$

$-1 \le a \le 1$

So $a \geq \frac{1}{2}$ or $a \leq -\frac{1}{2}$

$a \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

64. **(2)**

Sol: **(I)** $\tan 9° - \tan 27° - \tan 63° + \tan 81° = (\tan 9° + \tan 81°) - (\tan 27° + \tan 63°)$

$= \frac{\sin 90°}{\cos 9° \cos 81°} - \frac{\sin 90°}{\cos 27° \cos 63°}$

$= \frac{2}{2\sin 9° \cos 9°} - \frac{2}{2\sin 27° \cos 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = \frac{2}{\frac{\sqrt{5}-1}{4}} - \frac{2}{\frac{\sqrt{5}+1}{4}}$

$= \frac{8\ (\sqrt{5}+1-\sqrt{5}+1)}{4} = 4$

(II) $2(\text{cosec } 10° - \sqrt{3} \sec 10°) = 4$

$\left(\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right) \times \frac{1}{\sin 10° \cos 10°} \times \frac{2}{2} = 8$

(III) $\sqrt{2} \sin 10° \left(\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right)$

$= \sqrt{2} \left(\frac{2\sin 5° \cos 5° \sec 5°}{2} + \frac{2\sin 5° \cos 5° \cos 40°}{\sin 5°} - 2\sin 35° \sin 10°\right)$

$= \sqrt{2}\ (\sin 5° + 2\cos 45° + \cos 35° - \cos 25°) = \sqrt{2}\ (\sin 5° + 2\cos 45° + 2\sin 30° \sin(-5°))$

$= \sqrt{2}\ (\sqrt{2}) = 2$

(IV) $\cot 70° + 4 \cos 70° =$

$\frac{\cos 70°}{\sin 70°} + 4\cos 70° = \frac{\cos 70° + 4\cos 70° \sin 70°}{\sin 70°}$

$= \frac{\cos 70° + 2\sin 140°}{\sin 70°}$

$= \frac{(\cos 70° + \sin 140°) + \sin 140°}{\sin 70°}$

$= \frac{(\sin 20° + \sin 140°) + \sin 140°}{\sin 70°}$

$= \frac{2\sin 80° \times \cos 60° + \sin 140°}{\sin 70°} = \frac{2\sin 110° \times \cos 30°}{\sin 70°}$

$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

$\sqrt{3}\ (\cot 70° + 4\cos 70°) = 3$

65. **(3)**

Sol: (I) $\sin^2\theta + 3\cos\theta = 3 \Rightarrow 1 - \cos^2\theta + 3\cos\theta = 3$ $\cos^2\theta - 3\cos\theta + 2 = 0$ $\cos\theta = 1, 2$

$\cos\theta = 1$ ($\cos\theta \neq 2$) $\Rightarrow \theta = 0$ in $[-\pi, \pi]$

No. of solution = 1

(II) $\sin\theta + \text{cosec}\,\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow (\sin\theta - 1)^2 = 0 \quad \Rightarrow \sin\theta = 1$

$\therefore \sin^{2008}\theta + \text{cosec}^{2008}\theta = \sin^{2008}\theta + \frac{1}{\sin^{2008}\theta}$

$= 1 + 1 = 2$

(III) $\sin^4\theta + \cos^4\theta - 1 = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - 1 = -\frac{1}{2}\sin^2 2\theta$

$\because \quad 0 \leq \sin^2 2\theta \leq 1 \quad \therefore -\frac{1}{2} \leq -\frac{1}{2}\sin^2 2\theta \leq 0$

$\therefore$ maximum value = 0

(IV) $2\sin^2\theta + 3\cos^2\theta - 3 = 2\sin^2\theta + 3 - 3\sin^2\theta - 3 = -\sin^2\theta$

$\because 0 \leq \sin^2\theta \leq 1 \qquad \therefore -1 \leq -\sin^2\theta \leq 0$

$\therefore$ least value = –1

Integer Type Questions (66 to 75)

66. **(25)**

Sol: square & add

$a^2 + b^2 = 9 + 16 = 25$

67. **(1)**

Sol: $\tan 1° \tan 2° \tan 3° \ldots \tan 45° \ldots \tan 87° \tan 88° \tan 89°$
$= \tan 1° \tan 2° \tan 3° \ldots \tan 45° \ldots \cot 3° \cot 2° \cot 1° = 1$

68. **(1)**

Sol: $37° = 45° - 8°$

$\Rightarrow$ tan 37° = $\dfrac{1-\tan 8^o}{1+\tan 8^o}$ $\Rightarrow 1$ + tan 37° = $\dfrac{2}{1+\tan 8^o}$

Similarly 1 + tan 23° = $\dfrac{2}{1+\tan 22^o}$

$\therefore \dfrac{(1+\tan 8^o)(1+\tan 37^o)}{(1+\tan 22^o)(1+\tan 23^o)} = 1$

69. **(4)**

Sol: $12 \sin\theta - 9\sin^2\theta = 4 - (3\sin\theta - 2)^2$

whose maximum value is 4 when $\sin\theta = \dfrac{2}{3}$

70. **(0)**

Sol: $\tan^2\theta = 2\tan^2\phi + 1$ ... (i)

$\cos 2\theta + \sin^2\phi = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} + \sin^2\phi =$

$\dfrac{1-2\tan^2\phi - 1}{1+2\tan^2\phi+1} + \sin^2\phi = \dfrac{-2\tan^2\phi}{2\ (1+\tan^2\phi)} + \sin^2\phi$

$= -\sin^2\phi + \sin^2\phi = 0$. which is independent of ϕ

71. **(9)**

Sol: $f(\theta) = \sin^2\theta + \dfrac{1}{\sin^2\theta} + 2 + \cos^2\theta + \dfrac{1}{\cos^2\theta} + 2$

$= 5 + \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = 5 + \dfrac{4}{\sin^2 2\theta}$

minimum value of $f(\theta) = 5 + \dfrac{4}{1} = 9$

72. **(3)**

Sol: Since, tan 30° and tan 15° are the roots of equation $x^2 + px + q = 0$.

$\therefore$ tan 30° + tan 15° = $-p$ and tan 30° tan 15° $= q$

Therefore, $2 + q - p = 2 + \tan 30^o \tan 15^o + (\tan 30^o + \tan 15^o)$

$\Rightarrow 2 + q - p = 2 + \tan 30^o \tan 15^o + 1 - \tan 30^o.\tan 15^o \left(\because \tan 45^o = \dfrac{\tan 30^o + \tan 15^o}{1-\tan 30^o \tan 15^o}\right)$

$\Rightarrow 2 + q - p = 3$

73. **(2)**

Sol: $\dfrac{\cot 20° - \tan 20°}{\cot 40°} = \dfrac{2(1-\tan^2 20^o)}{2\tan 20^o} \times \tan 40^o$

$= \dfrac{2}{\tan 40^o} \times \tan 40^o$

74. **(0)**

Sol: $\sin\alpha \sin\beta - \cos\alpha\cos\beta + 1 = 0$

$\Rightarrow \cos(\alpha + \beta) = 1$

$\alpha + \beta = 2n\pi$

$\Rightarrow 1 + \cot\alpha\tan\beta = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \dfrac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = 0$

75. **(7)**

Sol: $\dfrac{1}{\sin\frac{\pi}{n}} - \dfrac{1}{\sin\frac{3\pi}{n}} = \dfrac{1}{\sin\frac{2\pi}{n}}$

$\dfrac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \dfrac{1}{\sin\frac{2\pi}{n}}$

$\sin\dfrac{4\pi}{n} = \sin\dfrac{3\pi}{n}$

$\dfrac{4\pi}{n} = (-1)^k \dfrac{3\pi}{n} + k\pi,\ k \in I$

If $k = 2m \Rightarrow \dfrac{\pi}{n} = 2m\pi$

$\dfrac{1}{n} = 2m$, not possible

If $k = 2m + 1 \Rightarrow \dfrac{7\pi}{n} = (2m+1)\pi$

$\Rightarrow n = 7,\ m = 0$

QUADRATIC EQUATIONS

Single Option Correct Type Questions (01 to 65)

1. (3)

Sol. One factor is common let $(x-\alpha)$ is the common factor

$a\alpha^2 + b\alpha + c = 0$

$b\alpha^2 + c\alpha + a = 0$

$$\frac{\alpha^2}{ab-c^2} = \frac{\alpha}{bc-a^2} = \frac{1}{ac-b^2}$$

$$\Rightarrow \frac{ab-c^2}{bc-a^2} = \frac{bc-a^2}{ac-b^2}$$

$\Rightarrow (ab - c^2)(ac - b^2) = (bc - a^2)^2$

$\Rightarrow a^2bc - ab^3 - ac^3 + b^2c^2 = b^2c^2 + a^4 - 2a^2bc$

$\Rightarrow a^4 - 3a^2bc + ab^3 + ac^3 = 0$

$\Rightarrow a[a^3 + b^3 + c^3 - 3abc] = 0$

2. (1)

Sol. $x^2 - 3x + 2m = 0$ with roots 2α, β

$x^2 - x + m = 0$ with roots α, γ

$2\alpha\beta = 2m$

$\alpha\beta = m$ ..... (i)

$\alpha\gamma = m$ ..... (ii)

$\frac{m}{\beta}\gamma = m$

$m\left(\frac{\gamma}{\beta} - 1\right) = 0$

$m = 0$ or $\gamma = \beta$

where $\gamma = \beta$

$2\alpha + \beta = 3,\ \alpha + \beta = 1$

So, $\alpha = 2,\ \beta = -1$

$m = \alpha\beta = -2$

3. (3)

Sol. $\because$ D of $x^2 + 4x + 5 = 0$ is less than zero

$\Rightarrow$ both the roots are imaginary $\Rightarrow$ both the roots of quadratic are same

$\Rightarrow b^2 - 4ac < 0$ & $\frac{a}{1} = \frac{b}{4} = \frac{c}{5} = k$

$\Rightarrow a = k,\ b = 4k,\ c = 5k.$

4. (1)

$(1 + m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has equal roots.

$\Rightarrow$ D = 0

$\Rightarrow 4(9m^2 + 6m + 1) - 4(1 + m)(1 + 8m) = 0$

$\Rightarrow 9m^2 + 6m + 1 - 8m^2 - 9m - 1 = 0$

$\Rightarrow m^2 - 3m = 0 \quad \Rightarrow \quad m(m - 3) = 0$

$\Rightarrow m = 0, 3$

5. (2)

Sol. $(a - c)(b - c)(a + d)(b + d)$

$= [ab - c(a + b) + c^2][ab + d(a + b) + d^2]$

$\because a + b = -q,\ ab = 1$

$= (1 + qc + c^2)(1 - qd + d^2)$

$\because c^2 + 1 = -pc,\ d^2 + 1 = -pd$ and $cd = 1$

$= (qc - pc)(-qd - pd)$

$= -cd(q - p)(q + p)$

$= cd(p^2 - q^2) = p^2 - q^2$

6. (1)

Sol. $x^2 - px + q = 0$ with roots α, β

$\Rightarrow p = -11,\ q = 24$

then correct equation will be $x^2 - 11x + 24 = 0$

$\Rightarrow (x - 8)(x - 3) = 0$

$\Rightarrow x = 3, 8$

7. **(2)**

Sol. $(3x+2)^2 + p(3x+2) + q = 0 \begin{cases} \nearrow \alpha \\ \searrow \beta \end{cases}$

$x^2 + px + q = 0 \begin{cases} \nearrow 3\alpha + 2 \\ \searrow 3\beta + 2 \end{cases}$

8. **(2)**

Sol. $\alpha + \beta + 2h = \dfrac{-q}{p}$

$\dfrac{-b}{a} + 2h = -\dfrac{q}{p}$

$h = \dfrac{1}{2}\left(\dfrac{b}{a} - \dfrac{q}{p}\right)$

9. **(3)**

Sol. $(x-a)(x-b) + c = 0 \begin{cases} \nearrow \alpha \\ \searrow \beta \end{cases}$

$(x - c - \alpha)(x - c - \beta) = c$

Let $x - c = t$

$(t - \alpha)(t - \beta) = c$

$t^2 - (\alpha + \beta)t + \alpha\beta = c$

$t^2 - (a+b)t + ab = 0$

roots are a & b

$x - c = a, x - c = b$

$x = a + c, b + c$

10. **(2)**

Sol. Clearly $(x-a)(x-b)(x-c) - d = (x-\alpha)(x-\beta)(x-\gamma)$

$\therefore$ if α, β, γ are the roots of given equation then $(x-\alpha)(x-\beta)(x-\gamma) + d = 0$ will have roots a, b, c.

11. **(3)**

Sol. $\alpha + \beta = \lambda - 3$ and $\alpha\beta = -\lambda$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (\lambda-3)^2 + 2\lambda$

$\Rightarrow \alpha^2 + \beta^2 = \lambda^2 - 4\lambda + 9$

Let $y = \alpha^2 + \beta^2 = \lambda^2 - 4\lambda + 9$

is minimum at $\lambda = -\dfrac{-4}{2} = 2$

$\therefore$ $\alpha^2 + \beta^2$ is minimum at $\lambda = 2$.

12. **(4)**

Sol. **(1)** $S = \alpha^2 + \beta^2 = a^2 - 2b$

$P = \alpha^2\beta^2 = b^2$

$\therefore$ equation is $x^2 - (a^2 - 2b)x + b^2 = 0$

(2) $S = \dfrac{1}{\alpha} + \dfrac{1}{\beta} = -\dfrac{a}{b}$, $P = \dfrac{1}{\alpha} \cdot \dfrac{1}{\beta} = \dfrac{1}{b}$

$\therefore$ $x^2 + \dfrac{a}{b}x + \dfrac{1}{b} = 0$

$\Rightarrow$ $bx^2 + ax + 1 = 0$

(3) $S = \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{a^2 - 2b}{b}$

$P = \dfrac{\alpha}{\beta} \cdot \dfrac{\beta}{\alpha} = 1$

$x^2 - \dfrac{a^2 - 2b}{b}x + 1 = 0$

$\Rightarrow$ $bx^2 - (a^2 - 2b)x + b = 0$

(4) $S = \alpha + \beta - 2 = -a - 2$

$P = (\alpha - 1)(\beta - 1)$

$= \alpha\beta - (\alpha + \beta) + 1$

$= b + a + 1$

$\therefore$ equation is

$x^2 + (a+2)x + (a+b+1) = 0.$

13. **(1)**

Sol. $D = 0$

$\Rightarrow$ $(k+1)^2 - 8k = 0$

$\Rightarrow$ $k = 3 \pm 2\sqrt{2}$

14. **(3)**

Sol. $x^2 + (a-b)x + (1-a-b) = 0$

$\because$ $D > 0$

$\Rightarrow$ $(a-b)^2 - 4 \times 1 \times (1-a-b) > 0$

$a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

$\because$ $b^2 + 2b(2-a) + (a^2 + 4a - 4) > 0$

$\because$ $4(2-a)^2 - 4 \times 1 \times (a^2 + 4a - 4) < 0$

$\Rightarrow$ $4 + a^2 - 4a - a^2 - 4a + 4 < 0$

$\Rightarrow$ $8a - 8 > 0$

$\Rightarrow$ $a > 1$

15. **(4)**

Sol. $x^2 + px + q = 0 \quad \Rightarrow \quad \alpha, \beta$

$\alpha + \beta = -p, \alpha\beta = q$ and $p^2 - 4q > 0$

$x^2 - rx + s = 0 \quad \Rightarrow \quad \alpha^4, \beta^4$ .....(1)

Now $\alpha^4 + \beta^4 = r$, $(\alpha\beta)^4 = s = q^4$

$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = r \quad \Rightarrow \quad [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = r$

$(p^2 - 2q)^2 - 2q^2 = r$

$(p^2 - 2q)^2 = 2q^2 + r > 0$

.....(2)

Now, $\quad x^2 - 4qx + 2q^2 - r = 0$

$D = 16q^2 - 4(2q^2 - r)$ by equation (2)

$= 8q^2 + 4r = 4(2q^2 + r) > 0$

$D > 0$ two real roots

Product of roots $= 2q^2 - r$

$= 2q^2 - [(p^2 - 2q)^2 - 2q^2]$

$= 4q^2 - (p^2 - 2q)^2$

$= -p^2(p^2 - 4q) < 0$ from (1)

So product of roots is – ve

hence roots are opposite in sign

16. **(1)**

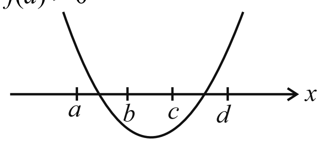
Sol. $\because$ Let $f(x) = (x - a)(x - c) + 2(x - b)(x - d)$

$\because \quad f(a) > 0$

$f(b) < 0$

$f(c) < 0$

$f(d) > 0$



$\therefore$ two real and distinct roots.

17. **(4)**

Sol. **Ist graph**

$x^2 + 2(k - 1)x + (k + 5) = 0$

(i) $\quad D \geq 0$

$4(k - 1)^2 - 4(k + 5) \geq 0$

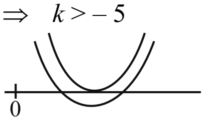
$(k + 1)(k - 4) \geq 0$

$k \in (-\infty, -1] \cup [4, \infty)$

(ii) $\quad f(0) > 0$

$k + 5 > 0$

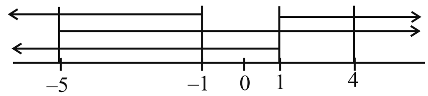
$\Rightarrow \quad k > -5$



(iii) $\quad -\dfrac{b}{2a} > 0$

$\Rightarrow \quad \dfrac{2(1-k)}{2} > 0$

$k < 1$

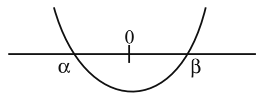


$k \in (-5, -1]$ ... (i)

IInd graph

$f(0) < 0$

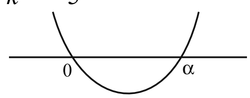
$k + 5 < 0 \quad \Rightarrow \quad k < -5$ ... (ii)



IIIrd graph

$f(0) = 0$

$k = -5$ ... (iii)



by (i) $\cup$ (ii) $\cup$ (iii)

$k \in (-\infty, -1]$

18. **(3)**

Sol. $x^2 - cx + d = 0 \rightarrow \alpha, \beta$

$x^2 - ax + b = 0 \rightarrow \alpha, \alpha$

$$\left(x - \frac{a}{2}\right)^2 = \left(\frac{a^2}{4} - b\right)$$

$\Rightarrow \quad a^2 = 4b$ .... (i)

$x = \dfrac{a}{2}$ .... (ii)

$\Rightarrow \quad \alpha^2 - c\alpha + d = 0$

$\Rightarrow \quad \alpha^2 - a\alpha + b = 0$

$\alpha = \left(\dfrac{d - b}{c - a}\right)$ and from (ii) is $\dfrac{a}{2}$

$\dfrac{a}{2} = \dfrac{d - b}{c - a} = 2d - 2b = ac - a^2$

{from (i) $a^2 = 4b$}

$2(b + d) = ac$

19. **(3)**

Sol. $3x^2 - 4x + 5 = 0$ ....(1)

Now to find the equation whose roots are 2α, 2β

Let $y = 2x$

Put $x = \frac{y}{2}$ in (1), we get

$3\left(\frac{y^2}{4}\right) - 4\left(\frac{y}{2}\right) + 5 = 0$

$\Rightarrow$ $3y^2 - 8y + 20 = 0$

$\therefore$ required equation is $3x^2 - 8x + 20 = 0$

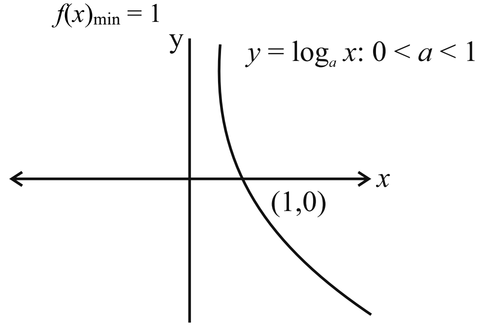
20. **(1)**

Sol. Statement-1 : $y = \log_{1/3}(x^2 - 4x + 5)$ is max. when $x^2 - 4x + 5$ is min.

Let $f(x) = x^2 - 4x + 5$

$\Rightarrow$ $(x - 2)^2 + 1$

$f(x)_{\min} = 1$



$y_{\max} = \log_{1/3}1 = 0$

Statement-1 is true

Statement-2 : $\log_a x \le 0$ for $x \ge 1$, $0 < a < 1$

$\therefore$ Statement-2 is true, correct explanation for statement-1

21. **(2)**

Sol. $x^2 + abx + c = 0$ (roots α, β) ...(1)

$\alpha + \beta = -ab$, $\alpha\beta = c$

$x^2 + acx + b = 0$ (roots α, δ) ...(2)

$\alpha + \delta = -ac$, $\alpha\delta = b$

$\alpha^2 + ab\,\alpha + c = 0$

$\alpha^2 + ac\,\alpha + b = 0$

$$\frac{\alpha^2}{ab^2 - ac^2} = \frac{\alpha}{c-b} = \frac{1}{a\ c-ab}$$

$$\Rightarrow \alpha^2 = \frac{a\ (b^2 - c^2)}{a(c-b)} = -(b+c)$$

$$\& \ \alpha = \frac{c-b}{a(c-b)} = \frac{1}{a} \quad \therefore \text{common root, } \alpha = \frac{1}{a}$$

$$\therefore\ -(b+c) = \frac{1}{a^2}$$

$\Rightarrow$ $a^2(b + c) = -1$

Product of the roots of equation (1) & (2) gives

$\beta \times \frac{1}{a} = c \Rightarrow \beta = ac$

$\& \ \delta \times \frac{1}{a} = b \Rightarrow \delta = ab.$

$\therefore$ equation having roots β, δ is

$x^2 - a(b + c)x + a^2bc = 0$

$a(b + c)x^2 - a^2(b + c)^2x + a.(b + c)a^2bc = 0$

$a(b + c)x^2 + (b + c)x - abc = 0.$

22. **(2)**

Sol. $x^2 + 3x + 1 = (x - \alpha)(x - \beta)$. Put $x = 2$

$\Rightarrow 11 = (2 - \alpha)(2 - \beta)$

23. **(2)**

Sol. Since, both roots of equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4.

$\therefore$ $D \ge 0$

$\Rightarrow$ $4m^2 - 4m^2 + 4 \ge 0 \Rightarrow m \in R$ ......(i)

and $-2 < -\frac{b}{2a} < 4$

$\Rightarrow$ $-2 < m < 4$ ....(ii)

Also, $f(4) > 0$

$\Rightarrow$ $16 - 8m + m^2 - 1 > 0$

$\Rightarrow$ $m^2 - 8m + 15 > 0$

$\Rightarrow$ $(m - 3)(m - 5) > 0$

$\Rightarrow\ m \in (-\infty, 3) \cup (5, \infty)$ .....(iii)

Also, $f(-2) > 0$

$\Rightarrow\ 4 + 4m + m^2 - 1 > 0$

$\Rightarrow\ m^2 + 4m + 3 > 0$

$\Rightarrow\ (m + 3)(m + 1) > 0$

$\Rightarrow\ m \in (-\infty, -3) \cup (-1, \infty)$ .....(iv)

$\therefore$ Intersection of (i), (ii), (iii) and (iv) gives $m \in (-1, 3)$

24. (1)

Sol. Let the correct equation be $ax^2 + bx + c = 0$ now

sachin's equation $\Rightarrow ax^2 + bx + c' = 0$ with roots 4, 3

Rahul's equation $\Rightarrow ax^2 + b'x + c = 0$ with roots 3, 2

$-\frac{b}{a} = 7$ .... (i)

$\frac{c}{a} = 6$ .... (ii)

correct equation is $x^2 - 7x + 6 = 0$ roots are 6 and 1

25. (1)

Sol. $x^2 + 2x + 3 = 0$ ...(i)

$ax^2 + bx + c = 0$ ...(ii)

Since equation (i) has imaginary roots

So equation (ii) will also have both roots same as (i). Thus

$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

$\Rightarrow\ a = \lambda, b = 2\lambda, c = 3\lambda$

Hence 1 : 2 : 3

26. (3)

Sol.

$a^2 = 3\{x\}^2 - 2\{x\}$ [$\because x - [x] = \{x\}$]

Let $\{x\} = t \quad \therefore t \in (0, 1)$

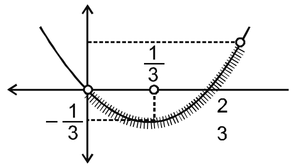
As x is not an integer

$\therefore\ a^2 = 3t^2 - 2t, f(t) = 3t$

$\Rightarrow\ a^2 = 3t$

Clearly by graph

$-\frac{2}{3} \le a^2 < 1$



$\therefore\ a \in (-1, 1) - \{0\}$ (As $x \ne$ integer)

27. (2)

Sol. $px^2 + qx + r = 0$ with roots α, β

$p, q, r \to$ A.P.

$2q = p + rv$

$\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$\frac{\alpha + \beta}{\alpha\beta} = 4$

$\Rightarrow\ \frac{-q}{r} = 4$

$q = -4r$ ..... (i)

$\therefore\ -8r = p + r$

$p = -9r$ ..... (ii)

$|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= \sqrt{\frac{q^2}{p^2} - \frac{4r}{p}}$ By (i) and (ii)

$= \frac{\sqrt{q^2 - 4pr}}{|p|} = \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$

28. (3)

Sol. $3x^2 + px + 3 = 0$ with roots α, α

$\because\ \alpha + \alpha^2 = -\frac{p}{3}$ .... (i)

$\alpha^3 = 1$

$\Rightarrow (\alpha - 1)(\alpha^2 + \alpha + 1) = 0$

$\Rightarrow \alpha = 1$ or $\alpha^2 + \alpha = -1$

so $p = -6$ $\quad p = 3$

29. (2)

Sol. $x^2 + bx + c = 0$ with roots α, β

$\because\ \alpha + \beta = -b$

$\alpha\beta = c$

$\because$ Sum is –ve and product is – ve.

$\therefore\ \alpha < 0 < \beta < |\alpha|$

30. **(4)**

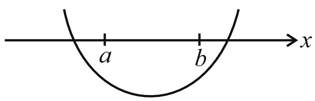
Sol. $(x - a)(x - b) - 1 = 0$

Let $f(x) = (x - a)(x - b) - 1$

$\because\ f(a) = -1$

$f(b) = -1$

and $\because$ the graph will be concave upwards.



31. **(4)**

Sol. $\min f(x) > \min g(x)$

$\Rightarrow\ -b^2 + 2c^2 > b^2 + c^2$

$\Rightarrow\ c^2 > 2b^2$

$\Rightarrow\ |c| > |b|\sqrt{2}$

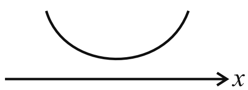
32. **(3)**

Sol. $x^2 + 2ax - (3a - 10) > 0 \quad \forall\ x \in R$

$\therefore\ D < 0$

$\Rightarrow\ 4a^2 + 4(3a - 10) < 0$

$\Rightarrow\ a^2 + 3a - 10 < 0$



$\Rightarrow\ (a + 5)(a - 2) < 0$

$\Rightarrow\ a \in (-5, 2)$

33. **(1)**

Sol. $x^2 + px + q = 0$

$\Rightarrow\ \alpha + \alpha^2 = -p$

and $\alpha^3 = q$

$\therefore\ (\alpha + \alpha^2)^3 = \alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2)$

$-p^3 = q + q^2 + 3q(-p).$

34. **(4)**

Sol. α, β be roots of equation $x^2 - px + r = 0$

So $\alpha + \beta = p,\ \alpha\beta = r$ .....(i)

and $\frac{\alpha}{2}$, 2β be roots of equation $x^2 - qx + r = 0$

then $\frac{\alpha}{2} + 2\beta = q,\ \alpha\beta = r$ .....(ii)

by (i) and (ii), $\beta = \frac{2q - p}{3}$

and $\alpha = \frac{2(2p - q)}{3}$

So $r = \alpha\beta = \frac{2}{9}(2p - q)(2q - p)$

35. **(2)**

Sol. Product = 1

Sum $= \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

Since $\alpha^3 + \beta^3 = q \Rightarrow -p(\alpha^2 + \beta^2 - \alpha\beta) = q$

$((\alpha + \beta)^2 - 3\alpha\beta) = -\frac{q}{p} \Rightarrow p^2 + \frac{q}{p} = 3\alpha\beta$

Hence sum $= \frac{\left\{p^2 - \frac{2}{3}\left(\frac{p^3 + q}{p}\right)\right\} 3p}{(p^3 + q)}$

$= \frac{p^3 - 2q}{p^3 + q}$

so the equation is $x^2 - \left(\frac{p^3 - 2q}{p^3 + q}\right) x + 1 = 0$

$\Rightarrow\ (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

36. **(2)**

Sol. $x^2 + bx - 1 = 0$

$x^2 + x + b = 0$

$\frac{x^2}{b^2 + 1} = \frac{x}{-1 - b} = \frac{1}{1 - b}$

$\Rightarrow\ x = \frac{b^2 + 1}{-(b + 1)} = \frac{-(b + 1)}{1 - b}$

$\Rightarrow\ (b^2 + 1)(1 - b) = (b + 1)^2$

$\Rightarrow\ b^2 - b^3 + 1 - b = b^2 + 2b + 1$

$\Rightarrow\ b^3 + 3b = 0$

$\Rightarrow\ b = 0\ ;\ b^2 = -3$

37. (4)

Sol. $p(x)$ will be of the form $ax^2 + c$. Since it has purely imaginary roots only.
Since $p(x)$ is zero at imaginary values while $ax^2 + c$ takes real value only at real 'x', no root is real.
Also $p(p(x)) = 0 \Rightarrow p(x)$ is purely imaginary
$\Rightarrow ax^2 + c$ = purely imaginary
Hence x can not be purely imaginary since x^2 will be negative in that case and $ax^2 + c$ will be real.

38. (3)

Sol. $x^2 - 2x\sec\theta + 1 = 0$

$$\Rightarrow x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2}$$

$\Rightarrow x = \sec\theta + \tan\theta, \sec\theta - \tan\theta$

$\Rightarrow \alpha_1 = \sec\theta - \tan\theta$

now $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x =$

$$\frac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$$

$\Rightarrow x = -\tan\theta \pm \sec\theta$

$\Rightarrow \alpha_2 = (\sec\theta - \tan\theta)$

$\Rightarrow \beta_2 = -(\sec\theta + \tan\theta)$

$\therefore \alpha_1 + \beta_2 = -2\tan\theta$

39. (3)

Sol. The given equation becomes an identity
$a(a^2 - 3a + 2) = 0$; $a^3 - 5a^2 + 6a = 0$ and $a^2 - 2a = 0$
$\Rightarrow a(a - 1)(a - 2) = 0$; $a(a - 2)(a - 3) = 0$ and $a(a - 2) = 0$
$\Rightarrow a = 0$ and $a = 2$

40. (1)

Sol. Given equation becomes
$a(1 - 2\sin^2 x) + b\sin^2 x + c = 0$
$(a + c) + (b - 2a)\sin^2 x = 0 \ \forall\, x \in R$

$a + c = 0$ and $b - 2a = 0 \Rightarrow \dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{-1}$

clearly a, b, c can not be natural numbers at a time

41. (2)

Sol. $D = 25(a + b)^2 + 8(a - b)^2 > 0 \ \forall\, a, b \in R$
Hence roots are real and unequal

42. (2)

Sol. $(2x + b)(m^2 + mb) = (x^2 + bx)(2m + b)$
$\Rightarrow x^2(2m + b) + (b^2 - 2m^2)x - b(m^2 + mb) = 0$
sum of roots must be zero
$\Rightarrow b^2 - 2m^2 = 0$
$\Rightarrow b^2 = 2m^2$

43. (2)

Sol. Let roots are $p\alpha$ and $q\alpha$ then
$(p + q)\alpha = -b/c$ ----(i) and $pq\alpha^2 = c/a$ -----(ii)
by (i) and (ii) $ac(p + q)^2 = b^2 pq$

44. (2)

Sol. $\alpha_2 = \dfrac{1}{\alpha_1}$ and $\beta_2 = \dfrac{1}{\beta_1}$ hence
$c_2x^2 + b_2x + a_2 = 0$ has root α_1, β_1
$(c_2x^2 + b_2x + a_2 = 0)$
which are common with $a_1x^2 + b_1x + c_1 = 0$

$$\Rightarrow \frac{c_2}{a_1} = \frac{b_2}{b_1} = \frac{a_2}{c_1}$$

45. (4)

Sol. Roots are opposite in sign

$$\Rightarrow \text{Product of roots} = \frac{a^2 - 3a + 2}{3} < 0$$

$\Rightarrow (a - 1)(a - 2) < 0 \Rightarrow a \in (1,2)$

46. (1)

Sol. $f(x) = ax^2 - bx + c$
$f(0) = c > 0$
$f(2) = 4a - 2b + c < 0$
Hence a roots lies in (0,2)

47. (3)

Sol. Let $y = \dfrac{x^2 - 6x + 5}{x^2 + 2x + 1} \Rightarrow (y - 1)x^2 + 2(y + 3)x + (y - 5) = 0$

as $x \in R \Rightarrow D \geq 0 \quad \Rightarrow 4(y + 3)^2 - 4(y - 1)(y - 5) \geq 0 \quad \Rightarrow y \geq -\dfrac{1}{3}$

48. (3)

Sol. Roots of the equation are α, β so $\alpha + \beta = p$ and $\alpha\beta = -p^2$
as $\alpha - p > 0$ and $\beta - p > 0$
$\Rightarrow \alpha + \beta - 2p > 0, (\alpha - p)(\alpha - p) > 0$
$\Rightarrow p - 2p > 0, -p^2 > 0$

$\Rightarrow p < 0$ and $p^2 < 0$, $p \in R$

$\Rightarrow p$ not exists

49. **(3)**

Sol. $ax^2 + 2x + 5 = a(x - \alpha)(x - \beta)$ .....(i)

Put $x = 1 \Rightarrow a(1-\alpha)(1-\beta) = a + 7$

$x = -1 \Rightarrow a(1+\alpha)(1+\beta) = a + 3$

$\Rightarrow \dfrac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)+\frac{4}{a}} = \dfrac{\left(\frac{a+7}{a}\right)}{\frac{a+3}{a}+\frac{4}{a}} = 1$

50. **(2)**

Sol. $3x^2 + 6cx + c^2 = 0 \Rightarrow \alpha^2 + 2\alpha = -c/3$ and $\beta^2 + 2\beta = -c/3$

Hence quadratic equation is $x^2 + \frac{2c}{3}x + \frac{c^2}{9} = 0$

$\Rightarrow 9x^2 + 6cx + c^2 = 0$

51. **(3)**

Sol. $D_1 = b^2 - 8c$ and $D_2 = b^2 + 4c$ hence at least one of D_1 and D_2 is > 0

52. **(3)**

Sol. $\alpha + \beta + \gamma = 0$ $\alpha\beta + \beta\gamma + \gamma\alpha = -3$

$\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = 0$

$\alpha^2 + \beta^2 + \gamma^2 = 6$

53. **(4)**

Sol. Given that $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$

$\Rightarrow (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = -56$

$\Rightarrow \alpha^2 + \beta^2 - \alpha\beta = 28$

Now $(\alpha + \beta)^2 = (-2)^2$

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 4$

$\Rightarrow 28 + 3\alpha\beta = 4 \Rightarrow \alpha\beta = -8$

$\therefore$ required equation is $x^2 - (-2)x + (-8) = 0$

$\Rightarrow x^2 + 2x - 8 = 0.$

54. **(1)**

Sol. $x^2 - 10x + (y^2 + 21) = 0$

for real roots of x, $D \geq 0$

$100 - 4(y^2 + 21) \geq 0$

$\Rightarrow y^2 \leq 4$

$\Rightarrow -2 \leq y \leq 2$

also $y^2 = -x^2 + 10x - 21$

for real roots of y, $-x^2 + 10x - 21 \geq 0$

$\Rightarrow (x - 7)(x - 3) \leq 0$

$3 \leq x \leq 7$

55. **(3)**

Sol. $2 - i$ and $+ 2i$ are other roots.

So, product is $= (2 + i)(2 - i)\left(\sqrt{5} + 2i\right)\left(\sqrt{5} - 2i\right) = 5 \times 9 = 45$

56. **(2)**

Sol. According to question

$\alpha + \alpha^2 = 6$ ....(i)

$\alpha^3 = c$ ....(ii)

$\Rightarrow \alpha^2 + \alpha - 6 = 0$ Form (i)

$\Rightarrow (\alpha + 3)(\alpha - 2) = 0 \Rightarrow \alpha = -3$ or $\alpha = 2$

$\therefore$ Form (ii) , we get c = – 27 or 8.

57. **(1)**

Sol. $a(p + q)^2 + 2apq + c = 0$

$a(p + r)^2 + 2apr + c = 0$

$\Rightarrow a(p + x)^2 + 2apx + c = 0$

Product of roots

$qr = p^2 + \frac{c}{a}$

58. **(1)**

Sol. $D = 4a^2 - 4[a - b + c](a + b - c)$

$= 4[a^2 - \{a^2 + ab - ac - ab - b^2 + bc + ac + bc - c^2]$

$= 4[b^2 + c^2 - 2bc] = 4(b - c)^2$

$D = \{2(b - c)\}^2$

Hence D is perfect square of a rational number

59. **(4)**

Sol. $(x^2 + ax - 3b)(x^2 - cx + b)(x^2 - dx + 2b) = 0$

there are 3 quadratic equation

Let D_1, D_2, D_3 be the discriminant

$D_1 = a^2 + 12b$

$D_2 = c^2 - 4b$

$D_3 = d^2 - 8b$

If b is less than zero D_2 & D_3 are greater than zero

If b is greater than zero then $D_1 > 0$

So the equation

will have atleast 2 real roots

60. **(1)**

Sol. Given that, roots of equation $x^2 - 10ax - 11b = 0$

are c, d

So $c + d = 10a$ and $cd = -11b$

and a, b are the roots of equation $x^2 - 10cx - 11d = 0$

$\therefore\ a + b = 10c,\ ab = -11d$

So $a + b + c + d = 10(a + c)$

and $(c + d) - (a + b) = 10(a - c)$

$(c - a) - (b - d) + 10(c - a) = 0$

$b + d = 9(a + c)$ ....(i)

$abcd = 121\, bd$

$ac = 121$ ....(ii)

$b - d = 11(c - a)$ ....(iii)

c & a satisfies the equation $x^2 - 10ax - 11b = 0$ and $x^2 - 10cx - 11d = 0$ respectively

$\therefore\ c^2 - 10ac - 11b = 0$

$a^2 - 10ca - 11d = 0$

$(c^2 - a^2) - 11(b - d) = 0$

$(c - a)(c + a) = 11(b - d) = 11.\ 11(c - a)$ (by equation (iii))

$c + a = 121$

$a + b + c + d = 10(c + a)$

$10.121 = 1210$

61. **(4)**

Sol. Let $y = \dfrac{x^2 - x + c}{x^2 + x + 2c}$; $x \in R$ and $y \in R$.

$\Rightarrow (y - 1)x^2 + (y + 1)x + 2yc - c = 0$

$\because\ x \in R$

$\Rightarrow D \geq 0$

$\Rightarrow (y + 1)^2 - 4c(y - 1)(2y - 1) \geq 0$

$\Rightarrow y^2 + 1 + 2y - 4c[2y^2 - 3y + 1] \geq 0$

$\Rightarrow (1 - 8c)y^2 + (2 + 12c)y + 1 - 4c \geq 0$ .... (1)

Now for all $y \in R$, (1) will be true if .

$1 - 8c > 0 \Rightarrow c < \dfrac{1}{8}$ and $D \leq 0$

$\Rightarrow 4(1 + 6c)^2 - 4(1 - 8c)(1 - 4c) \leq 0$

$\Rightarrow 1 + 36c^2 + 12c - 1 - 32c^2 + 12c \leq 0$

$\Rightarrow 4c^2 + 24c \leq 0 \Rightarrow -6 \leq c \leq 0$

But $c = -6$ and $c = 0$ will not satisfy given condition $\therefore\ c \in (-6, 0)$

62. **(2)**

Sol. Dis. of $x^2 + px + 3q$ is $p^2 - 12q \equiv D_1$

Dis. of $-x^2 + rx + q$ is $r^2 + 4q \equiv D_2$

Dis. of $-x^2 + sx - 2q$ is $s^2 - 8q \equiv D_3$

Case 1 : If $q < 0$, then $D_1 > 0$, $D_3 > 0$ and D_2 may or may not be positive

Case 2 : If $q > 0$, then $D_2 > 0$ and D_1, D_3 may or may not be positive

Case 3 : If $q = 0$, then $D_1 \geq 0, D_2 \geq 0$ and $D_3 \geq 0$

from **Case 1, Case 2** and **Case 3** we can say that the given equation has at least two real roots.

63. **(2)**

Sol. $(a - 1)(x^2 + x + 1)^2 - (a + 1)(x^4 + x^2 + 1) = 0$ ......(1)

$\because\ x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$

$\therefore$ (1) becomes

$\Rightarrow (x^2 + x + 1)[(x^2 + x + 1)(a - 1) - (a + 1)(x^2 - x + 1)] = 0$

$\Rightarrow (x^2 + x + 1)(x^2 - ax + 1) = 0$

Here two roots are imaginary and for other two roots to be real $D > 0$

$\Rightarrow a^2 - 4 > 0$

$\Rightarrow a \in (-\infty, -2) \cup (2, \infty)$

64. **(4)**

Sol. $(a + x)^{2/3} + (x - a)^{2/3} = 4(a^2 - x^2)^{1/3}$

$$\left[\frac{a + x}{a - x}\right]^{1/3} + \left[\frac{a - x}{a + x}\right]^{1/3}(-1)^{2/3} = 4$$

$$\left(\frac{a + x}{a - x}\right)^{1/3} + \left(\frac{a - x}{a + x}\right)^{1/3} = 4$$

$$t^{1/3} + \left(\frac{1}{t}\right)^{1/3} = 4,\ \frac{a + x}{a - x} = t$$

cube $\rightarrow$

$$t + \frac{1}{t} + 3\left[t^{1/3} + (t)^{-1/3}\right] = 64$$

$$t + \frac{1}{t} + 34 = 64$$

$t^2 + 1 + 12t = 64t$

$t^2 - 52t + 1 = 0$

$D > 0$

two solutions

65. (1)

Sol. $(3-x)^4 + (2-x)^4 = (5-2x)^4$

Let $(3-x) = a$, $(2-x) = b$

$a^4 + b^4 = (a+b)^4$

$\Rightarrow a^4 + b^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$\Rightarrow 4a^3b + 6a^2b^2 + 4ab^3 = 0$

$\Rightarrow 2ab\,[2a^2 + 3ab + 2b^2] = 0$

$\Rightarrow ab = 0$

$\Rightarrow (3-x)(2-x) = 0$

$\Rightarrow x = 2$ & 3

or $2a^2 + 3ab + b^2 = 0$

$\Rightarrow 2(3-x)^2 + 3(3-x)(2-x) + 2(2-x)^2 = 0$

$\Rightarrow 2[9 + x^2 - 6x] + 3[6 + x^2 - 5x] + 2[x^2 - 4x + 4] = 0$

$\Rightarrow 7x^2 - 35x + 44 = 0$

$D < 0$

Integer Type Questions (66 to 75)

66. (3)

Sol. $\sqrt{x^2-4} - (x-2) = \sqrt{x^2-5x+6}$

$\sqrt{x-2}\,(\sqrt{x+2} - \sqrt{x-2}) = \sqrt{x-2}\,\sqrt{x-3}$

so $x = 2$ or $\sqrt{x+2} - \sqrt{x-2} = \sqrt{x-3}$

squaring both side

$x + 2 + x - 2 - 2\sqrt{x^2-4} = x - 3$

$\Rightarrow x + 3 = 2\sqrt{x^2-4}$

$\Rightarrow x^2 + 6x + 9 = 4x^2 - 16$

$\Rightarrow 3x^2 - 6x - 25 = 0$

$D > 0$

Total 3 solution

67. (1)

Sol. Let α and β be the roots of equation $x^2 - (a-2)x - a - 1 = 0$, then $\alpha + \beta = a - 2$, $ab = -(a+1)$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = (a-2)^2 + 2(a+1)$

$\Rightarrow \alpha^2 + \beta^2 = a^2 - 2a + 6$

$\Rightarrow \alpha^2 + \beta^2 = (a-1)^2 + 5$

The value of $\alpha^2 + \beta^2$ will be least, if $a - 1 = 0$.

$\Rightarrow a = 1$

68. (3)

Sol. Since, tan 30º and tan 15º are the roots of equation $x^2 + px + q = 0$.

$\therefore$ $\tan 30^\circ + \tan 15^\circ = -p$ and $\tan 30^\circ \tan 15^\circ = q$

Therefore, $2 + q - p = 2 + \tan 30^\circ \tan 15^\circ + (\tan 30^\circ + \tan 15^\circ)$

$\Rightarrow 2 + q - p = 2 + \tan 30^\circ \tan 15^\circ + 1 - \tan 30^\circ . \tan 15^\circ$ $\left(\because \tan 45^\circ = \dfrac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ}\right)$

$\Rightarrow 2 + q - p = 3$

69. (41)

Sol. Now, $\dfrac{3x^2+9x+17}{3x^2+9x+7} = 1 + \dfrac{10}{3\left(x^2+3x+\dfrac{7}{3}\right)} = 1 + \dfrac{10}{3\left[\left(x+\dfrac{3}{2}\right)^2 + \dfrac{1}{12}\right]}$

$\Rightarrow$ Maximum value of $\dfrac{3x^2+9x+17}{3x^2+9x+17}$ occurs at $x = -\dfrac{3}{2}$.

$\therefore$ Maximum value of $\dfrac{3x^2+9x+17}{3x^2+9x+7} = 1 + \dfrac{10}{3\left(\dfrac{1}{12}\right)} = 1 + 40 = 41$

70. (2)

Sol. $x^2 + ax + 1 = 0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$

$\Rightarrow |\alpha - \beta| <$

$\Rightarrow (\alpha-\beta)^2 < 5$

$\Rightarrow (\alpha+\beta)^2 - 4\alpha\beta < 5$

$\Rightarrow a^2 - 4 < 5$

$\Rightarrow a \in (-3, 3)$

71. (18)

Sol. $P(x) = 0$

$\Rightarrow f(x) = g(x)$

$\Rightarrow ax^2 + bx + c = a_1x^2 + b_1x + C,$

$\Rightarrow \quad (a - a_1) x^2 + (b - b_1) x + (c - c_1) = 0.$

It has only one solution $x = -1$

$\Rightarrow \quad b - b_1 = a - a_1 + c - c_1$ .... (1)

vertex $(-1, 0) \Rightarrow \dfrac{b - b_1}{2(a - a_1)} = -1$

$\Rightarrow \quad b - b_1 = 2(a - a_1)$ .... (2)

$\Rightarrow \quad f(-2) - g(-2) = 2$

$\Rightarrow \quad 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$

$\Rightarrow \quad 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2$ ... (3)

by (1), (2) and (3) $(a - a_1) = (c - c_1) = \dfrac{1}{2}(b - b_1) = 2$

Now $P(2) = f(2) - g(2)$

$= 4(a - a_1) + 2(b - b_1) + (c - c_1)$

$= 8 + 8 + 2 = 18$

72. **(3)**

Sol. $x^2 - 6x - 2 = 0$

$a_n = \alpha^n - \beta^n$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} = \frac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)}$$

$$\frac{\alpha + \beta}{2} = \frac{6}{2} = 3$$

73. **(2)**

Sol. for $x^2 - 6x + 1 = 0$, $D_1 > 0$

and $x^2 + 3x + 6 = 0$, $D_2 < 0$

Hence only two real roots exists.

74. **(0)**

Sol. $a, b, c > 0 \Rightarrow ax^2 + 2b(x) + c > 0 \ \forall \ x \in R^+$

75. **(0)**

Sol. $27^{1/x} + 12^{1/x} = 2.(8^{1/x})$

$3^{3/x} + 3^{1/x}.2^{2/x} = 2.2^{3/x}$

$$\left(\frac{3}{2}\right)^{3/x} + \left(\frac{3}{2}\right)^{1/x} = 2$$

assume $\left(\dfrac{3}{2}\right)^{1/x} = t$

$t^3 + t - 2 = 0$

$(t - 1)(t^2 + t + 2) = 0$

$\Rightarrow t = 1$

so $\left(\dfrac{3}{2}\right)^{1/x} = 1$

No real solution

COMPLEX NUMBERS

Single Option Correct Type Questions (01 to 62)

1. **(4)**

Sol: $\dfrac{(1-\cos\theta)-2i\sin\theta}{(1-\cos\theta)^2+4\sin^2\theta}$

$= \dfrac{1-\cos\theta-2\,i\,\sin\theta}{(1-\cos\theta)(1-\cos\theta+4\,(1+\cos\theta))}$

real part $= \dfrac{(1-\cos\theta)}{(1-\cos\theta)(5+3\cos\theta)}$

$= \dfrac{1}{5+3\cos\theta}$

2. **(3)**

Sol: $z = \dfrac{1+i\sqrt{3}}{\sqrt{3}+i} = \dfrac{2e^{i\frac{\pi}{3}}}{2e^{i\frac{\pi}{6}}} = e^{i\frac{\pi}{6}}$

$\overline{z} = e^{-i\frac{\pi}{6}}$

$\overline{z}^{\,100} = e^{-i\frac{100\pi}{6}} = e^{-i\frac{50\pi}{3}} = e^{-\frac{2i\pi}{3}}$

which is in III quadrant

3. **(2)**

Sol: $z = 1 + \cos\dfrac{11\pi}{9} + i\sin\dfrac{11\pi}{9}$

$z = 1 - \cos\dfrac{2\pi}{9} - i\sin\dfrac{2\pi}{9}$

$z = 2\sin^2\dfrac{\pi}{9} - 2i\sin\dfrac{\pi}{9}\cos\dfrac{\pi}{9}$

$z = 2\sin\dfrac{\pi}{9}\left(\sin\dfrac{\pi}{9} - i\cos\dfrac{\pi}{9}\right)$

$= 2\cos\left(\dfrac{7\pi}{18}\right)\left(\cos\dfrac{7\pi}{18} - i\sin\dfrac{7\pi}{18}\right)$

$= 2\cos\left(\dfrac{7\pi}{18}\right)\left(\cos\left(-\dfrac{7\pi}{18}\right) + i\sin\left(\dfrac{-7\pi}{18}\right)\right)$

$arg\,(z) = \dfrac{-7\pi}{18}$

$|\,z\,| = 2\cos\dfrac{7\pi}{18}$

4. **(1)**

Sol: $(2+i)\,(2+2i)\,(2+3i)\,.....\,(2+9i) = x+iy$

$5.8.13.........\ 85 = (x^2+y^2)$

5. **(1)**

Sol: $|z_1+z_2| = |z_1-z_2|$

$z_1\overline{z}_1 + z_2\overline{z}_2 + z_1\overline{z}_2 + \overline{z}_1 z_2 = z_1\overline{z}_1 + z_2\overline{z}_2 - z_1\overline{z}_2 - \overline{z}_1 z_2$

$z_1\overline{z}_2 + \overline{z}_1 z_2 = 0$

$\dfrac{z_1}{z_2} + \dfrac{\overline{z}_1}{\overline{z}_2} = 0$

$\dfrac{z_1}{z_2}$ is purely imaginary $\left|\arg\left(\dfrac{z_1}{z_2}\right)\right| = \dfrac{\pi}{2}$

6. **(1)**

Sol: $|z_1| = |z_2| = |z_3| = 1$

$z_1\overline{z}_1 = z_2\overline{z}_2 = z_3\overline{z}_3 = 1$

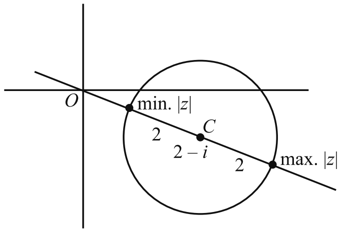
Given $1 = \left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3}\right| = |\overline{z}_1 + \overline{z}_2 + \overline{z}_3|$

$=\left|\overline{z_1+z_2+z_3}\right| = 1$

$1=|z_1+z_2+z_3|$

7. **(1)**

Sol:



$OC = \sqrt{5}$

min. $|z| = \sqrt{5}-2$

max. $|z| = \sqrt{5}+2$

8. **(1)**

Sol: $z = -4+5i$

$z_{new} = 1.5\,(-4+5i)\;e^{i\pi}$

$= \frac{3}{2}\;(4-5i) = 6 - \frac{15i}{2}$

9. **(1)**

Sol: $\therefore\ |z-(2-i)| = |z-(3+i)|$

Locus of z is the perpendicular bisector of

$(2, -1)$ and $(3, 1)$

i.e. $2x+4y=5$

10. **(1)**

Sol: Ray joining, z to $2+3i$ should make on angle of $\frac{\pi}{4}$ with $+ve$ direction of real axis.

11. **(4)**

Sol: $G \to$ Centroid of $\Delta = \frac{z_1+z_2+z_3}{3}$

$H \to$ Orthocentre $= z$ say, $O \to$ Circum centre $= 0$

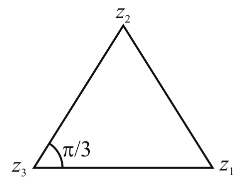
$\because$ G divides HO in ratio 2:1

$\frac{z_1+z_2+z_3}{3} = \frac{2\cdot 0+1\cdot z}{2+1}$

$\Rightarrow\ z = z_1+z_2+z_3$

12. **(3)**

Sol:



$\because\ \frac{z_1-z_3}{z_2-z_3} = \frac{1-i\sqrt{3}}{2} = \frac{(1-\sqrt{3}i)(1+\sqrt{3}i)}{2(1+\sqrt{3}i)} =$

$\frac{4}{2(1+\sqrt{3}i)} = \frac{2}{1+\sqrt{3}i}$

$\frac{z_2-z_3}{z_1-z_3} = \frac{1+i\sqrt{3}}{2} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$\Rightarrow \left|\frac{z_2-z_3}{z_1-z_3}\right| = 1$ and $arg\left(\frac{z_2-z_3}{z_1-z_3}\right) = \frac{\pi}{3}$

Hence triangle is equilatral

13. **(1)**

Sol: $\frac{(\cos 2\theta - i\sin 2\theta)^4(\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2}(\cos 3\theta - i\sin 3\theta)^{-9}}$

$= e^{(-8\theta-20\theta+6\theta-27\theta)i}$

$= e^{(-49\theta)i} = \cos 49\theta - i\sin 49\theta$

14. **(1)**

Sol: $\left[\frac{1+\cos(\pi/8)+i\sin(\pi/8)}{1+\cos(\pi/8)-i\sin(\pi/8)}\right]^8 =$

$\left(\frac{2\cos^2\frac{\pi}{16}+i.2\sin\frac{\pi}{16}\cos\frac{\pi}{16}}{2\cos^2\frac{\pi}{16}-i\,2\cos\frac{\pi}{16}\sin\frac{\pi}{16}}\right)^8$

$= \left(\frac{\cos\frac{\pi}{16}+i.\sin\frac{\pi}{16}}{\cos\frac{\pi}{16}-i\sin\frac{\pi}{16}}\right)^8 = \cos\pi + i\sin\pi = -1$

15. **(3)**

Sol: $E^{i\theta}.e^{2i\theta}.e^{3i\theta}.........e^{ni\theta} = 1$

$= e^{i\theta(1+2+........+n)} = 1$

$= e^{\frac{in(n+1)\theta}{2}} = 1$

$\Rightarrow \frac{n(n+1)\theta}{2} = 2k\pi \qquad k \in I$

$\Rightarrow \theta = \frac{4k\pi}{n(n+1)} \qquad k \in I$ or $\theta = \frac{4m\pi}{n(n+1)}$, $m \in I$

16. **(3)**

Sol: $4 + 5\omega^{334} + 3\,\omega^{365} = 4 + 5\omega + 3\omega^2 = 1 + 2\omega = 1 + 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = i\sqrt{3}$

17. **(3)**

Sol: $z_1z_2z_3z_4z_5 = e^{i\frac{2\pi}{5}[1+2......5]} = e^{\frac{2\pi}{5}\cdot\frac{5}{2}\cdot 6} = e^{i6\pi} = 1$

18. **(2)**

Sol: $z = 2e^{i\pi}\,e^{i\pi/6} = 2e^{-i5\pi/6}$

$|z| = 2, \qquad \text{Arg } z = -\frac{5\pi}{6}$.

19. **(1)**

Sol: $z = 1 + e^{i\frac{18\pi}{25}} = e^{i\frac{9\pi}{25}}\left[e^{i\frac{9\pi}{25}} + e^{-i\frac{9\pi}{25}}\right]$

$z = 2\cos\left(\frac{9\pi}{25}\right)\, e^{i\frac{9\pi}{25}}$

$|z| = 2\cos\left(\frac{9\pi}{25}\right) \qquad \text{Arg } z = \frac{9\pi}{25}$

20. **(1)**

Sol: $(a + ib)^5 = \alpha + i\beta$

$i^5\,(b - ia)^5 = \alpha + i\beta$

$(b - ia)^5 = -\,i\,\alpha + \beta$

$(b + ia)^5 = \beta + i\alpha$

21. **(4)**

Sol: $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$

$\Rightarrow \frac{z_1}{z_2} = -\frac{\bar{z}_1}{\bar{z}_2}$

$\frac{z_1}{z_2} + \overline{\left(\frac{z_1}{z_2}\right)} = 0 \Rightarrow \frac{z_1}{z_2}$ is purely imaginary

so amp$\left(\frac{z_1}{z_2}\right)$ may be $\frac{\pi}{2}$

22. **(1)**

Sol: max $(arg\ z) = \frac{\pi}{2}$

max $|\,z\,| = d + r$

min $|\,z\,| = d - r$

$d = OC = \sqrt{5}$

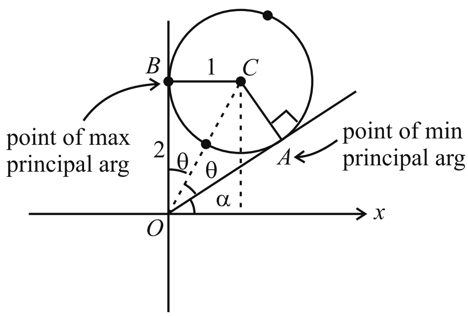
$r = 1$

$\tan\theta = \frac{1}{2}$

$\alpha = \frac{\pi}{2} - 2\theta$

$\tan\alpha = \cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$

$= \frac{1-\frac{1}{4}}{1} = \frac{3}{4}$



23. **(3)**

Sol: (I) $z_k z_j = 1 \Rightarrow z_j = z_{10-k}$

Hence for each $k \in \{1, 2, 3, \ldots, 9\}$ there exists z_j such that $z_k \cdot z_j = 1$ True

(II) $z_1 \cdot z = z_k \Rightarrow z = z_{k-1}$ for $k = 2, 3, 4, \ldots, 9$ & $z = 1$ for $k = 1$ False

(III) $z_1, z_2, \ldots, z_9$ are roots of the equation $z^{10} = 1$ other then unity, hence

$\frac{z^{10}-1}{z-1} = 1 + z + \ldots + z^9 = (z - z_1)(z - z_2) \ldots (z - z_9)$

Substituting $z = 1$, we get $\frac{(1-z_1)(1-z_2)\ldots(1-z_9)}{10} = \frac{10}{10} = 1$

(IV) $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right) = 1 - \{$sum of real parts of roots of $z^{10} = 1$ except $1\}$

$= 1 - (-1) = 2$

(as $1 + z_1 + z_2 + \ldots + z_9 = 0$)

$\Rightarrow \sum \text{Re}(z_k) + 1 = 0$

24. **(2)**

Sol: $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$

$\Rightarrow (a-1)^2 + (1-b)^2 = a^2 + 1 = b^2 + 1$

$\Rightarrow a = b$ and $a^2 - 2a + 1 + b^2 - 2b + 1 = a^2 + 1$

$\Rightarrow a^2 - 4a + 1 = 0$

$\Rightarrow \quad a = 2 - \sqrt{3} = b$

$\therefore \quad 0 < a, b < 1$

25. **(1)**

Sol: All the three vertices lies on circle $|z| = 1$

so there centroid at O i.e. $\frac{z_1 + z_2 + z_3}{3} = 0$

26. **(2)**

Sol: z_1, z_2, z_3 are in $A.P.$

$2z_2 = z_1 + z_3$

$\Rightarrow z_2 = \frac{z_1 + z_3}{2}$

$\Rightarrow$ straight line

27. **(1)**

Sol: $z\bar{z} + \alpha\bar{z} + \bar{\alpha}z + k = 0$ is equation of circle

centre $= -\alpha$

$= -4 - 3i$

radius $= \sqrt{\alpha\bar{\alpha} - k}$

$= \sqrt{25 - 5} = 2\sqrt{5}$

28. **(1)**

Sol: Given $= e^{\frac{\pi}{2}i + \frac{\pi}{2^2}i + \ldots + \infty} = e^{\frac{\pi i/2}{1-\frac{1}{2}}} = e^{\pi i} = -1$

29. **(1)**

Sol: $\left(\frac{e^{i\alpha}}{e^{-i\alpha}}\right)^n - \left[\frac{e^{in\alpha}}{e^{-in\alpha}}\right]$

$\Rightarrow e^{i2n\alpha} - e^{i2n\alpha} = 0$

30. **(1)**

Sol: $x = e^{i\theta} \qquad y = e^{\phi} \quad \Rightarrow x^n + \frac{1}{x^n} = 2\cos n\theta$

$\Rightarrow x^n - \frac{1}{x^n} = 2i\sin n\theta$

31. **(4)**

Sol: $h(\omega) = \omega f(\omega^3) + \omega^2 g(\omega^3) = 0$

and $h(\omega^2) = \omega^2 f(\omega^6) + \omega^4 g(\omega^6) = 0$

$\Rightarrow \omega f(1) + \omega^2 g(1) = 0$ and $\omega^2 f(1) + \omega g(1) = 0$

$\Rightarrow f(1) = 0$ and $g(1) = 0 \qquad \Rightarrow h(1) = 0$

32. **(4)**

Sol: $x^n - 1 = (x-1)(x-\omega)(x-\omega^2) \ldots\ldots\ldots (x - \omega^{n-1})$

put $x = 5$

$\frac{5^n - 1}{4} = (5-\omega)(5-\omega^2) \ldots (5 - \omega^{n-1})$

33. **(1)**

Sol: $(z-1)(z-\alpha_1) \ldots\ldots\ldots (z-\alpha_4) = z^5 - 1$

Put $z = \omega$, $z = \omega^2$ and divide

$$\frac{(\omega-1)(\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)(\omega-\alpha_4)}{(\omega^2-1)(\omega^2-\alpha_1)(\omega^2-\alpha_2)(\omega^2-\alpha_3)(\omega^2-\alpha_4)}$$

$$= \frac{\omega^5 - 1}{\omega^{10} - 1}$$

$$\frac{(\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)(\omega-\alpha_4)}{(\omega^2-\alpha_1)(\omega^2-\alpha_2)(\omega^2-\alpha_3)(\omega^2-\alpha_4)}$$

$$= \frac{(\omega^2-1)^2}{(\omega-1)^2} = (\omega+1)^2 = \omega^4 = \omega$$

34. **(3)**

Sol: Real $(1 + \alpha + \alpha^2 + \ldots\ldots + \alpha^{10}) = 0 \Rightarrow 1$ + Real $(\alpha + \alpha^2 + \ldots\ldots \alpha^5)$ + Real$(\alpha^6 + \alpha^7 + \ldots\ldots + \alpha^{10}) = 0$

$\Rightarrow 2$ Real $(\alpha + \alpha^2 + \ldots\ldots \alpha^5) = -1 \quad \Rightarrow (\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5) = -\dfrac{1}{2}$

35. **(4)**

Sol: Sum of root $= a + a^2 + a^3 + a^4 + a^5 + a^6 = -1$ product of root $= 3a^7 + (a + a^2 + a^3 + a^4 + a^5 + a^6) = 3 - 1 = 2 \Rightarrow$ quadratic equation is $x^2 + x + 2 = 0$

36. **(4)**

Sol: Statement-1 $\quad Arg\,(2 + 3i)$ is $\tan^{-1}\dfrac{3}{2}$

$Arg\,(2 - 3i)$ is $\tan^{-1}\left(-\dfrac{3}{2}\right)$

$Arg\,(2 + 3i) + Arg\,(2 - 3i) = 0$

Statement-2 Let $z = -2 + 0i$, then $\bar{z} = -2 - 0i$

$\therefore Arg\,(z) + Arg\,(\bar{z}) = 2\pi \neq 0$

$\therefore$ statement is wrong.

37. **(3)**

Sol: Statement -1 $\quad (1 + z)^6 = -z^6$

take modulus

$|1 + z|^6 = |z|^6$

$\left|\dfrac{1+z}{z}\right| = 1$ which is straight line

Statement -2 $\quad z_2 = \dfrac{z_1 + z_3}{2}$

$\therefore z_2$ is mid point of line joining z_1 & z_3. Hence z_1, z_2, z_3 are collinear

38. **(3)**

Sol: For statement-1

$$\frac{1}{z_1 - z_2} + \frac{1}{(z_2 - z_3)} + \frac{1}{(z_3 - z_1)} = 0$$

$$\Rightarrow \quad \frac{1}{z_1 - z_2} + \frac{1}{(z_2 - z_3)} + \frac{1}{(z_3 - z_1)} = 0$$

$\Rightarrow \quad z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

$\Rightarrow \quad z_1\, z_2\, z_3$ are vertices of equilateral triangle

For statement-2

$|z_1 - z_0| = |z_2 - z_0| = |z_3 - z_0|$

$\Rightarrow \quad z_0$ is circum-centre

39. **(4)**

Sol: If ω is a cube root of unity, then $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

Now,

$(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$

$(\because\ 1 + \omega + \omega^2 = 0) = (-2\,\omega^2)^7$

$= -2^7.\,\omega^{14}$

$= -128\,(\omega^3)^4\,\omega^2$

$= -128\,\omega^2 \qquad (\because\ \omega^3 = 1)$

40. **(3)**

Sol: If z_1, z_2 and z_3 are vertices of an equilateral triangle. Then,

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

Since, origin, z_1 and z_2 are the vertices of an equilateral triangle, then

$z_1^2 + z_2^2 = z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2 \qquad \ldots$(i)

Again z_1, z_2 are the roots of the equation

$z^2 + az + b = 0$,

Then, $z_1 + z_2 = -a$

and $z_1z_2 = b$

On putting these values in Eq. (i), we get

$(-a)^2 = 3b$

$\Rightarrow a^2 = 3b$.

41. **(4)**

Sol: Let $z = r_1\,e^{i\theta}$ and $w = r_2\,e^{i\phi}$

$\Rightarrow \quad \bar{z} = r_1\,e^{-i\theta}$

Given, $|z\omega| = 1$

$\Rightarrow \left|r_1e^{i\theta}.r_2e^{i\phi}\right| = 1$

$\Rightarrow r_1r_2 = 1 \qquad \ldots$(i) and $arg(z) - arg(\omega) = \frac{\pi}{2} \Rightarrow \theta - \phi = \frac{\pi}{2}$

Then, $\bar{z}\omega = r_1 e^{-i\theta}.r_2 e^{i\phi}$

$= r_1 r_2 \ e^{-i(\theta-\phi)}$

From Eqs. (i) and (ii), we get

$\bar{z}\ \omega = 1.\ e^{-i\pi/2}$

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} \Rightarrow \bar{z}\ \omega = -i.$

42. (1)

Sol: $\left(\frac{1+i}{1-i}\right)^x = \left[\frac{(1+i)\ (1+i)}{(1-i)\ (1+i)}\right]^x = \left[\frac{(1+i)^2}{1-i^2}\right]^x$

$= \left[\frac{1-1+2i}{2}\right]^x$

$\Rightarrow \left(\frac{1+i}{1-i}\right)^x = (i)^x = 1$ (given)

$\Rightarrow (i)^x = (i)^{4n}$,

where n is any positive integer. $\Rightarrow x = 4n$.

43. (3)

Sol: Since, $\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\ \bar{w}$

$\Rightarrow z = iw \Rightarrow w = -iz$

Also $arg(zw) = \pi$

$\Rightarrow arg(-iz^2) = \pi$

$\Rightarrow arg(-i) + 2\ arg(z) = \pi$

$\Rightarrow -\frac{\pi}{2} + 2arg(z) = \pi \left[\because \arg(-i) = -\frac{\pi}{2}\right]$

$\Rightarrow 2\ arg\ (z) = \frac{3\pi}{2}$

$\Rightarrow arg\ (z) = \frac{3\pi}{4}$

44. (4)

Sol: $z^{1/3} = p + iq$

$\Rightarrow (x - iy)^{1/3} = (p + iq)$ $(\because z = x - iy)$

$\Rightarrow (x - iy) = (p + iq)^3$

$\Rightarrow (x - iy) = p^3 + (iq)^3 + 3p^2qi + 3pq^2i^2$

$\Rightarrow (x - iy) = p^3 - iq^3 + 3p^2qi - 3pq^2$

$\Rightarrow (x - iy) = (p^3 - 3pq^2) + i(3p^2q - q^3)$

On comparing both sides, we get

$x = (p^3 - 3pq^2)$ and $-y = 3p^2q - q^3$

$\Rightarrow x = p(p^2 - 3q^2)$ and $y = q(q^2 - 3p^2)$

$\Rightarrow \frac{x}{p} = (p^2 - 3q^2)$ and $\frac{y}{q} = (q^2 - 3p^2)$

Now, $\frac{x}{p} + \frac{y}{q} = p^2 - 3q^2 + q^2 - 3p^2$

$\Rightarrow \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\Rightarrow \frac{\frac{x}{p} + \frac{y}{q}}{(p^2 + q^2)} = -2$

45. (2)

Sol: Since, $(x - 1)^3 + 8 = 0$

$\Rightarrow (x - 1)^3 = -8 = (-2)^3 \Rightarrow \left(\frac{x-1}{-2}\right)^3 = 1$

$\Rightarrow \left(\frac{x-1}{-2}\right) = (1)^{1/3}$

$\therefore$ Cube roots of $\left(\frac{x-1}{-2}\right)$ are 1, ω and ω^2.

$\Rightarrow$ Cube roots of $(x - 1)$ are -2, -2ω and $-2\ \omega^2$

$\Rightarrow$ Cube roots of x are -1, $1 - 2\omega$ and $1 - 2\omega^2$.

46. (2)

Sol: Since, $|z_1 + z_2| = |z_1| + |z_2|$

$\therefore\ |z_1 + z_2|^2 = (|z_1| + |z_2|)^2$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2Re(z_1\ \bar{z}_2) = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\Rightarrow Re(z_1\bar{z}_2) = |z_1||z_2|$

$\Rightarrow |z_1||z_2| \cos(\theta_1 - \theta_2) = |z_1|\ |z_2|$

$\Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow arg(z_1) - arg(z_2) = 0.$

47. **(2)**

Sol: Given that, $w = \dfrac{z}{z-\frac{i}{3}}$ and $|w| = 1$

$\Rightarrow \left|\dfrac{z}{z-\frac{i}{3}}\right| = 1 \Rightarrow |z| = \left|z-\dfrac{i}{3}\right|$

$\Rightarrow z$ lies on perpendicular bisector of (0, 0) and $\left(0, \dfrac{1}{3}\right)$.

So, z lies on a straight line.

48. **(3)**

Sol: $\sum_{k=1}^{10}\left(\sin\dfrac{2k\pi}{11} + i\cos\dfrac{2k\pi}{11}\right)$

$= i\sum_{k=1}^{10}\left(\cos\dfrac{2k\pi}{11} - i\sin\dfrac{2k\pi}{11}\right) = i\sum_{k=1}^{10}\left(e^{-\frac{2k\pi}{11}}\right) =$

$i\left\{\sum_{k=0}^{10}\left(e^{-\frac{2k\pi}{11}}\right) - 1\right\} = -i$

49. **(2)**

Sol: $(1+\omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega$

$-\omega^{14} = A + B\omega$

$-\omega^2 = A + B\omega$

$1 + \omega = A + B\omega$

$\therefore (A, B) = (1, 1)$

50. **(4)**

Sol: Let roots be $p + iq$ and $p - iq$ $p, q \in R$

root lie on line $Re(z) = 1$

$\Rightarrow p = 1$

product of roots $= p^2 + q^2 = \beta = 1 + q^2$

$\Rightarrow \beta \in (1, \infty)$ ($q \neq 0$, $\because$ roots are distinct)

51. **(3)**

Sol: $|z| = 1$, $\arg z = \theta$ $\quad z = e^{i\theta}$

$\bar{z} = \dfrac{1}{z}$

$arg\left(\dfrac{1+z}{1+\frac{1}{z}}\right) = arg\,(z) = \theta.$

52. **(1)**

Sol: $|z| = 1;\ |z_2| \neq 1$

$\left|\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$

$|z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

$(z_1 - 2z_2)\left(\bar{z}_1 - 2\bar{z}_2\right) = \left(2 - z_1\bar{z}_2\right)\left(2 - \bar{z}_1 z_2\right)$

$\Rightarrow |z_1|^2 + 4|z_2|^2 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 = 4 + |z_1|^2|z_2|^2 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2$

$\Rightarrow \left(|z_2|^2 - 1\right)\left(|z_1|^2 - 4\right) = 0$

$\because |z_2| \neq 1$

$\therefore |z_1| = 2$

53. **(3)**

Sol: $\dfrac{2+3i\sin\theta}{1-2i\sin\theta} \times \dfrac{1+2i\sin\theta}{1+2i\sin\theta}$

$2 - 6\sin^2\theta = 0$ (For purely imaginary)

$\sin^2\theta = \dfrac{1}{3}$

$\sin\theta = \dfrac{1}{\sqrt{3}}$

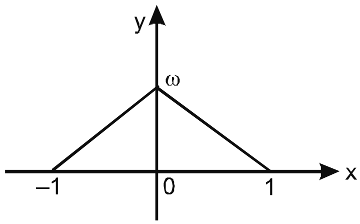
$\theta = \sin^{-1}\dfrac{1}{\sqrt{3}}$

54. **(1)**

Sol: $\omega = \dfrac{z-1}{z+1} \Rightarrow \omega z + \omega = z - 1$

$\Rightarrow (\omega - 1)z = -1 - \omega$

$\Rightarrow z = \dfrac{1+\omega}{1-\omega}$



Now $|z| = 1 \Rightarrow \left|\frac{1+\omega}{1-\omega}\right| = 1$

$\Rightarrow |\omega - (-1)| = |\omega - 1|$

$\Rightarrow$ ω lies on the perpendicular bisector of the segment joining – 1 and 1.

Thus, ω lies on the imaginary axis.

55. (1)

Sol: $\omega = -1 + 5z$, then $\omega + 1 = 5z$

$|\omega + 1| = 5|z| = 5 \times 2$

$|\omega + 1| = 10$

Thus ω lies on a circle

56. (3)

Sol: $z^2 - zz_1 - izz_1 + iz_1^2 = 0$

$(z - z_1)(z - iz_1) = 0$

$z = z_1, iz_1$

∴ triangle is right angled

$\therefore$ area $= \frac{1}{2}|z_1|\,|iz_1| = \frac{|z_1|^2}{2}$

57. (4)

Sol: $\frac{1+z+z^2}{1-z+z^2} = 1 + \frac{2z}{1-z+z^2}$ is real

$\Rightarrow 1 + \frac{2}{z+\frac{1}{z}-1}$ is real

$\Rightarrow z + \frac{1}{z}$ is real

$\Rightarrow z + \frac{1}{z} = \bar{z} + \frac{1}{\bar{z}} \Rightarrow (z - \bar{z}) = \frac{1}{\bar{z}} - \frac{1}{z}$

$\Rightarrow (z - \bar{z})\left(1 - \frac{1}{|z|^2}\right) = 0$

$\Rightarrow |z| = 1$

58. (1)

Sol: $\log_{1/2}\left(\frac{|z-1|+4}{3|z-1|-2}\right) > 1$

$0 < \frac{|z-1|+4}{3|z-1|-2} < \frac{1}{2}$; $|z - 1| = t$

$0 < \frac{t+4}{3t-2} < \frac{1}{2}$

$0 < \frac{t+4}{3t-2} < \frac{1}{2}$ $\Rightarrow t > 10$ So true

59. (3)

Sol: $|z_1 + z_2| \neq |z_1 - z_2|$

but $z_1 + z_2 \perp z_1 - z_2$

so it is a rhombus.

60. (4)

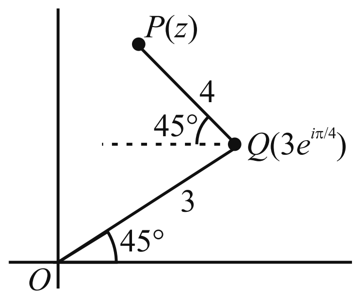
Sol: $\frac{w - \bar{w}z}{1-z} = \frac{\bar{w} - w\bar{z}}{1-\bar{z}}$

$\Rightarrow (z\bar{z} - 1)(\bar{w} - w) = 0$

$\Rightarrow z\bar{z} - 1 \Rightarrow |z|^2 = 1 \Rightarrow z| = 1$

61. (4)

Sol:



$Q = 3e^{i\pi/4}$

Let $P = z$

$\therefore \angle PQO = 90^\circ$

So by using rotation formula

$\frac{0 - 3e^{i\pi/4}}{z - 3e^{i\pi/4}} = \frac{3}{4}e^{i\pi/2}$

On solving $z = (3 + 4i)\, e^{i\pi/4}$

62. **(4)**

Sol: $|z| = 1,\ z \neq \pm 1$

Let $z = e^{i\theta}$

$\Rightarrow \dfrac{z}{1-z^2} = \dfrac{e^{i\theta}}{1-e^{i2\theta}} = \dfrac{1}{-2i\sin\theta}$ which is purely imaginary

Hence $\dfrac{z}{1-z^2}$ lies on the y-axis

Integer Type Questions (63 to 73)

63. **(0)**

Sol: $\sum_{n=1}^{200} i^n = i + i^2 + i^3 + ... + i^{200}$

$= \dfrac{i(1-i^{200})}{1-i} = \dfrac{i(1-1)}{1-i} = 0$

64. **(5)**

Sol: $z = 3 - 4i$

$(z-3)^2 = (-4i)^2$

$\Rightarrow z^2 - 6z + 25 = 0$

Now $z^4 - 3z^3 + 3z^2 + 99z - 95$

$= (z^2 - 6z + 25)(z^2 + 3z - 4) + 5$

$= 5$

65. **(1)**

Sol: $\dfrac{z-i}{z+i} = \lambda i$

$\dfrac{z-i}{z+i} + \dfrac{\bar{z}+i}{\bar{z}-i} = 0$

$\dfrac{(z-i)(\bar{z}-i)+(\bar{z}+i)(z+i)}{(z+i)(\bar{z}-i)} = 0$

$z\bar{z} - i\bar{z} - iz - 1 + z\bar{z} + iz + i\bar{z} - 1 = 0$

$2z\bar{z} - 2 = 0$

$z\bar{z} = 1$

66. **(4)**

Sol: $(3 + 5\omega + 3\omega^2)^2 + (3 + 3\omega + 5\omega^2)^2 = (2\omega)^2 + (2\omega^2)^2 = 4\omega^2 + 4\omega^4 = -4$

67. **(54)**

Sol:

$\left(\omega+\dfrac{1}{\omega}\right)^2 + \left(\omega^2+\dfrac{1}{\omega^2}\right)^2 + \left(\omega^3+\dfrac{1}{\omega^3}\right)^2 .....\left(\omega^{27}+\dfrac{1}{\omega^{27}}\right)^2$

there are 9 term which have ω^{3p}.

so sum $9 \times 4 = 36$

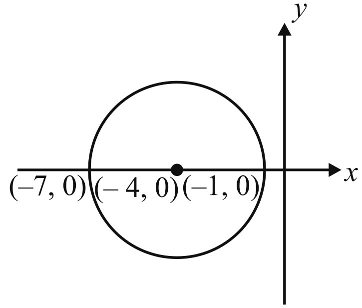
there are 18 term which not have ω^{3p}

so sum is $= 18$

total sum $= 18 + 36 = 54$

68. **(6)**

Sol: From the Argand diagram maximum value of $|z + 1|$ is 6.



69. **(1)**

Sol: $x^2 - x + 1 = 0 \Rightarrow x = -\omega, -\omega^2$

$\therefore \alpha^{2009} + \beta^{2009} = -\omega^{2009} - \omega^{4018} = -\omega^2 - \omega = 1.$

70. **(1)**

Sol: $|z-1|^2 = |z+1|^2 \quad \Rightarrow x = 0$

$|z-1|^2 = |z-i|^2$

$\Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2 \ \Rightarrow 1 + y^2 = (y-1)^2 \ (\because x = 0)$

$\therefore y = 0 \quad \Rightarrow (0, 0)$ satisfies

71. **(1)**

Sol: Let $p = |a + b\omega + c\omega^2|$

$p^2 = (a + b\omega + c\omega^2)(a + b\bar{\omega} + c\bar{\omega}^2) = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$p^2 = (a^2 + b^2 + c^2 - ab - bc - ca)$

$p^2 = \dfrac{1}{2}((a-b)^2 + (b-c)^2 + (c-a)^2)$

Since a, b, c are all integers but not all equal simultaneously

$a, b, c \in I$

so *RHS* will be minimum only when any two are zero and third is a minimum magnitude integer = 1

$a = b = 0, c = 1$

$a = b = 0, c = 1$

so $p^2 \geq \frac{1}{2}\ [0 + 1 + 1]$

$p^2 \geq 1$

minimum value of $|p| = 1$

$|p| = 1$

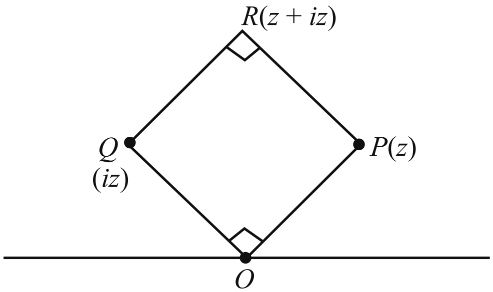
72. (0)

Sol: $\omega^4 + \omega^8 + \frac{1}{\omega^3} = \omega + \frac{1}{\omega} + 1$

73. (20)

Sol: $iz = ze^{i\frac{\pi}{2}}$, Q is obtained by rotating P about origin through an angle $\frac{\pi}{2}$

$R(z + iz)$ represents vertex of parallelogram(square) *OPRQ*.



$\Rightarrow \Delta PQR = 200$

$\Rightarrow \frac{1}{2}\ |z|\ |iz| = 200$

$|z|^2 = 400 \Rightarrow |z| = 20$

BINOMIAL THEOREM

Single Option Correct Type Questions (01 to 60)

1. **(2)**

Sol. $S = \sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$

$S = {}^{100}C_0 (x-3)^{100} + {}^{100}C_1 (x-3)^{99} \cdot 2 + \ldots\ldots + {}^{100}C_{100} \cdot 2^{100}$

$S = (2 + (x-3))^{100} = (x-1)^{100}$

Co-efficient of $x^{52} = {}^{100}C_{52} = {}^{100}C_{48}$

2. **(3)**

Sol. $(1+x)^{21}[1+(1+x)+\ldots.+(1+x)^9] = (1+x)^{21}$

$\times\left[\frac{(1+x)^{10}-1}{x}\right] = \frac{(1+x)^{31}-(1+x)^{21}}{x}$

Coefficient of $x_6 = {}^{31}C_6 - {}^{21}C_6$

3. **(3)**

Sol. $\int_0^1 (1-x)^n dx =$

$\int_0^1 (C_0 - C_1x + C_2x^2 - C_3x^3 + \ldots\ldots + (-1)^n C_n x^n)dx$

$\Rightarrow \frac{1}{n+1} = \left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots\ldots\ldots\ldots\ldots + (-1)^n \frac{C_n}{n+1}\right]$

$\Rightarrow \frac{1}{3}\left(\frac{1}{n+1}\right)$

$= \frac{1}{3}\left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots\ldots\ldots\ldots + (-1)^n \frac{C_n}{n+1}\right]$

4. **(3)**

Sol. $\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{k=0}^{10} (-1)^k \frac{{}^{10}C_k}{2^k}\right)$

$= ({}^{10}C_0 + \ldots\ldots + {}^{10}C_{10})\left({}^{10}C_0 - \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{2^2} \ldots\ldots\ldots + \frac{{}^{10}C_{10}}{2^{10}}\right)$

$= 2^{10} \times \left(1 - \frac{1}{2}\right)^{10} = 1$

5. **(4)**

Sol. $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$

$S = \sum_{r=0}^{n} \frac{n-2r}{{}^nC_r} = \sum_{r=0}^{n}\left(\frac{n-r}{{}^nC_r} - \frac{r}{{}^nC_r}\right)$

$S = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} - \sum_{r=0}^{n} \frac{r}{{}^nC_r} = 0$

6. **(3)**

Sol. $(1+x)^n \left(1 + \frac{1}{x}\right)^n$

$= [C_0 + C_1 .x + C_2 . x^2 + \ldots\ldots + C_n . x^n]$

$\left[C_0 + C_1 \cdot \frac{1}{x} + C_2 \cdot \frac{1}{x^2} + \ldots\ldots + C_n \cdot \frac{1}{x^n}\right]$

Coeff. of $x^0 = C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2$

7. **(3)**

Sol. $\left(x^3 - \frac{1}{x^2}\right)^n$

General term $= \frac{n!}{r!(n-r)!} (-1)^{n-r} x^{5r-2n}$

If $5r - 2n = 5$, then $5r = 2n + 5$

$\Rightarrow\ r = \frac{2n}{5} + 1$

If $5r - 2n = 10$, then $5r = 2n + 10 \Rightarrow r = \frac{2n}{5}$

$+ 2$

Let $n = 5k$

Now $\frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0$

$\Rightarrow\ \frac{1}{3k-1} - \frac{1}{2k+2} = 0$

$\Rightarrow\ k = 3 \Rightarrow\ n = 15$

8. (4)

Sol. $\left(4^{1/3} + \frac{1}{6^{1/4}}\right)^{20}$

$T_{r+1} = {}^{20}C_r (4^{1/3})^{20-r}(6^{-1/4})^r$

For rational terms

$20 - r = 3k$ & $r = 4k$, where $k, p \in N$

$\Rightarrow\ r = 20$ & $r = 8$

no. of rational terms = 2

no. of irrational terms = 19

9. (4)

Sol. $(27)^{27} = 3^{81} = 3 \cdot (9)^{40}$

$= 3(10-1)^{40} = 3(10^{40} - {}^{40}C_1 \cdot 10^{39} + \ldots.. + {}^{40}C_{38} \cdot 10^2 - {}^{40}C_{39} \cdot 10 + 1)$

$= 3(1000\,\lambda - 400 + 1)$

Last 3 digits of this number = 803.

10. (3)

Sol. $= a\sum_{r=1}^{n}(-1)^{r-1}.\ {}^nC_r - \sum_{r=1}^{n} r.\ {}^nC_r(-1)^{r-1} = a$

11. (2)

Sol. $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots.. + a_{2n}x^{2n}$

put $x = 1$

$3^n = a_0 + a_1 + a_2 + \ldots\ldots. + a_{2n}$ ....(i)

$x = -1$

$1 = a_0 - a_1 + a_2 + \ldots\ldots + a_{2n}$ ....(ii)

adding (i) & (ii)

$\frac{3^n + 1}{2} = a_0 + a_2 + \ldots\ldots + a_{2n}$.

12. (4)

Sol. $\left(1+2\sqrt{x}\right)^{40} = {}^{40}C_0 + {}^{40}C_1\ 2\sqrt{x} + \ldots.. + {}^{40}C_{40}\ (2\sqrt{x})^{40}$

$(1-2\sqrt{x})^{40} = {}^{40}C_0 - {}^{40}C_1\ 2\sqrt{x} + \ldots\ldots + {}^{40}C_{40}\ (2\sqrt{x})^{40}$

$(1+2\sqrt{x})^{40} + (1-2\sqrt{x})^{40}$

$= 2\,[{}^{40}C_0 + {}^{40}C_2\ (2\sqrt{x})^2 + \ldots\ldots + {}^{40}C_{40}\ (2\sqrt{x})^{40}]$

Putting $x = 1$

${}^{40}C_0 + {}^{40}C_2(2)^2 + \ldots\ldots. + {}^{40}C_{40}\ (2)^{40} = \frac{3^{40}+1}{2}$

13. (1)

Sol. $\sum_{r=0}^{n} (r+1)\ C_r^2$

$\because\ (1+x)^n = C_0 + C_1x + \ldots\ldots\ldots.. + C_nx^n$

Multiply by x & then differentiate

$(1+x)^n + x.\ n(1+x)^{n-1} = C_0 + 2C_1x + \ldots\ldots\ldots.. + (n+1)C_nx^n$ ....(i)

$(x+1)^n = C_0x^n + C_1x^{n-1} + \ldots\ldots\ldots.. + C_n$........(ii)

Multiply (i) & (ii) & equate the coefficient of x^n on

both side

$C_0^2 + 2C_1^2 + \ldots\ldots.. + (n+1)\ C_n^2 = {}^{2n}C_n + n.\ {}^{2n-1}C_{n-1} = 2.{}^{2n-1}C_{n-1} + n.{}^{2n-1}C_{n-1} = \frac{(n+2)(2n-1)!}{n!(n-1)!}$

14. (3)

Sol. $\left(\left(x+\frac{1}{x}\right)^2 - 1\right)^n = {}^nC_0\left(x+\frac{1}{x}\right)^{2n} - {}^nC_1\left(x+\frac{1}{x}\right)^{2n-2} + \ldots. + {}^nC_n(-1)^n$

Total number of terms = $2n + 1$

15. **(2)**

Sol. Coeff. of x^{10} in $(1-x^4)^5 (1-x)^{-5}$

$(1 - {}^5C_1 x^4 + {}^5C_2\, x^8 + ...) (1-x)^{-5}$

Coeff. of x^{10} in $(1-x)^{-5}$ – 5.coeff. of x^6 in $(1-x)^{-5}$ + 10 coeff. of x^2 in $(1-x)^{-5}$

${}^{14}C_4 - 5 \times {}^{10}C_4 + 10.{}^6C_4 = 101$

16. **(3)**

Sol. $(x+3)^n + (x+3)^{n-1}(x+2) + ... + (x+2)^n =$

$(x+3)^n \left(\dfrac{1-\left(\dfrac{x+2}{x+3}\right)^{n+1}}{1-\left(\dfrac{x+2}{x+3}\right)} \right) = [(x+3)^{n+1} - (x+2)^{n+1}]$

Coefficient of x^{n-1} is ${}^{n+1}C_2 \times 5$

17. **(1)**

Sol. Statement-1 : $\left(x + \dfrac{1}{x} + 2\right)^m = \dfrac{(x+1)^{2m}}{x^m}$

$\Rightarrow$ co-efficient of $x^0 = {}^{2m}C_m = \dfrac{(2m)!}{(m!)^2}$

(True)

Statement-2: Obviously true and correct explanation of statement-1

18. **(4)**

Sol. Statement-1: $({}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1})$ $+ ({}^{2n}C_{n+1} + ... + {}^{2n}C_{2n-1}) = 2^{2n-1}$

$\Rightarrow 2({}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1}) = 2^{2n-1}$

$\Rightarrow {}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1} = 2^{2n-2}$

Statement-1: false

Statement-2: ${}^{2n}C_1 + {}^{2n}C_3 + ... + {}^{2n}C_{2n-1} = (2)^{2n-1}$, True

19. **(4)**

Sol. $a_n = \dfrac{(1000)(1000)............(1000)}{1\cdot 2.........n}$

$a_{999} = a_{1000}$

a_n is maximum for $n = 999$ and $n = 1000$

20. **(3)**

Sol. $(101)^{50} - (99)^{50} = (100+1)^{50} - (100-1)^{50}$

$= 2\,({}^{50}C_1 (100)^{49} + {}^{50}C_3 (100)^{47} + {}^{50}C_5 (100)^{45}$ $+.......+ {}^{50}C_{49} (100)]$

$= 2.50 \,.\, (100)^{49} + 2[{}^{50}C_3 (100)^{49} + {}^{50}C_3 (100)^{47}$ $+ {}^{50}C_5 (100)^{45} +.........+ {}^{50}C_{49}\, 1(100)]$

$= 100^{50} + 2[{}^{50}C_3 (100)^{49} + {}^{50}C_3 (100)^{47} + {}^{50}C_5$ $(100)^{45} +.........+ {}^{50}C_{49} (100)]$

$\Rightarrow (101)^{50} - (99)^{50} > (100)^{50}$

or $(101)^{50} - (99)^{50} > (100)^{50}$

or $(101)^{50} - (100)^{50} > (99)^{50}$

Again $\left(\dfrac{1001}{1000}\right)^{999} = \left(1 + \dfrac{1}{1000}\right)^{999}$ $1 + {}^{999}C_1$ $\left(\dfrac{1}{1000}\right) + {}^{999}C_2 \left(\dfrac{1}{1000}\right)^2 + {}^{999}C_3 \left(\dfrac{1}{1000}\right)^3 + ...$

$< 1 + 1 + 1 + 1 + ...1000$ terms

$\left(\dfrac{1001}{1000}\right)^{999} < 1000 \Rightarrow (1001)^{999} < (1000)^{1000}$

21. **(3)**

Sol. $\because$ $(r+1)^{th}$ term in the expansion of $(1+x)^{27/5}$

$= \dfrac{\dfrac{27}{5}\left(\dfrac{27}{5}-1\right)....\left(\dfrac{27}{5}-r+1\right)}{r\,!} x^r$

Now this term will be negative, if the last factor in numerator is the only negative factor.

$\Rightarrow \dfrac{27}{5} - r + 1 < 0 \Rightarrow \dfrac{32}{5} < r$

$\Rightarrow 6\cdot 4 < r \Rightarrow$ least value of r is 7.

Thus first negative term will be 8th.

22. **(2)**

Sol. $(1+x)(1-x)^n = (1-x)^n + x(1-x)^n$

Coefficient of $x^n = (-1)^n + (-1)^{n-1}.(n)$

$= (-1)^n (1-n)$

23. (1)

Sol. $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$

$\Rightarrow t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$\Rightarrow t_n = \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$

$\Rightarrow \; 2t_n = n \sum_{r=0}^{n} \frac{1}{{}^nC_r} = ns_n$

$\Rightarrow \; \frac{t_n}{s_n} = \frac{n}{2}$

24. (2)

Sol. $(1+y)^m$

$T_r = {}^mC_{r-1} \,.\, y^{r-1}$

$\Rightarrow T_{r+1} = {}^mC_r \,.\, y^r$

$\Rightarrow T_{r+2} = {}^mC_{r+1} \,.\, y^{r+1}$

$\because \; {}^mC_{r-1} + {}^mC_{r+1} = 2\,{}^mC_r$

$\Rightarrow \; \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = 2 \Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$

25. (2)

Sol. $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$\Rightarrow \; \frac{\left(1+\frac{3}{2}x+\frac{3}{2}.\frac{1}{2}\frac{x^2}{2!}\right) - \left(1+\frac{3x}{2}+\frac{3.2}{2}.\frac{x^2}{4}\right)}{(1-x)^{1/2}}$

$= \left(-\frac{3}{8}x^2\right)(1-x)^{-1/2} = -\frac{3}{8}x^2$

26. (3)

Sol. $(1-ax)^{-1}(1-bx)^{-1}$

$= (1 + ax + (ax)^2 + \ldots\ldots)(1 + bx + (bx)^2 + \ldots\ldots\ldots\ldots)$

so, $a_n = a^n + a^{n-1}b + a^{n-2}b^2 + \ldots\ldots + b^n$

$= a^n \frac{\left(1-\left(\frac{b}{a}\right)^{n+1}\right)}{\left(1-\frac{b}{a}\right)} = \frac{b^{n+1}-a^{n+1}}{b-a}$

27. (3)

Sol. $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots\ldots\ldots$

$(1 - my + {}^mC_2y^2 \ldots\ldots\ldots)(1 + ny + {}^nC_2y^2 \ldots\ldots\ldots)$
$= 1 + a_1y + a_2y^2 + \ldots\ldots\ldots$

$a_1 = n - m = 10$ ..... (1)

$a_2 = {}^mC_2 + {}^nC_2 - mn = 10$ ..... (2)

solving (1) & (2), we get $(m, n) \equiv (35, 45)$

28. (1)

Sol. $T_5 + T_6 = 0$

${}^nC_4a^{n-4}.\, b^4 - {}^nC_5a^{n-5}.\, b^5 = 0$

$\Rightarrow \; \frac{a^{n-4}\, b^4}{a^{n-5}\, b^5} = \frac{{}^nC_5}{{}^nC_4}$

$\Rightarrow \; \frac{a}{b} = \frac{n\,!}{5\,!(n-5)\,!} \times \frac{4\,!(n-4)\,!}{n\,!} = \frac{n-4}{5}$

29. (1)

Sol. **Statement-1:** $\sum_{r=0}^{n}(r+1)\,{}^nC_r$

$= \sum_{r=0}^{n} r.{}^nC_r + \sum_{r=0}^{n}{}^nC_r$

$= n\,.\,2^{n-1} + 2^n = (n+2)\,2^{n-1}$

Statement-2: $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r$

$= \sum_{r=0}^{n} r.\,{}^nC_r\; x^r + \sum_{r=0}^{n}{}^nC_r\; x^r$

$= xn(1+x)^{n-1} + (1+x)^n$

30. (2)

Sol. $S_1 = \sum_{j=1}^{10} j(j-1)\,.\,\frac{10(10-1)}{j\,(j-1)}\;{}^8C_{j-2}$

$\Rightarrow \; S_1 = 9 \times 10 \sum_{j=2}^{10} {}^8C_{j-2}$

$\Rightarrow \; S_1 = 90\,.\,2^8$

$S_2 = \sum_{j=1}^{10} j \cdot \frac{10}{j}\, {}^9C_{j-1} = 10 \cdot 2^9$

$S_3 = \sum_{j=1}^{10} (j(j-1) + j)\, {}^{10}C_j =$

$\sum_{j=1}^{10} j(j-1)\, {}^{10}C_j + \sum_{j=1}^{10} j\, {}^{10}C_j = 90$

$\sum_{j=2}^{10} {}^8C_{j-2} + 10\sum_{j=1}^{10} {}^9C_{j-1}$

$= 90 \times 2^8 + 10 \times 2^9 = (45 + 10) \cdot 2^9 = (45 + 10) \cdot 2^9 = 55 \cdot 2^9$

so statement-1 is true and statement 2 is false.

31. **(3)**

Sol. $(1 - x - x^2 + x^3)^6$

$(1 - x)^6 (1 - x^2)^6$

$({}^6C_0 - {}^6C_1 x^1 + {}^6C_2 x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6)\ ({}^6C_0 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 + ... + {}^6C_6x^{12})$

Now coefficient of $x^7 = {}^6C_1{}^6C_3 - {}^6C_3{}^6C_2 + {}^6C_5{}^6C_1$

$= 6 \times 20 - 20 \times 15 + 36$

$= 120 - 300 + 36$

$= 156 - 300$

$= -144$

32. **(1)**

Sol. $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$

$=2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + ...]$

= which is an irrational number

33. **(3)**

Sol. $\left((x^{1/3} + 1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \quad \Rightarrow \quad 20 - 2r - 3r = 0$

$\Rightarrow \ r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

34. **(1)**

Sol. Coefficient of x^n in $\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2+.....\right)^2$

= Coefficient of x^n in e^{2x}

$e^{2x} = \left(1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!}....\right) = \frac{2^n}{n!}$

35. **(1)**

Sol. Let, $n = 1001$. We can write

$99^{1001} + 1 = (100 - 1)^n + 1$

$= {}^nC_0 100^n - {}^nC_1 100^{n-1} + {}^nC_2 100^{n-2} - ......$

$= 100\, m + {}^nC_{n-1}(100) - 1 + 1$

Where, $m = {}^nC_0 100^{n-1} - {}^nC_1 100^{n-2} + {}^nC_2 100^{n-3} - .... - {}^nC_{n-2}100 + {}^nC_{n-1}$

As, ${}^nC_{n-1} = 1001$, the units digit of m is different from 0. Thus, the number of zeros at the end of $99^{1001} + 1$ is two.

36. **(4)**

Sol. $({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + ...{}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + ...{}^{10}C_{10}) = S_1 - S_2$

$S_1 = {}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + ...{}^{21}C_{10}$

$S_1 = \frac{1}{2}({}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{20}) = \frac{1}{2}({}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{20} + {}^{21}C_{21} - 2)$

$S_1 = 2^{20} - 1$

$S_2 = ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + ...{}^{10}C_{10}) = 2^{10} - 1$

Therefore $S_1 - S_2 = 2^{20} - 2^{10}$

37. **(2)**

Sol. $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= (T_1 + T_2 + T_3 + T_4 + T_5 + T_6) + (T_1 - T_2 + T_3 - T_4 + T_5 - T_6)$

$= 2(T_1 + T_3 + T_5)$

$= 2(\ {}^5C_0(x)^5 + {}^5C_2(x)^3 \left(\sqrt{x^3-1}\right)^2 + {}^5C_4(x)^1 \left(\sqrt{x^3-1}\right)^4$

$= 2(x^5 + 10x^3 (x^3 - 1) + 5x (x^6 + 1 - 2x^3)]$

$= 2(x^5 + 10x^6 - 10x^3 + 5x^7 + 5x - 10x^4)$

$= 2(5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x)$

sum of odd degree terms = 10 + 2 –20 + 10 = 2

38. **(4)**

Sol. $(1 + t^2)^{12} (1 + t^{12} + t^{24} + t^{36}) = (1 + t^{12} + t^{24}) (1 + t^2)^{12}$

coefficient of $t^{24} = {}^{12}C_{12} + {}^{12}C_6 + {}^{12}C_0 = {}^{12}C_6 + 2$

39. **(4)**

Sol. ${}^{(n-1)}C_r = (k^2 - 3)\, {}^nC_{r+1}$

or ; ${}^{(n-1)}C_{n-(r+1)} = (k^2 - 3)\, {}^nC_{n-(r+1)}$

$1 \ge k^2 - 3 > 0 \Rightarrow k \in$

$[-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$

40. **(2)**

Sol. $S = {}^{30}C_0\, {}^{30}C_{20} - {}^{30}C_1\, {}^{30}C_{19} + {}^{30}C_2\, {}^{30}C_{18}.....$

S = Co-efficient of x^{20} in $(1-x)^{30} (1 + x)^{30}$

S = Co-efficient of x^{20} in $(1-x^2)^{30} = {}^{30}C_{10}$

41. **(4)**

Sol. $B_{10} \sum_{r=1}^{10} A_r B_r - C_{10} \sum_{r=1}^{10} (A_r)^2 = {}^{20}C_{10} ({}^{30}C_{20} - 1) - {}^{30}C_{10} ({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

42. **(3)**

Sol. $T_{r+1} = {}^{10}C_r (2x^2)^{10-r} \left(\frac{1}{3x^2}\right)^r$

$\Rightarrow T_{5+1} = T_6 = {}^{10}C_5 (2x^2)^5 \left(\frac{1}{3x^2}\right)^5$

$= \frac{896}{27} = \frac{a}{b} \quad \therefore a + b = 923$

43. **(3)**

Sol. $(ab + bc + ca)^6 = a^6 b^6 c^6 (a^{-1} + b^{-1} + c^{-1})^6$

General term in $= a^6 b^6 c^6$

$\frac{!6 (a^{-1})^{k_1} . (b^{-1})^{k_2} . (c^{-1})^{k_3}}{k_1 !\, k_2 !\, k_3 !}$

$\therefore k_1 = 3, k_2 = 2, k_3 = 1$

$\therefore$ Coefficient of $a^3 b^4 c^5$ is $\frac{6!}{3!\ 2!\ 1!} = 60$

44. **(4)**

Sol. sum of coefficient is zero

$\Rightarrow a^3 - 2a^2 + 1 = 0$

$\Rightarrow a^3 - a^2 - a^2 + a - a + 1 = 0$

$\Rightarrow a^2 (a-1) - a(a-1) - 1(a-1) = 0$

$\Rightarrow (a-1)(a^2 - a - 1) = 0$

$\Rightarrow a = 1, a = \frac{1 \pm \sqrt{1+4}}{2}$

45. **(1)**

Sol. Here $(1-2+3)^n = 128$

$\Rightarrow 2^n = 2^7 \Rightarrow n = 7$

$\therefore$ greatest coefficient of $(1+x)^{14}$ is ${}^{14}C_7$

46. **(2)**

Sol. $17^{256} = (17^2)^{128} = (290-1)^{128}$

$= 1000\, m + {}^{128}C_2 (290)^2 - {}^{128}C_1 (290) + 1$

$= 1000(m + 683527) + 681$

$\therefore$ last two digits = 81

47. **(2)**

Sol. Here $3^{2003} = 9 \times 3^{2001} = 9\, (28-1)^{667}$

$= 9[{}^{667}C_0 {}^{667} - {}^{667}C_1 28\ {}^{666} + ... - {}^{667}C_{667}]$

$= 9 \times 28k - 9 + 28 - 28$

$= (9 \times 28k - 28) + 19$

$\therefore \left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$

48. **(3)**

Sol. given $= \frac{2}{2^7 \sqrt{4x+1}}$

$\left[{}^7C_1 \sqrt{4x+1} + {}^7C_3 \left(\sqrt{4x+1}\right)^3 + {}^7C_5 \left(\sqrt{4x+1}\right)^5 + {}^7C_7 \left(\sqrt{4x+1}\right)^7 \right]$

$= \frac{1}{2^6} [{}^7C_1 + {}^7C_3 (4x+1) + {}^7C_5 (4x+1)^2 + {}^7C_7 (4x+1)^3]$

$\therefore$ degree = 3

49. **(2)**

Sol. $(x + a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + ... + {}^nC_n a^n$

$= T_0 + T_1 + T_2 + ... + T_n$

Replace a by ai

$(x + ai)^n = ({}^nC_0 x^n - {}^nC_2 x^{n-2} a^2 + ...) + i\, ({}^nC_1 x^{n-1} a - {}^nC_3 x^{n-3} a^3 + ...)$

$= (T_0 - T_2 + T_4 - T_6 + ...) + i(T_1 - T_3 + T_5 - T_7 + ...)$

taking mod both sides

$(T_0 - T_2 + T_4 - T_6 + ...)^2 + (T_1 - T_3 + T_5 - T_7 + ...)^2 = (x^2 + a^2)^n$

50. **(2)**

Sol. Here $\dfrac{n+1}{1+\left|\dfrac{x}{y}\right|}=\dfrac{15+1}{1+\left|\dfrac{3}{1}\right|}=4=r$

$\therefore$ T_4 & T_5 are numerically gratest terms in

$\therefore$ $|T_4|=|T_5|={}^{15}C_4.3^{11}=455\times 3^{12}$

$\therefore n=12 \quad \therefore \dfrac{{}^kC_2}{2}=\dfrac{12\cdot 11}{2\cdot 2}=33$

51. **(1)**

Sol. Let $1-\dfrac{1}{8}+\dfrac{1\cdot 3}{8\cdot 16}-\dfrac{1\cdot 3\cdot 5}{8\cdot 16\cdot 24}+ ....=(1+x)^n=$

$1+nx+\dfrac{n(n-1)}{2}x^2+....$

$\therefore nx=-\dfrac{1}{8}, \dfrac{n(n-1)}{2}x^2=\dfrac{3}{8\cdot 16}$

$\Rightarrow x=\dfrac{1}{4}, n=-\dfrac{1}{2} \Rightarrow$ sum =

$\left(1+\dfrac{1}{4}\right)^{-\frac{1}{2}}=\left(\dfrac{5}{4}\right)^{-\frac{1}{2}}=\dfrac{2}{\sqrt{5}}$

52. **(3)**

Sol. $(1-9x+20x^2)^{-1}=((1-5x)(1-4x))^{-1}$

$=\dfrac{1}{(1-5x)(1-4x)}=\dfrac{5}{1-5x}-\dfrac{4}{1-4x}$

$=5(1-5x)^{-1}-4(1-4x)^{-1}$

$\therefore$ coefficient of x^n is $5^{n+1}-4^{n+1}$

53. **(2)**

Sol. Here $T_{r+1}={}^{100}C_r\left(5^{\frac{1}{6}}\right)^r.\left(2^{\frac{1}{8}}\right)^{100-r}$

$={}^{100}C_r\, 5^{\frac{r}{6}}\, 2^{\frac{100-r}{8}}$

$\therefore$ Number of irrational terms = 101 – 4 = 97

54. **(1)**

Sol. Last term =

${}^nC_n.\left(2^{\frac{1}{3}}\right)^0.\left(-\dfrac{1}{\sqrt{2}}\right)^n=\dfrac{(-1)^n}{2^{n/2}}=\left(\dfrac{1}{3^{5/3}}\right)^{\log_2 8}=3^{-\frac{5}{3}.3}=2^{-5}$

$\Rightarrow \dfrac{n}{2}=+5\Rightarrow n=10$

$t_5={}^{10}C_4\left(2^{\frac{1}{3}}\right)^{10-4}\left(-\dfrac{1}{\sqrt{2}}\right)^4=210$

55. **(1)**

Sol. $R=(5\sqrt{5}+11)^{2n+1}=I+f, 0<f<1$

Let $f'=(5\sqrt{5}-11)^{2n+1}, 0<f'<1$

$\therefore (5\sqrt{5})^{2n+1}+{}^{2n+1}C_1(5\sqrt{5})^{2n}.11+....=I+f$

$\because -1<f-f'<1$

$\therefore (5\sqrt{5})2n+1-2n+1C1(5\sqrt{5})2n .11+....=f'$

$\therefore 2({}^{2n+1}C_1.(\sqrt{5}5)^{2n}.11+....)=I+f-f'$

$=I$

$\therefore Rf=Rf'=(5\sqrt{5}+11)^{2n+1}.(5\sqrt{5}-11)^{2n+1}$

$=(125-121)^{2n+1}=4^{2n+1}$

56. **(2)**

Sol. $3^n=6561$ (put $x=1$)

$\Rightarrow n=8$

$\dfrac{T_{r+1}}{T_r}=\dfrac{8-r+1}{r}\geq 1$

$\Rightarrow 8-r+1\geq r \Rightarrow r\leq\dfrac{9}{2}\Rightarrow r=4$ (5^{th} term is greatest)

57. **(2)**

Sol. $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+$

$\left(\dfrac{2\left(\sqrt{2x^2+1}-\sqrt{2x^2-1}\right)}{(2x^2+1-2x^2+1)}\right)^6$

$=\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+$

$\left(\sqrt{2x^2+1}-\sqrt{2x^2-1}\right)^6$

$=2\left[{}^6C_0(2x^2+1)^3+{}^6C_2(2x^2+1)^2(2x^2-1)+\right.$

$^{6}C_4\ (2x^2+1)(2x^2-1)^2 + {}^{6}C_6\ (2x^2-1)^3\Big]$

clearly '6'

58. **(1)**

Sol. Co-efficient of x^{15} in $(1+x+x^3+x^4)^n$

$=$ Co-efficient of x^{15} in $(1+x^3)^n(1+x)^n = {}^nC_0$
${}^nC_{15} + {}^nC_1\,{}^nC_{12} + {}^nC_2\,{}^nC_9 + {}^nC_3\,{}^nC_6 + {}^nC_4\,{}^nC_3$
$+ {}^nC_5\,{}^nC_0$

59. **(4)**

Sol. general term $= (1+x+2x^2)\ {}^4C_r(3x^2)^{4-r}$

$\left(\dfrac{-1}{3x^2}\right)^r = (1+x+2x^2)\,{}^4C_r\,3^{4-r}\left(\dfrac{-1}{3}\right)^r (x^{8-4r})$

$= {}^4C_r\,(3^{4-r})\left(\dfrac{-1}{3}\right)^r x^{8-4r} + {}^4C_r\ (3^{4-r})\left(\dfrac{-1}{3}\right)^r$

$(x^{9-4r}) + {}^4C_r\,3^{4-r}\,2\left(\dfrac{-1}{3}\right)^r (x^{10-4r})$

For independent term of x

$8-4r=0$

$\Rightarrow\ r=2$ and $\quad 9-4r=0$

$r=\dfrac{9}{4}$ Not possible and $10-4r=0$

$r=\dfrac{5}{2}$ Not possible

term $= {}^4C_2\cdot 3^2\times\dfrac{1}{3^2} = 6$

60. **(4)**

Sol. $y=(1-x)^{-1}(1+x)^n$

$y=(1+x+x^2+\ldots\infty)(1+x)^n$

$y=(1+x)^n + x(1+x)^n + x^2(1+x)^n + \ldots.$

Co-efficient of $x^r =$

${}^nC_r + {}^nC_{r-1} + \ldots.\ {}^nC_0 = 2^n$

$r\ge n \quad (\text{As } {}^nC_{n+1}) = 0$

Integer Type Questions (61 to 74)

61. **(2)**

Sol. $(2+3c+c^2)^{12}=0$

$c^2+3c+2=0$

$c=-2,-1$

62. **(2)**

Sol. Co-efficient of $x^n = {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots.. + {}^{2n+1}C_n = 2^{2n}$

63. **(49)**

Sol. ${}^{50}C_0\times{}^{50}C_1 + {}^{50}C_1\times{}^{50}C_2 + \ldots\ldots.+ {}^{50}C_{49}\times{}^{50}C_{50}$

$= {}^{50}C_0\times{}^{50}C_{49} + {}^{50}C_1\times{}^{50}C_{48} + \ldots\ldots.+ {}^{50}C_{49}\times{}^{50}C_0$

$=$ co-eff. of x^{49} in $(1+x)^{100} = {}^{100}C_{49}$

64. **(5)**

Sol. $(1+x)^n = {}^nC_0 + {}^nC_1\,x + {}^nC_2\,x^2 + \ldots\ldots.+ {}^nC_n\,x^n$

Multiply it by x

$x(1+x)^n = {}^nC_0\,x + {}^nC_1\,x^2 + {}^nC_2x^3 + \ldots\ldots\ldots\ {}^nC_nx^{n+1}$

Differentiate w.r. to x and put $x=-3$

$n\,x\,(1+x)^{n-1} + (1+x)^n = {}^nC_0 + 2\,{}^nC_1\,x + 3\,{}^nC_2\,x^2 + 4\,{}^nC_3\,x^3 + \ldots\ldots+ (n+1)\,{}^nC_n\,x^n$

So answer, $-3n(-2)^{n-1} + (-2)^n$

$= (-2)^n\left(1+\dfrac{3n}{2}\right)$

65. **(55)**

Sol. $\left(\dfrac{x}{4}-\dfrac{12}{x^2}\right)^{12}$

$T_{r+1} = (-1)^r.{}^{12}C_r\left(\dfrac{x}{4}\right)^{12-r}\left(\dfrac{12}{x^2}\right)^r$

$T_{r+1} = (-1)^r.{}^{12}C_r\left(\dfrac{1}{4}\right)^{12-r}(12)^r.(x)^{12-3r}$

Term independent of $x \Rightarrow 12-3r=0 \Rightarrow r=4$

$T_5 = (-1)^4.{}^{12}C_4\left(\dfrac{1}{4}\right)^8(12)^4 = \dfrac{3^6}{4^4}.k$

$k=55$

66. **(9)**

Sol. $f(n) = 10^n + 3\cdot 4^{n+2} + 5$

put $n=1$

$f(1) = 10+192+5 = 207$ this is divisible by 3 and 9

67. (101)

Sol. $3 \cdot {}^nC_0 - 8 \cdot {}^nC_1 + 13 \cdot {}^nC_2 - 18 \cdot {}^nC_3 + ...$ up to $(n + 1)$ terms

$(1 + x^5)^n = C_0 + C_1x^5 + C_2x^{10} + ... + C_nx^{5n}$

Multiplying by x^3 and differentiating w.r.t. x

$x^3 . n(1 + x^5)^{n-1} . 5x^4 + 3x^2 (1 + x^5)^n = 3C_0 x^2 + 8C_1 x^7 + 13C_2 x^{12} + ... + (5n + 3) C_n x^{5n+2}$

Now put $x = -1$

$3C_0 - 8C_1 + 13 C_2 + ...+ (n + 1)$ terms $= 0$

68. (35)

Sol. $(9x^2 + x - 8)^6 = (a_0 + a_1x + a_2x^2 + ... + a_{12} x^{12})$

$x = 1, \quad 2^6 = a_0 + a_1 + a_2 +...... + a_{12}$

$x = -1, \quad 0 = a_0 - a_1 + a_2 + ... + a_{12}$

$2^6 = 2(a_1 + a_3 + a_5 + ... + a_{11})$

subtracting (1) from (2)

$\Rightarrow a_1 + a_3 + ... + a_{11} = 2^5 = 32$

69. (15)

Sol. $S = \sum_{i=0}^{m} \binom{10}{i} \binom{20}{m-i}$

$= {}^{10}C_0 . {}^{20}C_m + {}^{10}C_1 . {}^{20}C_{m-1} + ...$

$\Rightarrow$ S = coefficient of x^m in $(1+x)^{10} (1+x)^{20} = {}^{30}C_m$

S is maximum when $m = 15$

70. (9)

Sol. $T_{r+1} = {}^nC_r \left(\sqrt[3]{2}\right)^{n-r} \left(\frac{1}{\sqrt[3]{3}}\right)^r$

$\therefore T_{6+1} = {}^nC_6 \left(\sqrt[3]{2}\right)^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6 = T_7$ from

begining, 7^{th} term from the end is $T_7' = {}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} \left(\sqrt[3]{2}\right)^6$

$\frac{T_7}{T_7'} = \frac{1}{6} \Rightarrow 6T_7 = T_7'$

$\Rightarrow 6 \cdot {}^nC_6 . \left(\sqrt[3]{2}\right)^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6 = {}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} \left(\sqrt[3]{2}\right)^6$

$\Rightarrow 6.\ 2^{\frac{n-6}{3}} \cdot 3^{\frac{-6}{3}} = 3^{-\frac{n-6}{3}} \cdot 2^{\frac{6}{3}}$

$\Rightarrow 2\times3 .\ (2^{\frac{n-6}{3}}) . 3^{-2} = (3^{-\frac{n-6}{3}}) 2^2$

$\Rightarrow 2^{\frac{n-6}{3}-1} = 3^{-\frac{n-6}{3}+1} \Rightarrow 2^{\frac{n-9}{3}} = 3^{\frac{9-n}{3}} \Rightarrow 9$

71. (7)

Sol. Given $= (7-1)^{83} + (7+1)^{83} = (7+1)^{83} - (1-7)^{83}$

$= 2 \cdot 7 \cdot 83 + 49I$, I is integer

$= 49\, I + 23 \times 49 + 35 \therefore R = 35 \quad \therefore \frac{R}{5} = 7$

72. (1)

Sol. Given $\sum_{k=0}^{4} \frac{3^{4-k}}{(4-k)!} . \frac{x^k}{k!} . \frac{4!}{4!} =$

$\sum_{x=1}^{4} \frac{{}^4c_k 3^{4-k}.x^k}{4!} = \frac{(3+x)^4}{4!} = \frac{32}{3}$

$\Rightarrow x = 1$

73. (8)

Sol. 9 = (0, 9) (1, 8), (2, 7), (3, 6), (4, 5) # 5 cases

9 = (1,2,6), (1,3,5), (2, 3, 4) # 3 cases

total = 8

74. (29)

Sol. The expression $(2 + x)^2 (3 + x)^3 (4 + x)^4 = (x + 2)(x + 2) (x + 3)(x + 3)(x + 3)(x + 4)(x + 4)(x + 4)(x + 4)$

$= x^9 + (2 + 2 + 3 + 3 + 3 + 4 + 4 + 4 + 4) x^8 + ...$

Co-efficient of $x^8 = 29$

PERMUTATIONS AND COMBINATIONS

Single Option Correct Type Questions (01 to 60)

1. **(2)**

Sol. Number of four digit no. in which atleast one digit is repeated (i.e. all digit are not different) is

$\boxed{9}\boxed{10}\boxed{10}\boxed{10} - \boxed{9}\boxed{9}\boxed{8}\boxed{7} = 9000 - 4536$

$= 4464$

2. **(2)**

Sol. The number of ways $= {}^{10}C_3 \times 3! = 720$ (Choosing 3 floors out of 10)

3. **(4)**

Sol. Even place

$E \quad E \quad E \quad E$ (arrows pointing to the four even places of a row of 9 boxes)

There are four even places and four odd digit number so total number of filling is $\frac{4!}{2!.2!}$. Rest occupy in $\frac{5!}{3!.2!}$ ways

Hence total number of ways $= \frac{4!}{2!.2!} \times \frac{5!}{3!.2!} = 60$

4. **(2)**

Sol. Total number of ways $= n! - 2! \times (n-1)!$

$= (n-1)!\,(n-2)$

5. **(2)**

Sol. Number of bowlers = 4

Number of wicketkeeper = 2

Total number of required selection

$= {}^4C_3 . {}^2C_1 . {}^{10}C_7 + {}^4C_4 . {}^2C_1 . {}^{10}C_6 + {}^4C_3 . {}^2C_2 . {}^{10}C_6 + {}^4C_4 . {}^2C_2 . {}^{10}C_5$

$= 960 + 420 + 840 + 252$

$= 2472$

6. **(2)**

Sol. $\frac{5!\,{}^6C_2 .2!}{6!2!} = \frac{5}{2}$

7. **(1)**

Sol. $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{2} = \frac{5!}{2!3!} = 10$

$\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{3} = \frac{5!}{3!} = 20$

$\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{4} = \frac{5!}{2!2!} = 30$

Hence sum of unit places is

$2 \times 10 + 3 \times 20 + 4 \times 30 = 200$

Hence required sum is

$= 200 \times (10^5 + 10^4 + 10^3 + 10^2 + 10^1 + 10^0)$

$= 200 \times (111111) = 22222200$

8. **(2)**

Sol. Required number of ways

${}^{36}C_9 . {}^{27}C_9 . {}^{18}C_9 . {}^9C_9 . 4! = \frac{36!}{(9!)^4} \times 4!$

9. **(2)**

Sol. $94864 = 2^4 . 7^2 . 11^2$ number of ways

$= \frac{(5.3.3)+1}{2} = 23$

10. **(1)**

Sol. Here $21600 = 2^5 . 3^3 . 5^2$

$(2 \times 5) \times 2^4 \times 3^3 \times 5^1$

Now numbers which are divisible by 10

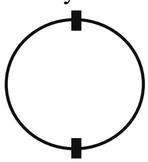
$= (4 + 1)(3 + 1)(1 + 1) = 40$

$(2 \times 3 \times 5) \times (2^4 \times 3^2 \times 5^1)$ now numbers which are

divisible by both 10 and 15
= (4 +1)(2 +1)(1+1) = 30
So the numbers which are divisible by only 10 but not by 15 = 40 – 30 = 10

11. **(1)**

Sol. First we seat first two specified person in $2\times10 = 20$ ways and remaining 10 person can be
arranged in 10! ways.



So total no. of ways is = $2\times10\times10! = 20.\ 10\,!$

12. **(3)**

Sol. As 4 particular flowers are together then the total number of ways is

$$\frac{4!\times4!}{2} = 288$$

13. **(2)**

Sol. Indians - 2
Americans - 3
Italians - 3
Frenchmen - 4
number of ways of arranged in a circle. while persons of same nationality are together is
$= 3\,! \times 2! \times 3! \times 3! \times 4\,! = 2\ .\ (3!)^3\ .\ 4!$

14. **(2)**

Sol. $xyz = 21600$
$= 2^5\ .\ 3^3\ .\ 5^2$
Here if $x + y + z = 5 \Rightarrow {}^7C_2 = 21$
$x + y + z = 3 \Rightarrow {}^5C_2 = 10$
$x + y + z = 2 \Rightarrow {}^4C_2 = 6$
Hence required no. = 21 × 10 × 6 = 1260

15. **(1)**

Sol. Using multinomial theorem
Total no. of ways of choosing 6 chocolates out of
8 different brand is = ${}^{8+6-1}C_6 = {}^{13}C_6$

16. **(1)**

Sol. Using multinomial theorem
co-efficient of x^{11} in the expansion of $(x + x^2 + x^3 + ...+ x^6)^3$
= coeff. of x^{11} in $x^3\,(1 - x^6)^3.(1-x)^{-3}$ is
$= {}^{10}C_8 - 3\ .\ {}^4C_2 = 45 - 18 = 27$

17. **(3)**

Sol. Total number of ways = ${}^8C_3 - {}^5C_3 - {}^3C_3 = 45$

18. **(4)**

Sol. Total number of points of intersection = ${}^8C_2 + 2 \times {}^4C_2 + 2 \times {}^8C_1 \times {}^4C_1$
= 104

19. **(1)**

Sol. Exponent of 2 in 45! is

$$\left[\frac{45}{2}\right] + \left[\frac{45}{2^2}\right] + \left[\frac{45}{2^3}\right] + \left[\frac{45}{2^4}\right] + \left[\frac{45}{2^5}\right] + \left[\frac{45}{2^6}\right]$$

$= 22 + 11 + 5 + 2 + 1 + 0 \quad = 41$

Exponent of 5 in 45! is

$$\left[\frac{45}{5}\right] + \left[\frac{45}{5^2}\right] + \left[\frac{45}{5^3}\right] \quad = 9 + 1 + 0 = 10$$

So no. of zeros at the end of 45! is 10

20. **(1)**

Sol. Required number of ways = ${}^5C_1\, D_4 + {}^5C_0\, D_5$

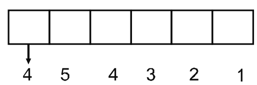
$$= 5 \times 4\,! \left(1 - \frac{1}{1\,!} + \frac{1}{2\,!} - \frac{1}{3\,!} + \frac{1}{4\,!}\right) + 1 \times 5\,! \left(1 - \frac{1}{1\,!} + \frac{1}{2\,!} - \frac{1}{3\,!} + \frac{1}{4\,!} - \frac{1}{5\,!}\right) = 89$$

21. **(1)**

Sol. possible outcomes are = ${}^{10}P_4$

22. **(4)**

Sol.



Hence total number of ways.
$= 4\times5! = 4 \times 120 = 480$

23. **(1)**

Sol. Number of one digit numbers = 9
Number of 2 digits numbers = 9 × 9 = 81
Number of 3 digits numbers = 9 × 9 × 8 = 648
total numbers = 9 + 81 + 648 = 738

24. **(2)**

Sol. Total number of ways is if there is no condition is

8 ! = 40320
Again if all vowels are together i.e.
A E U DGHTR.
so total number of ways = 6 ! × 3 ! = 4320
Hence total number of ways if all vowels do not together is
40320 – 4320 = 36000 ways

25. **(4)**
Sol. When 0 is not being used = 5!
When 3 is not being used = 5! – 4 !
Total number of ways = 5 ! + 5 ! – 4 ! = 216

26. **(1)**
Sol. Here $T_r = r.\ P_r = (r + 1 – 1)\ r\,!$
$= (r + 1)\ r! – r! = (r + 1)! – r!$
$\Rightarrow\ S_n = (n+1)! – 1$

27. **(2)**
Sol. Total number of 6 digit number that ends with 2, 1

				2	1
7	7	6	5		

i.e.
Hence total number of ways is
$7 \times 7 \times 6 \times 5 = 7 \times {}^7P_3$

28. **(4)**
Sol.

A	*B*	*A*	*B*	*A*	*B*

A	*B*	*A*	*B*	*A*	*B*

total number of required ways is ${}^{12}C_6 \times 6\,! \times 6\,! \times 2\,!$

$= \dfrac{12!}{6! \times 6!} \times 6\,! \times 6\,! \times 2\,! = 2 \times 12\,!$

29. **(1)**
Sol.

A				

= 24 ways
4!

G				

= 12 ways
4!/2!

I				

= 12 ways
$\dfrac{4!}{2!}$

N	A	A	G	I

and

N	A	A	I	G

30. **(1)**
Sol. We have arrange all the letter except '*CCC*' is $\dfrac{12!}{5!.3!.2!}$ now there are 13 place where '*C*' can be placed = ${}^{13}C_3$

Hence required number of ways is = $\dfrac{12!}{5!\,3!\,2!}$

${}^{13}C_3 = 11.\ \dfrac{13!}{6!}$

31. **(4)**
Sol. Total number of possible arrangements are
$\dfrac{8!}{3!4!} = 280$

32. **(2)**
Sol. Total no. of *M* are = 1
Total no. of *I* are = 4
Total no. of *P* are = 2
Total no. of *S* are = 4
First we arrange all the words other than I's are

Number of ways = $\dfrac{7!}{2!\,4!} = \dfrac{7 \times 6 \times 5}{1 \times 2} = 105$

Now, there are 8 places in between arranged letter where *I* can be placed to keep separated from each other. For doing this no. of ways = ${}^8C_4$
Total required no. = $105 \times {}^8C_4$
$= \dfrac{105 \times 8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}$
= 105 × 70 = 7350

33. **(1)**
Sol. Case - I when all 5 match win by the India then total no. of ways = ${}^5C_5$
Case - II when 6th match win by the India then total no. of ways = ${}^5C_4$
Case – III when 7th match win by the India then total no. of ways = ${}^6C_4$
Case – IV when 8th match win by the India then total no. of ways = ${}^7C_4$

Case - V when 9^{th} match win by the India then total no. of ways = ${}^8C_4$
Hence required no. of ways
= 1 + 5 + ${}^6C_4 + {}^7C_4 + {}^8C_4$
= 1 + 5 + 15 + 35 + 70 = 126

34. **(4)**
Sol. Total No. of bowlers = 6
Now, (i) If 4 bowlers are including the no. of ways selecting 11 players out of 15 players
$= {}^6C_4 \times {}^9C_7 = 15 \times 36 = 540$
(ii) If 5 bowlers are selected
$= {}^6C_5 \times {}^9C_6 = 6 \times 84 = 504$
(iii) If all 6 bowlers are selected
$= {}^6C_6 \times {}^9C_5 = 1 \times 126 = 126$
Hence total no. of ways = 540 + 504 + 126 = 1170

35. **(2)**
Sol. Total number is ${}^8C_5 = 56$
not required is ${}^6C_5 + {}^6C_5 = 12$
Hence required no of arrangement = 56 – 12 = 44

36. **(4)**
Sol. First we select 3 places out of 10 for speakers S_1, S_2 and S_3 put them in order S_1, S_3, S_2 or S_3, S_1, S_2 then arrange rest seven speakers at seven place without any restriction i.e. total number of ways = ${}^{10}C_3 \,.\, 7!.2\,! = \frac{10!}{3}$

37. **(1)**
Sol. First we choose any three place out of 11 place i.e. ${}^{11}C_3$ ways and rest 8 places are arranged by 8!
ways.
Hence required no. is ${}^{11}C_3 .\, 8! = 11!/3!$

38. **(2)**
Sol. Possible size of group of coins is
1, 2, 4
so the number of ways is
${}^7C_1 \,.\, {}^6C_2 \,.\, {}^4C_4 \,.\, 3! = 7 \times 15 \times 6 = 630$

39. **(4)**
Sol. $\frac{{}^{200}C_2 . {}^{198}C_2 . {}^{196}C_2 ...... {}^2C_2}{100!}$

$= \frac{200!}{2^{100}.100!} = \frac{101.102.103.....200}{2^{100}}$

$= \left(\frac{101}{2}\right).\left(\frac{102}{2}\right).\left(\frac{103}{2}\right).....\left(\frac{200}{2}\right)$

And $\frac{1.2.3.4.5.6.7.8.....200}{2^{100}.100!}$

$= \frac{(1.3.5.7.....199)(2.4.6.8......200)}{2^{100}.100!}$

$= \frac{(1.3.5......199).2^{100}.100!}{2^{100}.100!} = 1.3.5\,.199$

40. **(2)**
Sol. $\frac{(8+8)!}{8!8!} = {}^{16}C_8$

41. **(2)**
Sol. $14400 = 2^6 \times 3^2 \times 5^2$
Number of ways ; $= \frac{1}{2}[(6+1)(2+1)(2+1)+1]$
= 32

42. **(3)**
Sol. $10080 = 2^5 \times 3^2 \times 5^1 \times 7^1$
coprime factors = $2^{4-1} = 2^3 = 8$

43. **(1)**
Sol. First we select 5 beads from 8 different beads.
No. of ways = ${}^8C_5$
Now total number of arrangement is ${}^8C_5 \times \frac{4!}{2!}$
= 672

44. **(1)**
Sol. First find if all the person are sitting in a round table is 4! = 24 ways
if two of the person are sitting together i.e.
3! × 2! = 12 ways
Hence required number of ways = 24 – 12 = 12 ways

45. **(1)**
Sol. If women are sit together then total number of person is 5 hence required ways = 4! × 3!
= 24 × 6 = 144

46. **(4)**
Sol. Here B is always between *A* and *C* so i.e. either *ABC* or *CBA*
so total required number of ways is 4! × 2! = 24 × 2 = 48 ways

47. (3)

Sol. Desired no. of ways coefficient of x^{14} in the expansion of $(x^0 + x^2 + x^4)^5$

= coefficient of x^{14} in $(1 + x^2 + x^4)^5$

= coefficient of x^{14} in $\left(\frac{1-x^6}{1-x^2}\right)^5$

= coefficient of x^{14} in $(1 - x^6)^5 (1 - x^2)^{-5}$

= coefficient of x^{14} in $(1 - 5x^6 + 10x^{12}............)$

$(1 + {}^5C_1x^2 + {}^6C_2x^4 + {}^7C_3x^6 + ...)$

$= {}^{11}C_7 - 5 \,.\, {}^8C_4 + 10.5$

$= \frac{11\times10\times9\times8}{1\times2\times3\times4} - 5.\frac{8\times7\times6\times5}{1\times2\times3\times4} + 50$

$= 330 - 350 + 50 = 30$

Alternative Method:

0, 2, 4, 4, 4 $\frac{\lfloor 5}{\lfloor 3 \cdot \lfloor 1 \cdot \lfloor 1} = 20$

+

2, 2, 2, 4, 4 $\frac{\lfloor 5}{\lfloor 2 \cdot \lfloor 3} = 10$

48. (4)

Sol. Here $-10 \le x, y, z \le -1$

Using multinomial theorem

Find the coefficient of x^{12} in this expansion of $(x + x^2 + ...+ x^{10})^3$

= coeff of x^{12} in $x^3 (1 + x + x^2 + ...+ x^9)^3$

= coeff of x^{12} in $x^3 (1 - x^{10})^3 \,.\, (1 - x)^{-3} = {}^{11}C_9 = \frac{11\times10}{2} = 55$

49. (3)

Sol. $x_1 + x_2 + x_3 = 20 - t$

$t = 0, 1, 2, 3, 4$

Required value = $\sum_{t=0}^{4} {}^{19-t}C_2$

$= {}^{20}C_3 - {}^{15}C_3 = 1140 - 455 = 685$

50. (3)

Sol. Using multinomial theorem

Total no. of ways = ${}^{15+3-1}C_{15} \times {}^{10+3-1}C_{10}$

$= {}^{17}C_{15} \times {}^{12}C_{10} = \frac{17\times16}{2}\times\frac{12\times11}{2} = 8976$

51. (2)

Sol. $x_1 + x_2 + x_3 + x_4 \le n$ $\quad x_1 + x_2 + x_3 + x_4 + y = n$

(where y is known as pseudo variable) Total no. of required solution is

$= {}^{n+5-1}C_n = {}^{n+4}C_n$ or ${}^{n+4}C_4$

52. (3)

Sol. Let number be $x_1\, x_2\, x_3\, x_4\, x_5\, x_6$

But Here $x_1 + x_2 + ... x_6 = 12$

so required no. of integers = coefficient of x^{12} in expansion $(1 + x + x^2 + ... + x^9)^6$

$= (1 - x^{10})^6 \,.\, (1 - x)^{-6}$

$\Rightarrow {}^{17}C_{12} - {}^6C_1 \,.\, {}^7C_2$

$= 6188 - 126 = 6062$

53. (4)

Sol. $D_4 \times D_3$

$= 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) \times 3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)$

$= (12 - 4 + 1) \times (3 - 1)$

$= 9 \times 2 = 18$

54. (4)

Sol. If power of P is l in n! then $l = \left[\frac{n}{P}\right] + \left[\frac{n}{P^2}\right] + ...$

Power of 5 in 100! is

$= \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] + \left[\frac{100}{5^3}\right] = 20 + 4 + 0 = 24$

Power of 2 in 100 ! is

$= \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right] = 50 + 25 + 12 + 6 + 3 + 1 = 97$

So power of 50 in 100 ! is 12.

Maximum value of k is 12.

55. (1)

Sol. $N = 249480 = 2^3 \times 3^4 \times 5^1 \times 7^1 \times 11^1$

(A) Number of ways N is divisible by 3 but not by 5 = 4 × 4 ×2 × 2 = 64

(B) Number of ways N is divisible by 5 but not by 7 = 4 × 5 × 1 × 2 = 40

(C) Number of ways N is divisible by 3 but not by 21 = 4 × 4×2×2 = 64

(D) Number of ways N is divisible by 35 but not by 77 = 4×5 × 1× 1= 20

56. **(4)**

Sol. $x_1 + x_2 + x_3 + x_4 + x_5 = 6$

$\frac{(6+4)!}{6!4!} = {}^{10}C_4$

Statement - 1 is false while statement-2 is true

Ans. (4)

57. **(4)**

Sol. M I I I I P P

${}^{8}C_4 \,.\, \frac{7!}{4!.2!} = 7.\,{}^{6}C_4 \,.\, {}^{8}C_4$

58. **(2)**

Sol.

$B_1 + B_2 + B_3 + B_4 = 10$

= coefficient of x^{10} in $(x^1 + x^2 + ...+ x^7)^4$

= coefficient of x^6 in $(1 - x^7)^4 (1 - x)^{-4}$

$= {}^{4+6-1}C_6 = {}^{9}C_3$

Statement - 2 :

Obviously ${}^{9}C_3$

59. **(3)**

Sol. Number of ways: $x = {}^{6}C_4 \times {}^{3}C_1 \times 4! = 15 \times 3 \times 24 = 1080$

60. **(1)**

Sol. Required sum = 3! × 20 × (1111)

= 133320

Integer Type Questions (61 to 75)

61. **(20)**

Sol. There are 4 balls marked even digits {*i.e.* 2, 4, 6, 8}

and 5 balls marked odd digits {1, 3, 5, 7, 9}

sum is odd $\Rightarrow$ one ball with even digit and other one is with odd digit

$\Rightarrow$ no. of ways = ${}^{4}C_1 \,.\, {}^{5}C_1 = 20$

62. **(18)**

Sol. Let there are n teams in championship

No of matches played $= {}^{n}C_2 = 153$

$\Rightarrow \frac{n(n-1)}{2} = 153$

$\Rightarrow n(n - 1) = 306$

$\Rightarrow (n - 18)(n + 17) = 0$

$\Rightarrow n = 18$

63. **(21)**

Sol. upperdeck - 13 seats → 8 in upper deck.

lowerdeck - 7 seats → 5 in lower deck

Remaining passengers = 7

Now Remains 5 seats in upper deck and 2 seats in lower deck

for upper deck number of ways = $7C5$

for lower deck number of ways = $2C2$

So total number of ways = $7C5 \times 2C2 = \frac{7.6}{2}$

= 21

64. **(7)**

Sol. The number of triangles can be formed using n non-collinear points is nC3.

Since, $Tn = nC3$

Given, $Tn+1 - Tn = 21$

$\Rightarrow n+1C3 - nC3 = 21$

$\Rightarrow nC2 + nC3 - nC3 = 21$

$(\because nC2 + nC3 = n+1C3\}$

$\Rightarrow nC2 = 21$

$\Rightarrow \frac{n(n-1)}{2} = 21 \Rightarrow n2 - n - 42 = 0$

$\Rightarrow (n - 7)(n + 6) = 0$

$\Rightarrow n = 7 \quad (\because n \neq -6)$

65. **(196)**

Sol. $5C4 \,.\, 8C6 + 5C5 \,.\, 8C5$

= 140 + 56 = 196

66. **(21)**

Sol. 1 1 1

$\frac{(5+2)!}{5!2!} = 21$

67. **(108)**

Sol. ${}^{3}C_2 \times {}^{9}C_2 = 3 \times \frac{9\times 8}{2\times 1} = 12 \times 9 = 108$

68. **(192)**

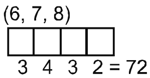
Sol. 

Number of integer greater than 6000 may be 4 digit or 5 digit

C-1 when number is of 4 digit

C-2 when number is of 5 digit = 5! = 120

total = 120 + 72 = 192 digit



69. **(485)**

Sol.

$^3C_3 \times {}^3C_3 + {}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1 + {}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2 + {}^4C_3 \times {}^4C_3 = 1 + 144 + 324 + 16 = 485$

70. **(40)**

Sol. Total 6 letters word that can be formed $= \dfrac{6!}{2!3!} = 60$

total 6 letters word in which both N comes together $= \dfrac{5!}{3!} = 20$

number of arrangement in which both N do not appear together = 60 – 20 = 40

71. **(420)**

Sol. Total number of required quadrilateral

$^7C_4 + {}^7C_3 \times {}^5C_1 + {}^7C_2 \times {}^5C_2$

$= \dfrac{7\times6\times5\times4}{1\times2\times3\times4} + \dfrac{7\times6\times5}{1\times2\times3}.5 + \dfrac{7\times6}{1\times2}\times\dfrac{5\times4}{1\times2}$

$= 35 + 175 + 210 = 420 = 2\,.{}^7P_3$

72. **(77)**

Sol. There are two possible cases

Case 1 **:** Five 1's, one 2's, one 3's

Number of numbers $= \dfrac{7!}{5!} = 42$

Case 2 **:** Four 1's, three 2's

Number of numbers $= \dfrac{7!}{4!3!} = 35$

Total number of numbers = 42 + 35 = **77**

73. **(150)**

Sol.

	B_1	B_2	B_3
Case-1:	1	1	3
Case-2:	2	2	1

Ways of distribution $\dfrac{5!}{1!\,1!\,3!\,2!} = \ .3!$

$+ \dfrac{5!}{2!\,2!\,1!\,2!}.3!$

$= 150$

Alternative Method:

$3^5 - {}^3C_1\, 2^5 + {}^3C_2 \cdot 1^5$

74. **(456)**

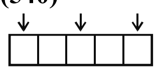
Sol. maximum size of square can be 8×8

so required no. of square be

$= \sum_{r=1}^{8}(16-r)(9-r)$

$= \sum_{r=1}^{8}\left(r^2 - 25r + 144\right) = 456$

75. **(540)**

Sol. 

There are 2M, 2T, 2A and 1 H, E, I, C, S

First find the number of ways if odd's no. position place be filled is $^5P_3 = 60$

Now Case I If even place letters is same i.e no. of ways = 3

Case II If even place letters is different i.e no. of ways $= {}^3C_2 \times 2! = 6$

Hence total no. of arrangement is

$60 \times (3 + 6) = 540$

SEQUENCE AND SERIES

Single Option Correct Type Questions (01 to 60)

1. **(4)**

Sol. $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

$a - 4(a + d) + 6(a + 2d) - 4(a + 3d) + (a + 4d)$
$= 0 - 0 = 0$

Like wise we can check other options

2. **(1)**

Sol. $2\log_5(2^x - 5) = \log_5 2 + \log_5 (2^x - 7/2)$

$\Rightarrow (2^x - 5)^2 = 2\left(2^x - \dfrac{7}{2}\right)$

$\Rightarrow t^2 - 10t + 25 = 2t - 7 \quad \{\text{put } 2^x = t\}$

$\Rightarrow t^2 - 12t + 32 = 0$

$\Rightarrow t = 8, 4$

$\Rightarrow 2^x = 4 \text{ or } 2^x = 8$

$\Rightarrow x = 2, 3$

$(\because 2^x - 5 > 0 \Rightarrow 2^x > 5)$

so only solution $x = 3$

$\Rightarrow 2x = 6$

3. **(3)**

Sol. $S = \log a + \log \dfrac{a^2}{b} + \log \dfrac{a^3}{b^2} + \ldots .\ n \text{ terms} = \log a + (2 \log a - \log b) + (3 \log a - 2 \log b) + \ldots ..\ n \text{ terms}$

Which is an A.P. with $d = \log a - \log b = \log \dfrac{a}{b}$ and $A = \log a$

$\therefore S_n = \dfrac{n}{2}\left[2 \log a + (n - 1) \log \dfrac{a}{b}\right]$

$= \dfrac{n^2}{2} \log \dfrac{a}{b} + \dfrac{n}{2}[2 \log a - \log a + \log b]$

$= \dfrac{n^2}{2} \log \dfrac{a}{b} + \dfrac{n}{2} \log ab$

4. **(2)**

Sol. Let the first term and common difference of first A.P. is a_1 and d_1 and of second A.P. is a_2 and d_2

$$\therefore \frac{S_1}{S_2} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]}$$

$$= \frac{3n+8}{7n+15}$$

$$\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{3n+8}{7n+15}$$

Ratio of 12th terms $= \dfrac{a_1 + 11d_1}{a_2 + 11d_2}$

So $\dfrac{n-1}{2} = 11,\ n = 23$

So ratio of 12th terms $= \dfrac{3 \times 23 + 8}{7 \times 23 + 15}$

$= \dfrac{77}{176} = \dfrac{7}{16}$

5. **(2)**

Sol. a and be first and l last term

$\therefore\ S = \frac{n}{2}\ (a+\ell)$ or $\frac{2S}{a+\ell} = n$ ......(1)

$l = a + (n-1)d$ so $d = \frac{\ell - a}{n-1}$ ....(2)

putting the value of n from (1) in (2)

$d = \frac{\ell - a}{\frac{2S}{a+\ell} - 1}$, $d = \frac{\ell^2 - a^2}{2S - \ell - a}$

6. **(3)**

Sol. b_1, b_2, b_3 are in G.P.

$\therefore\ b_3 > 4b_2 - 3b_1$

$\Rightarrow r^2 > 4r - 3$

$\Rightarrow r^2 - 4r + 3 > 0$

$\Rightarrow (r-1)(r-3) > 0$

So $0 < r < 1$ and $r > 3$

7. **(2)**

Sol. Let $T_{k+1} = ar^k$ and $T'_{k+1} = br^k$

Since $T''_{k+1} = ar^k + br^k = (a+b)r^k$

$\therefore\ T''_{k+1}$ is general term of a G.P.

8. **(1)**

Sol. Let $b + c - a,\ c + a - b,\ a + b - c$ are in A.P.

$\Rightarrow\ 2(c + a - b) = (b + c - a) + (a + b - c)$

$\Rightarrow\ 2c + 2a = 4b$

$\Rightarrow\ 2b = a + c$

$\Rightarrow\ a, b, c$ are in A.P.

9. **(2)**

Sol. x, y, z are in G.P.

$\therefore\ y^2 = xz$ ....(i)

$\Rightarrow\ x(x+z),\ y(x+z),\ z(x+z)$ are in G.P.

$\Rightarrow\ x^2 + xz,\ xy + yz,\ z^2 + xz$ are in G.P.

$\Rightarrow\ x^2 + y^2,\ xy + yz,\ y^2 + z^2$ are in G.P.

[putting $y^2 = xz$ from (i)]

10. **(2)**

Sol. Let $b = ar$, $c = ar^2$ and $d = ar^3$

So, $a^2(1-r^2),\ a^2(r^2)(1-r^2),\ a^2r^4(1-r^2)$

these are in G.P.

So, $(a^2 - b^2), (b^2 - c^2), (c^2 - d^2)$ are in G.P.

11. **(1)**

Sol. Let three positive numbers which are in G.P. be a, b, c

$\therefore\ b^2 = ac$ .....(i)

By first condition $a, b + 8$, c are in A.P.

$\therefore\ b = \frac{a+c}{2} - 8$ ....(ii)

By second condition, $a, b + 8, c + 64$ are in G.P.

$\therefore\ (b+8)^2 = a(c+64)$

$\therefore\ b^2 + 64 + 16b = ac + 64a$

$\therefore\ b^2 = ac$

$\therefore\ 64 + 16b = 64a$

$\Rightarrow\ b = 4(a-1)$ ....(iii)

putting the value of b in (ii)

$4(a-1) = \frac{a+c}{2} - 8$

$\Rightarrow\ c = 7a + 8$ ....(iv)

Now putting the value of both b and c in (i)

$16(a-1)^2 = a(7a+8)$

$\Rightarrow\ 9a^2 - 40a + 16 = 0$

$(9a-4)(a-4) = 0$

$\Rightarrow\ a = \frac{4}{9}, a = 4$

$a = \frac{4}{9}$ gives negative value of b, but it is given that b is positive, so $a = 4$ is acceptable value

$b = 4(4-1) = 12$

$\therefore$ common ratio $= \frac{12}{4} = 3$

12. (2)

Sol. If a be the side of a square then $d = a\sqrt{2}$

by given condition $\rightarrow a_n = \sqrt{2}\ a_{n+1}$ or a_{n+1}

$$= \frac{a_n}{\sqrt{2}} = \frac{a_{n-1}}{(\sqrt{2})^2} = \frac{a_{n-2}}{(\sqrt{2})^3} = ... = \frac{a_1}{(\sqrt{2})^n}$$

Replacing n by $n - 1$, we get

$$a_n = \frac{a_1}{(\sqrt{2})^{n-1}} = \frac{10}{2^{\frac{(n-1)}{2}}}$$

Area of $S_n < 1 \quad \Rightarrow \quad a_n^2 < 1$

$$\Rightarrow \frac{100}{2^{n-1}} < 1 \text{ or } 200 < 2^n \text{ or } 2^n > 200$$

Now $2^7 = 128 < 200,\ 2^8 = 256 > 200$

$\therefore \quad n = 8, 9, 10$

13. (2)

Sol. $\because \quad a, b, c$ are in H.P.

$$\therefore \quad b = \frac{2ac}{a+c}$$

Now, $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$

$$a - \frac{ac}{a+c}, \frac{ac}{a+c}, c - \frac{ac}{a+c}$$

$$\frac{a^2}{a+c}, \frac{ac}{a+c}, \frac{c^2}{a+c}$$

$$\therefore \quad \left(\frac{ac}{a+c}\right)^2 = \left(\frac{a^2}{a+c}\right)\left(\frac{c^2}{a+c}\right)$$

so given numbers are in G.P.

14. (2)

Sol. $$\frac{a+be^y}{a-be^y} + 1 = \frac{b+ce^y}{b-ce^y} + 1 = \frac{c+de^y}{c-de^y} + 1$$

$$\frac{2a}{a-be^y} = \frac{2b}{a-ce^y} = \frac{2c}{c-de^y}$$

$$1 - \frac{be^y}{a} = 1 - \frac{c}{b}\ e^y = 1 - \frac{d}{c}\ e^y$$

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$\Rightarrow a, b, c, d$ are in G.P.

15. (4)

Sol. $\frac{1}{a_1}, \frac{1}{a_2}, ..., \frac{1}{a_n}$, are in A.P.

$$\Rightarrow \frac{1}{a_2} - \frac{1}{a_1} = ... = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$$

$$\Rightarrow a_1a_2 = \frac{a_1 - a_2}{d},\ a_2a_3 = \frac{a_2 - a_3}{d}\ ....,$$

$$\frac{a_{n-1} - a_n}{d} = a_{n-1}.a_n$$

$$\Rightarrow a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$$

$$= \frac{a_1 - a_2}{d} + ... + \frac{a_{n-1} - a_n}{d}$$

$$= \frac{a_1 - a_n}{d} = (n-1)\ a_1a_n$$

16. (2)

Sol. In an A.P. $t_1 = \log_{10}a$, t_{n+1}

$= \log_{10}b$ and $t_{2n+1} = \log_{10}c$

t_1, t_{n+1}, t_{2n+1} form an A.P. of common difference nd as they are $a, a + nd, a + 2nd$

$$2t_{n+1} = t_1 + t_{2n+1}$$

or $\quad 2 \log b = \log a + \log c$

or $\quad \log b^2 = \log ac$

$\therefore \quad b^2 = ac$

$\therefore \quad a, b, c$ are in G.P.

17. (1)

Sol. $a_1, a_2, a_3 \ldots\ldots a_{2n+1}$ are in A.P.

Let common difference is d

$a_{2n+1} = a_1 + 2nd \Rightarrow a_{2n+1} - a_1 = 2nd$

Similarly, $a_{2n} - a_2 = 2(n-1)d$

In denominator, by properties of A.P. $a_{2n+1} + a_1$

$= a_{2n} + a_2 = a_{n+2} + a_n$

So, all terms in denominator is same

$$\text{sum} = \frac{2d[n+(n-1)+(n-2)+....1]}{a_{2n+1}+a_1}$$

$$\text{sum} = \frac{2dn(n+1)}{2(a_1+a_{2n+1})} = \frac{n(n+1)(a_2-a_1)}{a_1+a_{2n+1}}$$

But $a_1 + a_{2n+1} = a_1 + a_1 + 2nd = 2(a_1 + nd)$

$\because \quad a_1 + nd = a_{n+1}$

$\therefore \quad a_1 + a_{2n+1} = 2a_{n+1}$

$$\therefore \quad \text{sum} = \frac{n(n+1)}{2a_{n+1}}(a_2 - a_1)$$

18. (1)

Sol. $x + y + z = 15$ ....(i)

a, x, y, z, b are in AP

Suppose d is common difference

$$d = \frac{b-a}{4}$$

$$\therefore \quad x = a + \frac{b-a}{4} = \frac{b+3a}{4}, y = \frac{2b+2a}{4}$$

and $z = \dfrac{3b+a}{4}$

on substituting the values of x, y and z in (i), we get

$$\Rightarrow \frac{6a+6b}{4} = 15$$

$\Rightarrow a + b = 10$ …(ii)

$$\because \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3} \quad \text{…(iii)}$$

and a, x, y, z, b are in H.P.

$\therefore \dfrac{1}{a}, \dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}, \dfrac{1}{b}$ are in A.P.

$$\therefore \frac{1}{x} = \frac{1}{a} + \frac{\left(\frac{1}{b}-\frac{1}{a}\right)}{4}, \quad \frac{1}{y} = \frac{1}{a} + \frac{2}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$$

and $\dfrac{1}{z} = \dfrac{1}{a} + \dfrac{3}{4}\left(\dfrac{1}{b}-\dfrac{1}{a}\right)$

on substituting the value of $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$ in (iii), we get

$$\frac{3}{a} + \frac{6}{4}\left(\frac{1}{b}-\frac{1}{a}\right) = \frac{5}{3}$$

$$\Rightarrow \frac{3}{2a} + \frac{3}{2b} = \frac{5}{3}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{10}{9} \quad \text{..... (iv)}$$

By equations (ii) & (iv), we get

$a = 9, b = 1$ or $a = 1, b = 9$

19. (3)

Sol.

A.P.	T_1	$T_n = a$	T_{2n-1}
G.P.	T_1	$T_n = b$	T_{2n-1}
H.P.	T_1	$T_n = c$	T_{2n-1}

The n^{th} term is equidistant from the first and $(2n-1)^{th}$ term. In other words it is the middle term of a series of $(2n-1)$ terms. Also it is given that T_1 is same and T_{2n-1} is same for all the series. If they be p and q respectively, then p, a, q ; p, b, q ; p, c, q are in A.P., G.P. and H.P. respectively.

Therefore a, b, c are A.M., G.M. and H.M. respectively of the same quantities p and q.

We know that A.H. $= G^2$

$\therefore \quad ac = b^2$

$\therefore \quad ac - b^2 = 0$

20. (1)

Sol. $g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \ldots\ldots + (n-1)^2) = n^2$

21. (1)

Sol. $x_1 + x_2 + x_3 + ... + x_{50} = 50$

AM $\geq$ HM

$$\frac{x_1 + x_2 + ... + x_{50}}{50} \geq \frac{1}{\dfrac{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+....+\frac{1}{x_{50}}}{50}}$$

$$\Rightarrow \frac{x_1 + x_2 + ... + x_{50}}{50} \geq \frac{50}{\frac{1}{x_1}+\frac{1}{x_2}+........+\frac{1}{x_{50}}}$$

$$\Rightarrow \frac{1}{x_1}+\frac{1}{x_2}+........+\frac{1}{x_{50}} \geq 50$$

so minimum value of

$$\frac{1}{x_1}+\frac{1}{x_2}+........+\frac{1}{x_{50}} = 50$$

22. (4)

Sol. $0 < x$, y and $a,b < 1$ given series is

$\sqrt{x}(\sqrt{a}+\sqrt{x})+\sqrt{x}(\sqrt{ab}+\sqrt{xy})+$

$\sqrt{x}(b\sqrt{a}+y\sqrt{x})$ $+......\infty.$

$\therefore S = (\sqrt{ax}+x+\sqrt{axb}+x\sqrt{y}+\sqrt{axb}+xy ... \infty).$

$\Rightarrow S = (\sqrt{ax}+\sqrt{axb}+\sqrt{axb}.......\infty)$

$+ (x+x\sqrt{y}+xy+.......\infty).$

$$\Rightarrow S = \frac{\sqrt{ax}}{1-\sqrt{b}}+\frac{x}{1-\sqrt{y}}$$

23. (2)

Sol. $S = \frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+......$ n terms

$$= \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{2^2}\right)+\left(1-\frac{1}{2^3}\right)+............ + \left(1-\frac{1}{2^n}\right)$$

$$= n-\left(\frac{1}{2}+\frac{1}{2^2}+......\frac{1}{2^n}\right)$$

$$= n - \frac{1}{2}\frac{\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\left(\frac{1}{2}\right)\right)} = n + 2^{-n} - 1$$

24. (1)

Sol. Let a, b, c in G.P. then b^2 = ac

Now $a + b$, $2b$, $b + c$ in HP

$$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c} \text{ in AP}$$

$$\frac{2}{2b} = \frac{1}{a+b}+\frac{1}{b+c}$$

$(a + b)(b + c) = (a + c + 2b)\, b$

$ab + b^2 + ac + bc = ab + bc + 2b^2$

$\therefore \quad b^2 = ac$

So, statement (1) and (2) is true

25. (3)

Sol. $b_1 = a_1$, $b_2 = a_1(1 + r)$, $b_3 = a_1(1 + r + r^2)$,

$b_4 = a_1(1 + r + r^2 + r^3)$

Statement 1 is correct as the numbers are neither in A.P. nor in G.P.

Now, $$\frac{2b_1b_3}{b_1+b_3} = \frac{2a_1\ a_1(1+r+r^2)}{a_1(2+r+r^2)} \neq b_2.$$

Hence statement ‘2’ is false

26. **(3)**

Sol. $$\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

$$\Rightarrow \frac{a+5d}{a+20d} = \frac{11}{41}$$

27. **(4)**

Sol. $a = ar + ar^2$

$\Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow r = \frac{\sqrt{5}-1}{2}$

(–ve not permissible)

28. **(4)**

Sol. a = Rs. 200

d = Rs. 40

savings in first two months = Rs. 400

remained savings = 200 + 240 + 280 + ... upto n terms

$= \frac{n}{2}[400+(n-1)40] = 11040 - 400$

$200n + 20n^2 - 20n = 10640$

$\Rightarrow 20n^2 + 180n - 10640 = 0$

$\Rightarrow n^2 + 9n - 532 = 0$

$\Rightarrow (n + 28)(n - 19) = 0$

$\Rightarrow n = 19$

$\therefore$ no. of months = 19 + 2 = 21.

29. **(2)**

Sol. Let A.P. $a, a + d, a + 2d$, ...

$a_2 + a_4 + ... + a_{200} = \alpha$

$\Rightarrow \frac{100}{2}[2(a + d) + (100 - 1)d] = \alpha$ .... (i)

And $a_1 + a_3 + a_5 + ... + a_{199} = \beta$

$\Rightarrow \frac{100}{2}[2a + (100 - 1)d] = \beta$ .....(ii)

on solving (i) and (ii)

$d = \frac{\alpha - \beta}{100}$

30. **(4)**

Sol. $100(a + 99d) = 50(a + 49d)$

$2a + 198d = a + 49d$

$a + 149d = 0$

$T_{150} = a + 149d = 0$

31. **(3)**

Sol. $\frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + \ldots\ldots\ldots +$ up to 20 terms

$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + \ldots\ldots \textit{up to 20 terms}\right]$

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots \textit{up to 20 terms}\right]$

$= \frac{7}{9}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{10^2}\right) + \left(1-\frac{1}{10^3}\right) + \ldots\ldots \text{ up to 20 terms}\right]$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{\left(1-\frac{1}{10}\right)}\right]$$

$$= \frac{7}{9}\left[20 - \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^{20}\right)\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right]$$

$$= \frac{7}{81}\left[179 + (10)^{-20}\right]$$

32. **(2)**

Sol. $a \quad a_r \quad ar^2 \rightarrow$ G. P.

$a \quad 2ar \quad ar^2 \rightarrow$ A. P.

$2(2ar) = a + ar^2$

$4r = 1 + r^2$

$r^2 - 4r + 1 = 0$

$r = \frac{4 \mp 2\sqrt{3}}{2} = 2 + \sqrt{3}, \quad 2 - \sqrt{3}$

But $r > 1$

$r = 2 + \sqrt{3}$

33. **(2)**

Sol. $m = \frac{\ell + n}{2}$

ℓ, G_1, G_2, G_3, n are in G.P.

$r = \left(\frac{n}{\ell}\right)^{\frac{1}{4}}$

$G_1 = \ell\left(\frac{n}{\ell}\right)^{\frac{1}{4}} \quad G_2 = \ell\left(\frac{n}{\ell}\right)^{\frac{1}{2}} \quad G_3 = \ell\left(\frac{n}{\ell}\right)^{\frac{3}{4}}$

$G_1^4 + 2G_2^4 + G_3^4$

$= \ell^4 \times \frac{n}{\ell} + 2\ell^4 \frac{n^2}{\ell^2} + \ell^4 \times \frac{n^3}{\ell^3} = \ell^3 n + 2\ell^2 n^2 + \ell n^3 = n\ell(\ell^2 + 2n\ell + n^2)$

$= n\ell(\ell + n)^2 = 4m^2 n\ell$

34 **(1)**

Sol. $a + d, a + 4d, a + 8d \rightarrow$ G.P

$\therefore \quad (a + 4d)^2 = a^2 + 9ad + 8d^2$

$\Rightarrow \quad 8d^2 = ad \qquad \Rightarrow \quad a = 8d$

$\therefore \quad 9d, 12d, 16d \rightarrow$ G.P.

common ratio $r = \frac{12}{9} = \frac{4}{3}$

35. **(2)**

Sol. $225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$(15a)^2 + (3b)^2 + (5c)^2 - (15a)(3b) - (3b)(5c) - (15a)(5c) = 0$

$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

$15a = 3b, 3b = 5c, 5c = 15a$

$5a = b, 3b = 5c, c = 3a$

$\frac{a}{1} = \frac{b}{5} = \frac{c}{3} = \lambda$

$a = \lambda, b = 5\lambda, c = 3\lambda$

a, c, b are in AP

b, c, a are in AP

36. **(1)**

Sol. $a_1 \cdot a_2 \cdot a_3 \cdot ....a_n = c$

$\frac{a_1 + a_2 + a_3 + ...2a_n}{n} \geq (a_1 a_2 a_3 \ldots 2a_n)^{1/n}$

$\geq \quad (2c)^{1/n}$

$\Rightarrow \quad a_1 + a_2 + a_3 + ...2a_n \geq n(2c)^{1/n}$

37. **(4)**

Sol. $2b = a + c$ and $\qquad b^2 = \pm ac$

Case- I

if $b^2 = ac$ and $\qquad a + c + b = \frac{3}{2}$

$\Rightarrow \quad b = \frac{1}{2}$

$a + c = 1 \quad \Rightarrow \quad ac = \frac{1}{4} \quad \Rightarrow \quad (1 - c)\, c = \frac{1}{4}$

$c^2 - c + \frac{1}{4} = 0$

$\Rightarrow \quad c = \frac{1}{2} \qquad \Rightarrow \quad a = \frac{1}{2}$

$a = b = c$ so not valid $\quad a = b = c$

Case- II

$b^2 = -ac$ and $b = \frac{1}{2}$

$a + c = 1 \quad \Rightarrow \qquad ac = -\frac{1}{4}$

$(1 - c)\, c = -\frac{1}{4} \qquad \Rightarrow c^2 - c - \frac{1}{4} \quad = 0$

$c = \frac{1 \pm \sqrt{1+1}}{2} = \frac{1 + \sqrt{2}}{2}$

$c = \frac{1 + \sqrt{2}}{2} \quad \Rightarrow a = \frac{1 - \sqrt{2}}{2}$

38. (1)

Sol. If $\alpha \in \left(0, \frac{\pi}{2}\right)$, $\tan\alpha$ is positive

$\because$ AM $\geq$ GM

$$\therefore \quad \frac{\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2} \geq \sqrt{\sqrt{x^2+x}\cdot\frac{\tan^2\alpha}{\sqrt{x^2+x}}}$$

$$\Rightarrow \quad \sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha$$

39. (3)

Sol. $S_\infty = 5, \quad a = x$

$$\Rightarrow S_\infty = \frac{x}{1-r} = 5$$

$x = 5 - 5r$

$$r = \frac{5-x}{5}$$

$\because \quad -1 < r < 1$

$-5 < 5 - x < 5$

$\Rightarrow \quad -10 < -x < 0$

$0 < x < 10$

40. (3)

Sol. $4\alpha x^2 + \frac{1}{x} \geq 1$

$\Rightarrow$ Let $y = 4\alpha x^2 + \frac{1}{x}$

$$\Rightarrow y' = \frac{dy}{dx} = 8\alpha x - \frac{1}{x^2} = 0 \Rightarrow x = \left(\frac{1}{8\alpha}\right)^{1/3}$$

$$\Rightarrow f(x) = \frac{4\alpha x^3+1}{x} = \frac{1/2+1}{1/(8\alpha)^{1/3}}$$

$$\Rightarrow \frac{3}{2}\times(8\alpha)^{1/3} \geq 1$$

$$\Rightarrow \alpha^{1/3} \geq 1/3 \Rightarrow \alpha \geq \frac{1}{27}$$

41. (4)

Sol. Common terms are 31, 41, 51, 61, ...

the largest term in the sequence 1,11,21,31,..... is 991

the largest term in the sequence 31,36,41,46,..... is 526

Hence the largest term common in both is 521

42. (2)

Sol. $a - 2b + c = 0$

$\Rightarrow \quad a^3 - 8b^3 + c^3 = +3\,(a)\,(-2b)\,(c)$

$= -6\,abc$

43. (4)

Sol. $2\log\left(\frac{3b}{5c}\right) = \log\left(\frac{5c}{a}\right) + \log\left(\frac{a}{3b}\right)$

$$\Rightarrow \left(\frac{3b}{5c}\right)^2 = \frac{5c}{a}\cdot\frac{a}{3b}$$

$\Rightarrow \quad 3b = 5c$

$\Rightarrow \quad \frac{b}{5} = \frac{c}{3}$ ....(1)

also $\Rightarrow \quad \frac{a}{25} = \frac{c}{9}$ ....(2)

by (1) & (2) $\quad \frac{a}{25} = \frac{b}{15} = \frac{c}{9}$

Now $b + c < a$

44. (2)

Sol. $x, 2y, 3z$ in AP

$\Rightarrow \quad 4y = x + 3z$ ....(1)

also x, y, z in GP $\Rightarrow y^2 = xz$ ....(2)

by (1) $\quad 16y^2 = x^2 + 9z^2 + 6\,xz$

$16xz = x^2 + 9z^2 + 6\,xz$

$10\,xz = x^2 + 9z^2$

$\Rightarrow \quad (x - z)\,(x - 9z) = 0$

$x \neq z \qquad \frac{z}{x} = \frac{1}{9} \Rightarrow$

$\Rightarrow$ common ratio = $\frac{1}{3}$

45. (2)

Sol. Let 1025th term is = 2

$\Rightarrow 1 + 2 + 4 + 8 + ...+ 2^{n-1} < 1025 < 1 + 2 + 4 + 8 + ...+ 2^n$

$\Rightarrow 2^n - 1 < 1025 < 2^{n+1} - 1$

$\Rightarrow n = 10$

46. (1)

Sol. $(1 + x)(1 + x^2)(1 + x^4) ... (1 + x^{128})$

$= \left(\frac{1-x^{256}}{1-x}\right)$ ...(1)

also $\sum_{r=0}^{n} x^r = 1 + x + x^2 + ... + x^n = \frac{1-x^{n+1}}{1-x}$

Hence $n = 255$

47. (3)

Sol. $\frac{t_{10}}{t_1} = r^9$

$\Rightarrow r^9 = -\frac{1536}{3} = -512 = -2^9$

$\Rightarrow r = -2$

48. (3)

Sol. $\frac{a_1 + a_4}{a_1 a_4} = \frac{a_2 + a_3}{a_2 a_3}$

$\Rightarrow \frac{1}{a_1} + \frac{1}{a_4} = \frac{1}{a_2} + \frac{1}{a_3}$ ...(1)

also $\frac{a_1 - a_4}{a_1 a_4} = \frac{3(a_2 - a_3)}{a_2 a_3}$

$\Rightarrow \frac{1}{a_4} - \frac{1}{a_1} = 3\left(\frac{1}{a_3} - \frac{1}{a_2}\right)$ ...(2)

by (1) & (2)

$\frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3}$

Hence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}$ are in AP

$\Rightarrow a_1, a_2, a_3, a_4$ are in HP

49. (3)

Sol. 4(GM) = 5(HM)

$\Rightarrow 4\sqrt{ab} = 5\left(\frac{2ab}{a+b}\right)$

$\Rightarrow 4(a+b)^2 = 25\, ab$

$\Rightarrow 4a^2 - 17\, ab + 4b^2 = 0$

$\Rightarrow (4a - b)(a - 4b) = 0$

$\Rightarrow a = 4b \qquad (a = \frac{b}{4}$ neglecting)

50. (3)

Sol. Let roots are $a - d$, a, $a + d$ with common diff. = d

$\Rightarrow a + d + a + a - d = 12 \quad \Rightarrow a = 4$

also $a(a^2 - d^2) = 28$

$\Rightarrow 16 - d^2 = 7$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

51. (2)

Sol. $(x^2 + y^2 + z^2)(y^2 + z^2 + w^2) \leq (xy + yz + zw)^2$

$\Rightarrow x^2(z^2 + w^2) + y^4 + y^2w^2 + y^2z^2 + z^4 \leq 2xy^2z + 2yz^2w + 2xywz$

$\Rightarrow (xz - y^2)^2 + (yw - z^2)^2 + (xw - yz)^2 \leq 0$

$\Rightarrow xz = y^2$, $yw = z^2$ and $xw = yz$

$\Rightarrow \frac{x}{y} = \frac{y}{z} = \frac{z}{w}$

$\Rightarrow x, y, z, w$ are in G.P.

52. (4)

Sol. First number = 1

Last number = 100

Sum of integer 1 to 100

$S = \frac{100}{2}[101]$

$= 5050$

numbers which are divisible by 3 are 3, 6, 9 99

$S_1 = \frac{33}{2}[3 + 99] = 33 \times 51 = 1683$

numbers which are divisible by 5 are 5, 10 .... , 100

$S_2 = \frac{20}{2}[105] = 1050$

numbers which are divisible by 3 and 5 both are 15, 30 90

$S_3 = \frac{6}{2}[15 + 90] = 3(105) = 315$

Now sum of integers which are not divisible by 3 or 5

$= S - S_1 - S_2 + S_3$

$= 5050 - 1683 - 1050 + 315 = 2632$

53. (3)

Sol. $2b = a + c$ and $\frac{2}{b^2} = \frac{1}{a^2} + \frac{1}{c^2}$

$b^2 = \frac{2a^2c^2}{a^2 + c^2}$

$\Rightarrow (a^2 + c^2)(a + c)^2 = 8a^2c^2$

$(a^2 + c^2)(a^2 + c^2 + 2ac) = 8a^2c^2.$

$\Rightarrow a^4 + a^2c^2 + 2a^3c + a^2c^2 + c^4 + 2ac^3 = 8a^2c^2$

$\Rightarrow a^4 + c^4 + 2ac^3 = 6a^2c^2 - 2a^3c$

$\Rightarrow (a^4 + c^4 - 2a^2c^2) = 2ac(2ac - a^2 - c^2)$

$\Rightarrow (a^2 - c^2)^2 = -2ac(a - c)^2$

$\Rightarrow (a - c)^2[(a + c)^2 + 2ac] = 0$

$\Rightarrow (a - c)^2[2b^2 + ac] = 0$

either $a = c$ or $2b^2 = -ac$

$\because 2b = a + c$

either $a = b = c$ or $a, b, -\frac{c}{2}$ are in G.P.

54. (4)

Sol. a, b, c, d are four different real numbers in A.P. and are in decreasing A.P. and $a - b = m$, $(b - c)^2 = m^2$, $(c - a)^3 = -8m^3$

$a - d = 3m$, $(b - d) = 4m^2$, $(c - d)^3 = m^3$

$\therefore$ Given expression is

$2m + xm^2 - 8m^3 = 6m + 4m^2 + m^3$

$xm^2 = 9m^3 + 4m + 4m^2$

$x = \frac{9m^2 + 4m + 4}{m}$

$\therefore 9m^2 + (4 - x)m + 4 = 0$

For real m, $(x - 4)^2 - 144 \geq 0$

$\Rightarrow (x + 8)(x - 16) \geq 0$

$\Rightarrow x \leq -8, x \geq 16$

55. (3)

Sol. Let a and b are two numbers

$\frac{2ab}{a+b} = \frac{16}{5}$ ..... (1)

$\frac{a+b}{2} = A$ and $\sqrt{ab} = G$

$\because 2A + G^2 = 26$

$\Rightarrow (a + b) + ab = 26$ ..... (2)

$\Rightarrow \frac{10ab}{16} + ab = 26$

$\Rightarrow 26\,ab = 26 \times 16$

$\Rightarrow ab = 16$

$\therefore$ from (1), we get

$a + b = 10$

So a, b are (2, 8)

56. (4)

Sol. $a = 5^{1+x} + 5^{1-x} + 25^x + 25^{-x}$

$a = 5(5^x + 5^{-x}) + (25^x + 25^{-x})$

$a \geq 12$ $\{\because\ t^x + t^{-x} \geq 2\}$

57. **(2)**

Sol. If $a, b, c, x \in R$ and $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$

This is quadratic in x, for equal root $D = 0$

$\Rightarrow 4b^2(a + c)^2 = 4(a^2 + b^2)(b^2 + c^2)$

$\Rightarrow b^2a^2 + b^2c^2 + 2acb^2$

$= a^2b^2 + a^2c^2 + b^4 + b^2c^2$

$\Rightarrow b^4 - 2acb^2 + a^2c^2 = 0$

$\Rightarrow (b^2 - ac)^2 = 0$

$\Rightarrow b^2 = ac$

$\therefore$ a, b, c are in G.P.

58. **(3)**

Sol. $\dfrac{S_7}{S_{11}} = \dfrac{6}{11}$

$$\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11},$$

Given, $130 < a + 6d < 140$

$$\Rightarrow \frac{7(a+3d)}{11(a+5d)} = \frac{6}{11}$$

$7a + 21d = 6a + 30d$

$\Rightarrow 130 < 15d < 140$

$a = 9d$

Hence $d = 9$

$a = 81$

Hence $d = 9$

59. **(1)**

Sol. $\left(x+\dfrac{1}{x}\right)^2 + \left(x^2+\dfrac{1}{x^2}\right)^2 + \left(x^3+\dfrac{1}{x^3}\right)^2 + \ldots\ldots$ upto 10 terms

$$= \left(x^2+\frac{1}{x^2}+2\right)+\left(x^4+\frac{1}{x^4}+2\right)+\left(x^6+\frac{1}{x^6}+2\right) + \ldots\ldots\text{upto 10 terms}$$

$= (x^2 + x^4 + x^6 + \ldots\ldots\text{upto 10 terms})$

$$+\left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}\ldots\ldots\text{upto 10 terms}\right) + 2 \times 10$$

$$= \frac{x^2(x^{20}-1)}{x^2-1} + \frac{1}{x^2}\cdot\frac{\frac{1}{x^{20}}-1}{\frac{1}{x^2}-1} + 20$$

$$= \frac{x^2(x^{20}-1)}{x^2-1} + \frac{1}{x^{20}}\cdot\frac{x^{20}-1}{x^2-1} + 20$$

$$= \frac{x^{20}-1}{x^2-1}\left(x^2+\frac{1}{x^{20}}\right) + 20$$

$$= \left(\frac{x^{20}-1}{x^2-1}\right)\left(\frac{x^{22}+1}{x^{20}}\right) + 20$$

60. **(3)**

Sol. $a, a_1, a_2, a_3 \ldots\ldots a_{2n-1}\ b$ are in A.P.

$a, b_1, b_2, b_3 \ldots\ldots b_{2n-1}, b$ are in G.P.

$a, c_1, c_2, c_3 \ldots\ldots c_{2n-1}, b$ are in H.P.

There are $2n + 1$ terms in A.P., G.P. and H.P. If common difference is d for A.P. then $d = \dfrac{b-a}{2n}$

$\therefore$ a_n is $(n + 1)^{th}$ term;

$$\therefore\ a_n = a + \frac{n(b-a)}{2n} = \frac{a+b}{2}$$

If r is common ratio for G.P. $b = a(r)^{2n}$

$$\therefore\ r = \left(\frac{b}{a}\right)^{\frac{1}{2n}}$$

$$\therefore\ b_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{1}{2n}\cdot n}$$

$\therefore$ $b_n = \sqrt{ab}$

similarly $c_n = \dfrac{2ab}{a+b}$

in equation $a_n x^2 - b_n x + c_n = 0$

$$D = b_n^{\,2} - 4a_n c_n = ab - 4\left(\frac{a+b}{2}\right).\ \frac{2ab}{a+b}$$

$$= ab - 4ab = -3ab$$

Integer Type Questions (61 to 75)

61. **(8)**

Sol. Let given three terms be $br,\ b,\ \dfrac{b}{r}$

$$\therefore 12 = \frac{2(br)b}{br+b} = \frac{2br}{r+1} \qquad \ldots(1)$$

$$\text{and } 36 = \frac{2b\left(\frac{b}{r}\right)}{b+\left(\frac{b}{r}\right)} = \frac{2b}{r+1} \qquad \ldots(2)$$

$$(1) \div (2) \Rightarrow r = \frac{1}{3}$$

Then from (2) $b = 24$

$\therefore\quad a = br = 8$

62. **(3)**

Sol. $S_\infty = \sqrt{3} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{3\sqrt{3}} + \ldots$ is a sum of infinite term of a G.P. with $a = \sqrt{3}$, $r = \dfrac{1}{3}$

$$S_\infty = \frac{\sqrt{3}}{1-\frac{1}{3}} = \frac{3\sqrt{3}}{2} \qquad \Rightarrow \lambda = 3$$

63. **(3)**

Sol. $(1+p)\left(1+3x+9x^2+27x^3+81x^4+243x^5\right)$

$= 1 - p^6$

$\Rightarrow (1+p)\left(1+3x+(3x)^2+(3x)^3+(3x)^4+(3x)^5\right)$

$= 1 - p^6$

$$\Rightarrow (1+p)\frac{\left(1\left((3x)^6-1\right)\right)}{(3x-1)} = 1-p^6;\ \text{for } x \neq 1/3$$

$$\Rightarrow (1+p)\left(\frac{(3x)^6-1}{3x-1}\right) = 1-p^6$$

$$\Rightarrow (1+p)\frac{\left((3x)^3-1\right)\left((3x)^3+1\right)}{(3x-1)} = 1-p^6$$

$$\Rightarrow (1+p)\left(9x^2+1+3x\right)\left((3x)^3+1\right) = \left(1-p^6\right)$$

$$\Rightarrow \left(9x^2+3x+1\right)\left((3x)^3+1\right)$$

$$= \left(1-p^3\right)\left(1+p^2-p\right)$$

By comparison $p = -3x \Rightarrow p/x = -3$

64. **(3)**

Sol. Let roots be α, β, γ and $\alpha = a-d, \beta = a, \gamma = a+d$

then $\alpha + \beta + \gamma = 3a = -(-12) \Rightarrow a = 4$

$\alpha\beta\gamma = a(a^2 - d^2) = -(-28) \Rightarrow d = \pm 3$

65. **(1)**

Sol. $= 2\sin 2° + 4\sin 4° + \ldots + 178\sin 178° + 0$

$= (2\sin 2° + 4\sin 4° + \ldots + 176\sin 4° + 178\sin 2°) + 90$

$= 180\,[\sin 2° + \sin 4° + \ldots + \sin 88°] + 90$

$$= 180\left[\frac{\sin\frac{88°}{2}}{\sin\frac{2°}{2}} \times \sin\left(\frac{2°+88°}{2}\right)\right] + 90$$

$$= 180\left[\frac{\sin 44° \sin 45°}{\sin 1°}\right] + 90$$

$$= 90\,\frac{[\cos 1° - \cos 89°]}{\sin 1°} + 90$$

$= 90\,[\cot 1° - 1] + 90 = 90\cot 1° - 90 + 90$

or sum $= 90\cot 1°$

Since there are 90 terms, therefore average of 90 terms

$$= \frac{90\cot 1°}{90} = \cot 1°$$

66. **(34)**

Sol. $a_1 + a_2 + \ldots. + a_n = 4500$ notes and $a_1 + a_2 + ... + a_{10} = 150 \times 10 = 1500$ notes

$a_{11} + a_{12} + a_{13} + ... + a_n$

$= 4500 - 1500 = 3000$

$\Rightarrow a_{11} + a_{12} + ... + a_n = 3000$

$\Rightarrow 148 + 146 ...... = 3000$

$\Rightarrow \frac{(n-10)}{2} [2 \times 148 + (n - 10 - 1)(-2)]$

$= 3000$

$n = 34, 135$

$a_{34} = 148 + (34 - 1)(-2) = 148 - 66 = 82$

$a_{135} = 148 + (135 - 1)(-2)$

$= 148 - 268 = -120 < 0$

So, answer is 34 minutes are taken

67. (0)

Sol. $\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$

$(\alpha + \beta), \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in G.P

then $(\alpha^2 + \beta^2)^2 = (\alpha + \beta)(\alpha^3 + \beta^3)$

$\Rightarrow 2\alpha^2\beta^2 = \alpha\beta^3 + \beta\alpha^3$

$\Rightarrow \alpha\beta[\alpha^2 + \beta^2 - 2\alpha\beta] = 0$

so, $\alpha\beta(\alpha - \beta)^2 = 0$

$\Rightarrow \frac{c}{a} \cdot \frac{b^2 - 4ac}{a^2} = 0, a \neq 0 \Rightarrow c \cdot \Delta = 0$

68. (35)

Sol. $\frac{3 + 5 + 7 + ...n \text{ terms}}{5 + 8 + 11 + ...10 \text{ terms}} = 7$

$\Rightarrow \frac{n(n+2)}{185} = 7$

$\Rightarrow n^2 + 2n - 1295 = 0$

$\Rightarrow (n + 37)(n - 35) = 0$

$\Rightarrow n = 35$

69. (1)

Sol. $(f(2x))^2 = f(x) . f(4x)$

$\Rightarrow (4x+1)^2 = (2x+1)(8x+1)$

$\Rightarrow 16x^2 + 1 + 8x = 16x^2 + 10x + 1$

$\Rightarrow 2x = 0$

$\Rightarrow x = 0$

70. (4)

Sol. Let no. of terms $= 2n$

According to the question.

sum of all terms = 5 (sum of terms at odd places)

$\Rightarrow \frac{a(r^{2n} - 1)}{r - 1} = 5. \frac{a(r^{2n} - 1)}{r^2 - 1}$

$\Rightarrow r + 1 = 5 \quad \Rightarrow \quad r = 4$

71. (3)

Sol. By AM $\geq$ GM is

$x^{\log y - \log z} + y^{\log z - \log x} + z^{\log x - \log y}$

$\geq 3(x^{\log y - \log z} . y^{\log z - \log x} . z^{\log x - \log y})^{1/3}$

$\Rightarrow x^{\log y - \log z} + y^{\log z - \log x} + z^{\log x - \log y} \geq 3$

72. (12)

Sol. Let the edges are $\frac{a}{r}$, a, ar, where $r > 1$ from the question

$\frac{a}{r} . a. ar = 216 \Rightarrow a = 6$

and $2\left(\frac{a}{r} . a + a. ar + ar. \frac{a}{r}\right) = 252$

$\Rightarrow 72(1 + r^2 + r) = 252r$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow r = 2, \frac{1}{2}$

73. (0)

Sol. AM $\geq$ GM

$\Rightarrow \frac{(2+\sqrt{2})^{x/2} + (2-\sqrt{2})^{x/2}}{2}$

$\geq \left[(2+\sqrt{2})^{x/2}.(2-\sqrt{2})^{x/2}\right]^{1/2}$

$\Rightarrow (2+\sqrt{2})^{x/2} + (2-\sqrt{2})^{x/2} \geq 2(2)^{x/4}$

Equality holds only if

$$(2+\sqrt{2})^{x/2}=(2-\sqrt{2})^{x/2}$$

$\Rightarrow\ x=0$

74. (0)

Sol. $a_1 = 15$

$$\frac{a_k+a_{k-2}}{2}=a_{k-1} \text{ for } k = 3, 4, ..., 11$$

$\Rightarrow\ a_1, a_2, ..., a_{11}$ are in AP

$a_1 = a = 15$

$$\frac{a_1^2+a_2^2+.....+a_{11}^2}{11}=90$$

$$\Rightarrow\ \frac{(15)^2+(15+d)^2+....+(15+10d)^2}{11}=90$$

$\Rightarrow\ 7d^2 + 30d + 27 = 0$

$\Rightarrow\ d = -3 \text{ or } -\frac{9}{7}$

Since $27 - 2a_2 > 0$

$\Rightarrow\ a_2<\frac{27}{2} \qquad \Rightarrow d = -3$

$$\therefore\ \frac{a_1+a_2+....+a_{11}}{11}$$

$$=\frac{11}{2}\frac{[30+10(-3)]}{11}=0$$

75. (11)

Sol. In the given series first term is 1 and common ratio is r.

$$\therefore\ S=\frac{1}{1-r}=2\left(r=\frac{1}{2}\right)$$

S_n is sum of n terms

$$\therefore\ S_n=\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=2-\frac{1}{2^{n-1}}$$

$$S-S_n=\frac{1}{2^{n-1}}$$

$$S-S_n<\frac{1}{1000} \text{ (Given)}$$

$$\therefore\ \frac{1}{2^{n-1}}<\frac{1}{1000}\Rightarrow n=11$$

STRAIGHT LINES

Single Option Correct Type Questions (01 to 60)

1. **(2)**

Sol. $AB = \sqrt{4+9} = \sqrt{13}$

$BC = \sqrt{36+16} = 2\sqrt{13}$

$CD = \sqrt{4+9} = \sqrt{13}$

$AD = \sqrt{36+16} = 2\sqrt{13}$

$AC = \sqrt{64+1} = \sqrt{65}$

$BD = \sqrt{16+49} = \sqrt{65}$

its rectangle

2. **(1)**

Sol. $h = \dfrac{20\cos\theta + 15}{5} = 4\cos\theta + 3$

$k = \dfrac{20\sin\theta}{5} = 4\sin\theta$

Locus is $\left(\dfrac{h-3}{4}\right)^2 + \left(\dfrac{k}{4}\right)^2 = 1$

$(x-3)^2 + y^2 = 16$

3. **(2)**

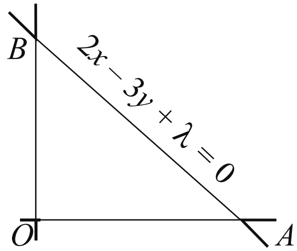
Sol. here $\tan\theta = \dfrac{1}{5}$

$\tan 2\theta = \dfrac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2} = \dfrac{5}{12}$

$\therefore$ required line $y = \dfrac{5x}{12}$

4. **(2)**

Sol.



Any line parallel to $2x - 3y = 4$ is $2x - 3y + \lambda = 0$

$A \equiv \left[-\dfrac{\lambda}{2}, 0\right], B \equiv \left[0, \dfrac{\lambda}{3}\right]$

area of $\Delta AOB = \dfrac{1}{2}\left|\left(\dfrac{\lambda}{2}\right)\left(\dfrac{\lambda}{3}\right)\right| = 12$

$\lambda^2 = 144$

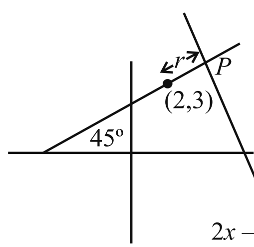
$\Rightarrow \quad \lambda = 12, -12$

line is $2x - 3y + 12$

$= 0$ and $2x - 3y - 12 = 0$

5. **(2)**

Sol.



Let coordinates of point P by parametric form

$P(2 + r\cos 45°, 3 + r\sin 45°)$

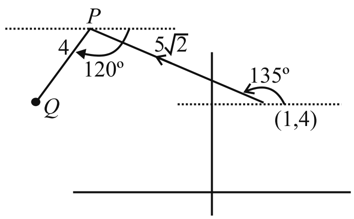
It satisfies the line $2x - 3y + 9 = 0$

$$\therefore 2\left(2+\frac{r}{\sqrt{2}}\right) - 3\left(3+\frac{r}{\sqrt{2}}\right) + 9 = 0$$

$\Rightarrow r = 4\sqrt{2}$

6. **(2)**

Sol.



For point $P \dfrac{x-1}{\cos 135°} = \dfrac{y-4}{\sin 135°} = 5\sqrt{2}$

$\Rightarrow \quad x = -4, y = 9$

For point Q

$$\frac{x+4}{\cos(-120°)} = \frac{y-9}{\sin(-120°)} = 4$$

$\Rightarrow \quad x = -6, y = 9 - 2\sqrt{3}$

$\therefore \quad Q\left(-6,\ 9-2\sqrt{3}\right)$.

7. **(2)**

Sol. condition for (x_1, y_1) & (x_2, y_2) lying on the same side w.r.t. $ax + by + c = 0$

$$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0$$

$$\Rightarrow \frac{1}{a^2 + ab + 1} > 0$$

$a^2 + ab + 1 > 0$ ... (i)

It is quadratic in a

$\therefore$ (i) will be true $\forall\ a \in R$, if

$b^2 - 4 < 0$

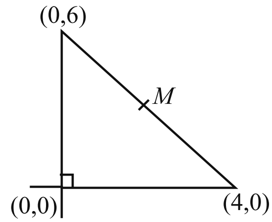
but $b > 0$

$\Rightarrow b \in (0, 2)$

8. **(3)**

Sol. In a right triangle circumcentre is the mid point of the hypotenuse

$$\therefore M \equiv \left(\frac{4+0}{2}, \frac{0+6}{2}\right) \equiv (2, 3)$$



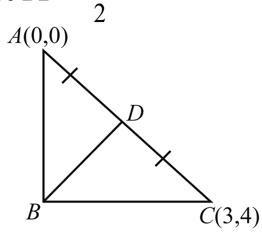
9. **(3)**

Sol.

In ΔABC right angle at B we have

$BD = AD = DC = \dfrac{AC}{2}$

Hence $BD = \dfrac{5}{2}$

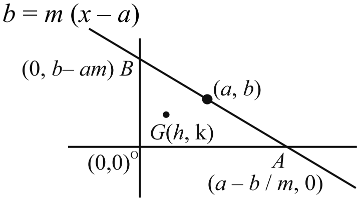


10. **(1)**

Sol.

equation of line AB

$y - b = m(x - a)$



$$\therefore \quad G\left(\frac{a-\frac{b}{m}}{3}, \frac{b-am}{3}\right)$$

$$\Rightarrow \quad h = \frac{a-\frac{b}{m}}{3}, \qquad k = \frac{b-am}{3}$$

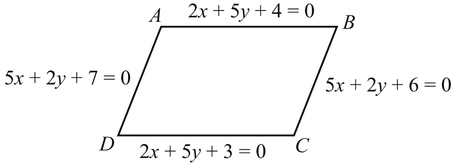
on eleminating 'm' we get required locus

$bh + ak - 3hk = 0$

$\Rightarrow \quad bx + ay - 3xy = 0$

11. **(3)**

Sol. Distance between lines AB & CD = Distance between lines AD and BC



$\Rightarrow$ $ABCD$ is a rhombus, also side AD is not perpendicular to DC hence not a square.

12. **(2)**

Sol.

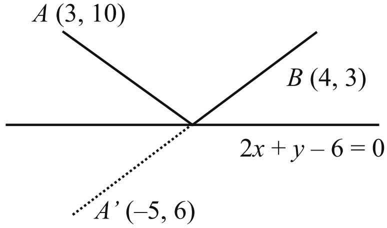


Image of A (3, 10) in $2x + y - 6 = 0$

$$\frac{x-3}{2} = \frac{y-10}{1} = -2\left(\frac{6+10-6}{2^2+1^2}\right)$$

$$\frac{x-3}{2} = \frac{y-10}{1} = -4$$

$A' = (-5, 6)$

Equation of $A'B$ is

$$y - 3 = \left(\frac{6-3}{-5-4}\right)(x-4)$$

$$y - 3 = -\frac{1}{3}(x-4)$$

$3y - 9 = -x + 4$

$\Rightarrow$ $x + 3y - 13 = 0$

13. **(1)**

Sol. $\frac{3a}{8} = -4$

$$a = -\frac{32}{3}$$

$$\frac{5b}{8} = 3$$

$$b = \frac{24}{5}$$

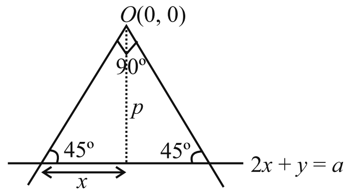
$$-\frac{3x}{32} + \frac{5y}{24} = 1$$

$\Rightarrow$ $-9x + 20y = 96$

$\Rightarrow$ $9x - 20y + 96 = 0$

14. **(3)**

Sol.



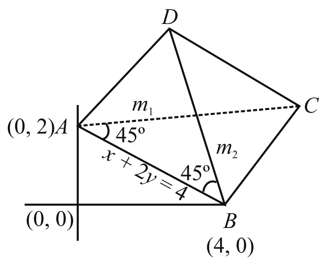
$$p = \left|\frac{0+0-a}{\sqrt{5}}\right| = \frac{a}{\sqrt{5}}$$

$$\tan 45^\circ = \frac{p}{x} \Rightarrow p = x$$

Hence area = $\frac{1}{2}(2x)(p) = px = p^2 = a^2/5$

15. **(3)**

Sol.



$$\tan 45^\circ = \left|\frac{m+\frac{1}{2}}{1-\frac{m}{2}}\right|$$

$$\Rightarrow \quad \pm 1 = \frac{2m+1}{2-m}$$

$$\Rightarrow \quad m = \frac{1}{3}, -3$$

$\therefore$ Equation of AC

$$y - 2 = \frac{1}{3}(x)$$

$\Rightarrow x - 3y + 6 = 0$ .... (i)

Equation of BD

$y = -3(x - 4)$

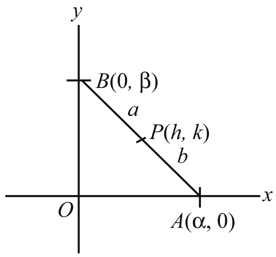
$\Rightarrow 3x + y - 12 = 0$ .... (ii)

From (i) & (ii) ls

$x = 3$ & $y = 3$

16. **(1)**

Sol.



By geometry

$\alpha^2 + \beta^2 = (a + b)^2$ ....(i)

By section formula

$$h = \frac{a\alpha}{a+b}$$

$$\Rightarrow \quad \alpha = \frac{h(a+b)}{a}$$

$$k = \frac{b\beta}{a+b}$$

$$\Rightarrow \quad \beta = \frac{k(a+b)}{b}$$

Put value of α and β in (i)

$$\frac{h^2(a+b)^2}{a^2} + \frac{k^2(a+b)^2}{b^2} = (a+b)^2$$

$$\Rightarrow \quad \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$$

Locus of P is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

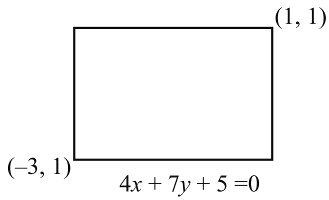
17. **(3)**

Sol. Required point is foot of perpendicular from (0, 0) on the given line which is

$$\frac{\alpha - 0}{3} = \frac{\beta - 0}{4} = \frac{-(-1)}{25}$$

18. **(4)**

Sol.



Line ⊥ to $4x + 7y + 5 = 0$ is

$7x - 4y + \lambda = 0$

It passes through $(-3, 1)$ and $(1, 1)$

$-11 - 4 + \lambda = 0 \Rightarrow \lambda = 25$

$7 - 4 + \lambda = 0 \Rightarrow \lambda = -3$

Hence lines are $7x - 4y + 25 = 0$, $7x - 4y - 3 = 0$

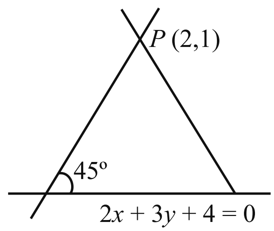
line || to $4x + 7y + 5 = 0$ passing through (1, 1) is $4x + 7y + \lambda = 0$

$\Rightarrow \quad \lambda = -11$

$\Rightarrow \quad 4x + 7y - 11 = 0$

19. **(3)**

Sol.



Let slope of required line is m

Now, $y - 1 = m(x - 2)$

$$\tan 45^\circ = \left|\frac{m + \frac{2}{3}}{1 - \frac{2m}{3}}\right| = \left|\frac{3m+2}{3-2m}\right|$$

$$\Rightarrow \quad \frac{3m+2}{3-2m} = \pm 1$$

$\Rightarrow \quad 3m + 2 = \pm(3 - 2m)$

$\Rightarrow \quad m = \frac{1}{5}, -5$

Hence, $y - 1 = \frac{1}{5}(x - 2) \Rightarrow x - 5y + 3 = 0$

$y - 1 = -5(x - 2) \Rightarrow 5x + y - 11 = 0$

20. **(1)**

Sol.

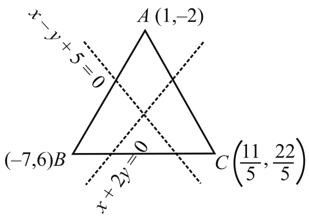


Image of A in $x - y + 5 = 0$ is

$$\frac{x-1}{1} = \frac{y+2}{-1} = -2\left(\frac{1+2+5}{2}\right) = -8$$

$x = -7, y = 6$

Image of $A(1, -2)$ in $x + 2y = 0$

$$\frac{x-1}{1} = \frac{y+2}{2} = -2\left(\frac{1-4}{5}\right) = \frac{6}{5}$$

$$x = \frac{11}{5}, y = \frac{2}{5}$$

Hence equation of BC is

$$y - 6 = \frac{(6-2/5)}{(-7-11/5)}(x+7)$$

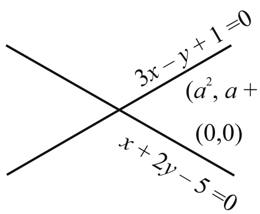
$$y - 6 = \frac{28}{-28}(x+7)$$

$$y - 6 = \frac{-14}{23}(x+7)$$

$\Rightarrow 14x + 23y - 40 = 0$

21. **(2)**

Sol.



Origin, $R(a^2, a + 1)$ lies same side w.r.t. to given lines

$a^2 + 2a + 2 - 5 < 0$

$\Rightarrow \quad a^2 + 2a - 3 < 0$

$\Rightarrow \quad (a + 3)(a - 1) < 0$

$\Rightarrow \quad a \in (-3, 1)$

$3a^2 - (a + 1) + 1 > 0 \Rightarrow 3a^2 - a > 0$

$\Rightarrow \quad a(3a - 1) > 0$

$\Rightarrow a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

take intersection we get $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

22. **(2)**

Sol. Let line be $x + 7y + \lambda = 0$

Distance of this line from

$(1, -1)$ is $= \left|\frac{1-7+\lambda}{\sqrt{50}}\right|$.

As per question $\left|\frac{1-7+\lambda}{\sqrt{50}}\right| = 1$.

$\Rightarrow \quad \lambda = 6 \pm 5\sqrt{2}$

23. **(1)**

Sol. Any point on the line $x + y = 4$ can be taken as $(t, 4 - t)$ the $\perp$ distance of the point $(t, 4 - t)$ from the line $4x + 3y = 10$ is 1

$$\Rightarrow \left|\frac{4t+3(4-t)-10}{5}\right| = 1$$

$$\left|\frac{t+2}{5}\right| = 1$$

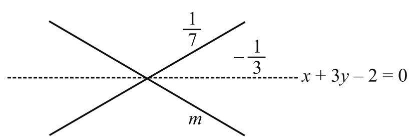
$\Rightarrow \quad |t + 2| = 5$

$\Rightarrow \quad t = 3$ and $t + 2 = -5 \Rightarrow \quad t = -7$

$P \equiv (3, 1)$ and $Q\ (-7, 11)$

24. **(3)**

Sol.



point of intersection of $x + 3y - 2 = 0$ and $x - 7y + 5 = 0$ is $\left(-\frac{1}{10}, \frac{7}{10}\right)$

$$\left(\frac{-\frac{1}{3}-m}{1-\frac{m}{3}}\right) = -\left(\frac{-\frac{1}{3}-\frac{1}{7}}{1-\frac{1}{21}}\right)$$

$$\Rightarrow \quad \frac{-1-3m}{3-m} = \frac{10}{20} = \frac{1}{2}$$

$\Rightarrow \quad -2 - 6m = 3 - m$

$\Rightarrow \quad m = -1$

Hence required equation

$y - \dfrac{7}{10} = -1\left(x + \dfrac{1}{10}\right)$

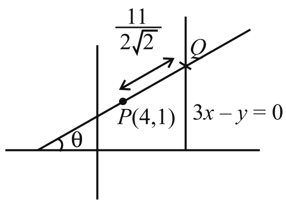
$\Rightarrow \quad 10y - 7 = -10x - 1$

$\Rightarrow \quad 10x + 10y = 6 \Rightarrow 5x + 5y = 3$

25. **(1)**

Sol. By parametric form Q

$\left(4 + \dfrac{11}{2\sqrt{2}}\cos\theta, 1 + \dfrac{11}{2\sqrt{2}}\sin\theta\right)$



it lies on $3x - y = 0$

$\Rightarrow 12 + \dfrac{33}{2\sqrt{2}}\cos\theta - 1 - \dfrac{11}{2\sqrt{2}}\sin\theta = 0$

$\Rightarrow 1 + \dfrac{3}{2\sqrt{2}}\cos\theta - \dfrac{\sin\theta}{2\sqrt{2}} = 0$

$\Rightarrow 3\cos\theta - \sin\theta = -2\sqrt{2}$

squaring both sides

$9\cos^2\theta + \sin^2\theta - 6\sin\theta\cos\theta = 8(\sin^2\theta + \cos^2\theta)$

$\Rightarrow \cos^2\theta - 6\sin\theta\cos\theta - 7\sin^2\theta = 0$ $7\tan^2\theta + 6\tan\theta - 1 = 0$

$\Rightarrow \quad \tan\theta = -1, \dfrac{1}{7}.$

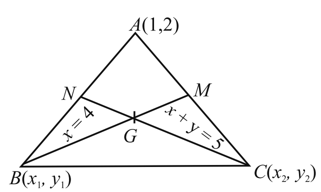
Hence required line are $x + y = 5$, $x - 7y + 3 = 0$

26. **(4)**

Sol. If H is orthocentre of triangle ABC, then orthocentre of triangle BCH is point A

27. **(2)**

Sol.



$x_1 + y_1 = 5$ ... (i)

$x_2 = 4$ ... (ii)

co - ordinates of G are $\equiv (4, 1)$

$\Rightarrow \quad \dfrac{1 + x_1 + x_2}{3} = 4$ ....(iii)

and $\dfrac{y_1 + y_2 + 2}{3} = 1$ ... (iv)

solving above equations, we get B & C.

28. **(4)**

Sol. $a^2 + 9b^2 - 4c^2 = 6ab$

then $a^2 + 9b^2 - 6ab = 4c^2$

$(a - 3b)^2 = (2c)^2$

$a - 3b = 2c$ and $a - 3b = -2c$

line $ax + by + c = 0$ is concurrent at

$ax + by + \left(\dfrac{a - 3b}{2}\right) = 0$ and $ax + by + \left(\dfrac{3b - a}{2}\right) = 0$

$x = -\dfrac{1}{2}$; $y = \dfrac{3}{2}$ and $x = \dfrac{1}{2}$; $y = -\dfrac{3}{2}$

$P\left(-\dfrac{1}{2}, \dfrac{3}{2}\right)$ and $\left(\dfrac{1}{2}, -\dfrac{3}{2}\right)$

29. **(2)**

Sol. S_1 is true because given quadrilateral is a rhombus.

S_2 is also standard rule but S_2 does not explain S_1.

30. **(3)**

Sol.

equation of line $y - 2 = m(x - 8)$ where $m < 0$

$\Rightarrow P \equiv \left(8 - \dfrac{2}{m}, 0\right)$ and $Q \equiv (0, 2 - 8m)$

Now $OP + OQ = 8 - \dfrac{2}{m} + 2 - 8m$

$= 10 + \dfrac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\dfrac{2}{-m} \times 8 \ (-m)}$

≥ 18

Statement 2 is false when number are negative

31 **(1)**

Sol.

(A) $AH \perp BC. \Rightarrow \left(\dfrac{k}{h}\right)\left(\dfrac{3-2}{-2-5}\right) = -1$

$4k = 7h$

$BH \perp AC. \Rightarrow \left(\frac{0+1}{0-5}\right)\left(\frac{k-3}{h+2}\right) = -1$

$k - 3 = 5(h + 2)$

$\Rightarrow$ $7h - 12 = 20h + 40$

$13h = -52$

$h = -4 \therefore k = -7$

$\therefore$ $A(-4, -7)$

(B) $x + y - 4 = 0$

$4x + 3y - 10 = 0$

Let $(h, 4 - h)$ be the point on(i)

Then $\left|\frac{4h + 3(4-h) - 10}{5}\right| = 1$

i.e. $h + 2 = \pm 5$ i.e. $h = 3$; $h = -7$

$\therefore$ reuired point is either (3, 1) or (–7, 11)

(C) orthocentre of the triangle is the point of intersection of the lines

$x + y - 1 = 0$ and $x - y + 3 = 0$

i .e., (–1, 2)

(D) Since a, b, c are in $A.P.$

$\therefore$ $b = \frac{a+c}{2}$

$\therefore$ the family of lines is $ax + \frac{a+c}{2}y = c$

i.e. $a\left(x + \frac{y}{2}\right) + c\left(\frac{y}{2} - 1\right) = 0$

$\therefore$ point of concurrency is (–1, 2)

32. **(1)**

Sol. For concurrency

(A) $\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix} = 12\,k + 2 - 3\,(-3i) - 5$

$(6 + 5k) = 0$

$\Rightarrow$ $-13k + 2 + 93 - 30 = 0$

$\Rightarrow$ $-13k + 65 = 0 \Rightarrow k = 5$

(B) For L_1 & L_2 to be parallel,

$\frac{1}{3} = \frac{3}{-k} \Rightarrow k = -9.$

Also, $\frac{3}{5} = \frac{-k}{2}$ for L_2, L_3 to be parallel

$\Rightarrow$ $k = -\frac{6}{5}$

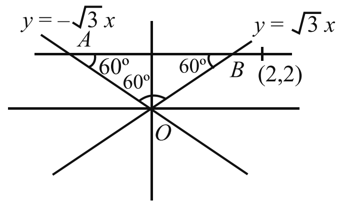
(C) They form a triangle when lines are non-concurrent & non-parallel.

Hence $k = \frac{5}{6}$ from the given options.

(D) L_1, L_2, L_3 will not from a triangle when they are concurrent or any two of them are parallel.

33. **(2)**

Sol.



$y - 2 = m(x - 2)$

$\tan 60^\circ = \left|\frac{m - \sqrt{3}}{1 + m\sqrt{3}}\right|$

$\frac{m - \sqrt{3}}{1 + m\sqrt{3}} = \pm\sqrt{3} \Rightarrow m - \sqrt{3} = \pm(\sqrt{3} + 3m)$

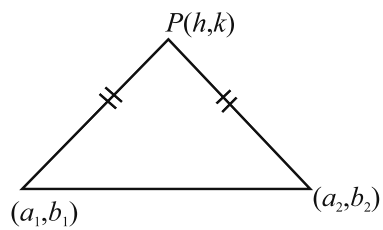
$\Rightarrow m = -\sqrt{3}, 0$

Hence, $y - 2 = 0$

$y - 2 = -\sqrt{3}\,(x - 2).$

34. **(1)**

Sol.



$(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$2h(a_1 - a_2) + 2k(b_1 - b_2) +$

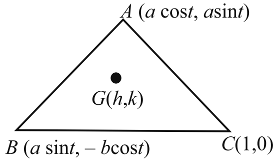
$\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right) = 0$

compare with $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$

$$c = \frac{\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right)}{2}.$$

35. **(2)**

Sol.



$3h - 1 = a\cos t + b\sin t$

$3k = a\sin t - b\cos t$

squaring and add. (Locus)

$(3x - 1)^2 + 9y^2 = a^2 + b^2$

36. **(1)**

Sol. $G\left(\frac{h}{3}, \frac{k-2}{3}\right)$

$\Rightarrow \frac{2h}{3} + (k - 2) = 1 \Rightarrow 2h + 3k = 9$

Locus $2x + 3y = 9$.

37. **(4)**

Sol. Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

it passes through (4, 3) $\frac{4}{a} + \frac{3}{b} = 1$

sum of intercepts is –1

$\Rightarrow a + b = -1 \Rightarrow a = -1 - b$

$\Rightarrow \frac{4}{-1-b} + \frac{3}{b} = 1$

$\Rightarrow 4b - 3 - 3b = -b - b^2$

$\Rightarrow b^2 + 2b - 3 = 0$

$\Rightarrow b = -3, 1$

$b = 1, a = -2$ $\frac{x}{-2} + \frac{y}{1} = 1$

$b = -3, a = 2$ $\frac{x}{2} + \frac{y}{-3} = 1.$

38. **(4)**

Sol. $ax + 2by + 3b = 0$

$bx - 2ay - 3a = 0$

$$\frac{x}{-6ab + 6ab} = \frac{y}{3b^2 + 3a^2} = \frac{1}{-2a^2 - 2b^2}$$

Hence point of intersection (0, –3/2)

Line parallel to x-axis $y = -3/2$.

39. **(2)**

Sol. $\because$ a, b, c are in H.P. $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

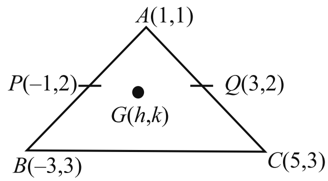
$\Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

given line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$

Clearly line passes through (1, –2).

40. **(2)**

Sol. Centroid is $\left(1, \frac{7}{3}\right)$



41. **(2)**

Sol. Let equation of line is

$\frac{x}{a} + \frac{y}{b} = 1.$

By section formula

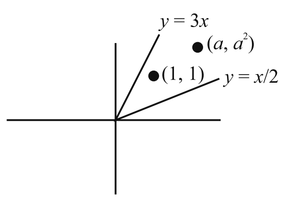
$\frac{a}{2} = 3 \Rightarrow a = 6$

$\frac{b}{2} = 4 \Rightarrow b = 8$

$\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24.$

42. **(2)**

Sol.



Since (1, 1) and (a, a^2) Both lies same side with respect to both lines

$a - 2a^2 < 0 \Rightarrow 2a^2 - a > 0$

$\Rightarrow a(2a-1) > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$

$3a - a^2 > 0 \Rightarrow a^2 - 3a < 0 \Rightarrow a \in (0, 3)$

Hence after taking intersection $a \in \left(\frac{1}{2}, 3\right)$.

43. **(3)**

Sol. $AB = \sqrt{(h-1)^2 + (k-1)^2}$

$BC = 1$

$AC = \sqrt{(h-2)^2 + (k-1)^2}$

$AB^2 + BC^2 = AC^2 \Rightarrow (h-1)^2 + (k-1)^2 + 1 = (h-2)^2 + (k-1)^2$

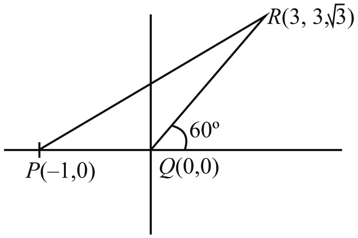
$\Rightarrow 2h = 2 \Rightarrow h = 1$

Area of $\Delta ABC = \frac{1}{2}\sqrt{(h-1)^2 + (k-1)^2} \times 1 = 1$

$(K-1)^2 = 4 \Rightarrow k - 1 = \pm 2 \Rightarrow k = 3, -1.$

44. **(1)**

Sol.



The line segment QR makes an angle 60º with the positive direction of x-axis.
hence bisector of angle PQR will make 120º with +ve direction of x-axis.
Its equation

$y - 0 = \tan 120^\circ (x - 0)$

$y = -\sqrt{3}x$

$x\sqrt{3} + y = 0$

45. **(1)**

Sol. $p(p^2+1)x - y + q = 0$

$(p^2+1)^2 x + (p^2+1) y + 2q = 0$ are perpendicular to a common line

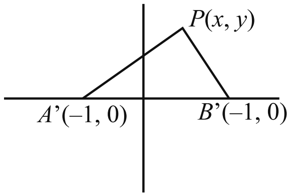
$\Rightarrow$ lines are parallel

$\Rightarrow$ slopes are equal

$\therefore \frac{p(p^2+1)}{1} = -\frac{(p^2+1)^2}{(p^2+1)} \Rightarrow p = -1$

46. **(1)**

Sol.



$\therefore \frac{PA'}{PB'} = \frac{3}{1}$

$\therefore (x+1)^2 + y^2 = 9((x-1)^2 + y^2)$

$x^2 + 2x + 1 + y^2 = 9x^2 + 9y^2 - 18x + 9$

$8x^2 + 8y^2 - 20x + 8 = 0$

$x^2 + y^2 - \frac{10}{4}x + 1 = 0$

$\therefore$ circumcentre is $\left(\frac{5}{4}, 0\right)$.

47. **(3)**

Sol. $\frac{x}{5} + \frac{y}{b} = 1$

$\frac{13}{5} + \frac{32}{b} = 1$

$\Rightarrow \frac{32}{b} = -\frac{8}{5}$

$\Rightarrow b = -20$

$\frac{x}{5} - \frac{y}{20} = 1$

$\Rightarrow 4x - y = 20$

Line K has same slope $\Rightarrow \frac{-3}{c} = 4$

$c = -\frac{3}{4}$

$\Rightarrow 4x - y = -3$

distance $= \frac{23}{\sqrt{17}}$

Hence correct option is (3)

48. **(2)**

Sol. $x + y = |a|$

$ax - y = 1$

if $a > 0$

$x + y = a$

$ax - y = 1$

$x(1 + a) = 1 + a \Rightarrow x = 1$

$y = a - 1$

It is in the first quadrant

so $a - 1 \geq 0$

$a \geq 1$

$a \in [1, \infty)$

If $a < 0$

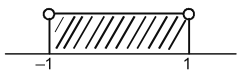
$x + y = -a$

$ax - y = 1$

+

$x(1 + a) = 1 - a$

$x = \dfrac{1-a}{1+a} > 0 \Rightarrow \dfrac{a-1}{a+1} < 0$

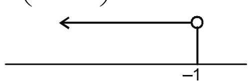


.............(1)

$y = -a - \dfrac{1-a}{1+a}$

$= \dfrac{-a-a^2-1+a}{1+a} > 0$

$-\left(\dfrac{a^2+1}{a+1}\right) > 0 \Rightarrow \dfrac{a^2+1}{a+1} < 0$

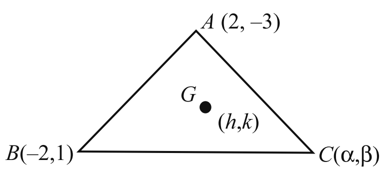


.............(2)

from (1) and (2) $a \in \{\phi\}$

49. **(2)**

Sol.



$\alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

third vertex on the line $2x + 3y = 9$

$2\alpha + 3\beta = 9$

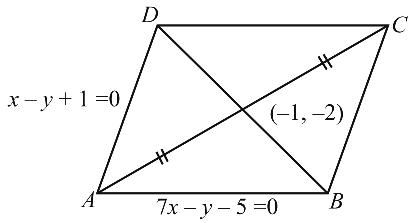
$2(3h) + 3(3k + 2) = 9$

$2h + 3k = 1$

$2x + 3y - 1 = 0$

50. **(2)**

Sol.



On solving equation of AB & AD

vertex A(1, 2)

$\because$ P is mid point of AC. Hence vertex C is (–3, –6).

So equation of other two sides are $7x - y + 15 = 0$ and $x - y - 3 = 0$.

Hence other vertices are $\left(\dfrac{1}{3}, -\dfrac{8}{3}\right)$ and $\left(-\dfrac{7}{3}, -\dfrac{4}{3}\right)$

51. **(4)**

Sol.

$$\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$$

$k(k - 2) - 5(-3k - 2) - k(-3k - k) = \pm 56$

$k^2 - 2k + 15k + 10 + 3k^2 + k^2 = \pm 56$

$5k^2 + 13k + 10 \pm 56 = 0$

$5k^2 + 13k + 66 = 0$ or $5k^2 + 13k - 46 = 0$

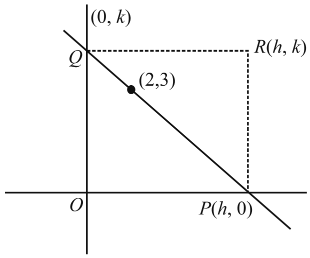
No solution or $k = \dfrac{-13 \pm \sqrt{169+920}}{10}$

$k = \dfrac{-13 \pm 33}{10} \Rightarrow k = 2$ or $k = -\dfrac{46}{10}$ (which is not an integer)

$\therefore$ vertices $A(2, -6)$, $B(5,2)$, $C(-2,2)$

Equation of altitude dropped from vertex A is
$x = 2$ ..... (i)
Equation of altitude dropped from vertex C is
$3x + 8y - 10 = 0$ ....(ii)
solving both (i) and (ii)
orthocentre $\left(2, \frac{1}{2}\right)$

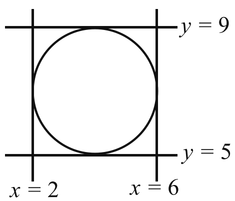
52. **(1)**
Sol.



$$\begin{vmatrix} 0 & k & 1 \\ 2 & 3 & 1 \\ h & 0 & 1 \end{vmatrix} = 0$$

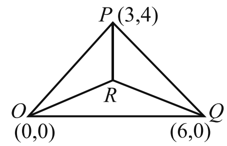
$- k(2 - h) + 1(-3h) = 0$
$-2y + xy - 3x = 0$
$3x + 2y = xy$

53. **(1)**
Sol.



The lines given by $x^2 - 8x + 12 = 0$ are $x = 2$ and $x = 6$.
The lines given by $y^2 - 14y + 45 = 0$ are $y = 5$ and $y = 9$
Centre of the required circle is the centre of the square.
$\therefore$ Required centre is
$$\left(\frac{2+6}{2}, \frac{5+9}{2}\right) = (4, 7).$$

54. **(3)**
Sol.



R is centroid hence $R \equiv \left(3, \frac{4}{3}\right)$

55. **(2)**
Sol. Let slope of line $L = m$

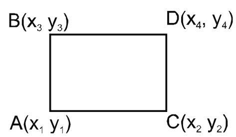
$$\therefore \left|\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}\right| = \tan 60^o = \sqrt{3}$$

$$\Rightarrow \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$$

taking positive sign, $m + \sqrt{3} = \sqrt{3} - 3m$
$m = 0$
taking negative sign
$m + \sqrt{3} + \sqrt{3} - 3m = 0$
$m = \sqrt{3}$
As L cuts x-axis
$\Rightarrow m = \sqrt{3}$
so L is $y + 2 = \sqrt{3}(x - 3)$

56. **(1)**
Sol. $x\cos\theta - y\sin\theta = \lambda = a\cos^4\theta - \sin^4\theta = a\cos 2\theta$

57. **(2)**
Sol.



(x_1, y_1), $(x_2\ y_2)$, (x_3, y_3) and (x_4, y_4)
Given $\sum_{i=1}^{4}(x_i^2 + y_i^2) \le 2(x_1 x_3 + x_2x_4 + y_1y_2 + y_3y_4)$
$x_1^2 + y_1^2 + x_2^2 + y_2^2 + x_3^2 + y_3^2 + x_4^2 + y_4^2 \le 2(x_1x_3 + x_2 x_4 + y_1y_2 + y_3y_4)$
$(x_1^2 + x_3^2 - 2x_1x_3) + (x_2^2 + x_4^2 - 2x_2 x_4) + (y_1^2 + y_2^2 - 2y_1y_2) + (y_3^2 + y_4^2 - 2y_3 y_4) \le 0$
$(x_1 - x_3)^2 + (x_2 - x_4)^2 + (y_1 - y_2)^2 + (y_3 - y_4)^2 \le 0$

Only possible when $x_1 = x_3$; $x_2 = x_4$
$y_1 = y_2$; $y_3 = y_4$
hence it is a rectangle

58. **(1)**

Sol. The x-coordinate of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is $x = \frac{5}{3+4m}$

For x being an integer 3 + 4m should be divisor of 5

i.e. 1, –1, 5 or ; k –5

$3 + 4m = 1 \Rightarrow m = -\frac{1}{2}$ (Not integer)

$4m + 3 = -1 \Rightarrow m = -1$ (Interger)

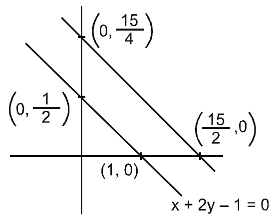
$3 + 4m = 5 \Rightarrow m = -\frac{1}{2}$ (Not an integer)

$3 + 4m = -5 \Rightarrow m = -2$ (integer)

$\therefore$ there are two integral value of m

59. **(1)**

Sol.



Point $P\left(1+\frac{t}{\sqrt{2}},\ 2+\frac{t}{\sqrt{2}}\right)$ lies between given line

Hence $L_1(P) = \left(1+\frac{t}{\sqrt{2}}\right) + 2\left(2+\frac{t}{\sqrt{2}}\right) - 1 > 0$

$5 + \frac{3t}{\sqrt{2}} - 1 > 0 \Rightarrow t > -\frac{4\sqrt{2}}{3}$

Now, $L_2(P) = 2\left(1+\frac{t}{\sqrt{2}}\right) + 4\left(2+\frac{t}{\sqrt{2}}\right) - 15 < 0$

$\Rightarrow 10 + \frac{6t}{\sqrt{2}} - 15 < 0 \Rightarrow t < \frac{5\sqrt{2}}{6}$

Hence $t \in \left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}\right)$.

60. **(4)**

Sol. (i) After reflection about line $y = x$ position of point will be (1, 4)

(ii) After this step (4, 4)

(iii) $h = 4\sqrt{2}\cos 150°$, $k = 4\sqrt{2}\sin 150°$

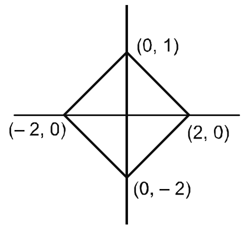
$h = -2\sqrt{6}$, $k = 2\sqrt{2}$

Integer Type Questions (61 to 70)

61. **(8)**

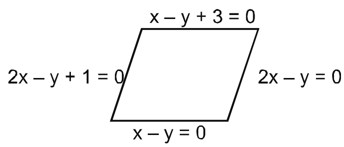
Sol. $|x| + |y| = 2$ represent a square of side = $2\sqrt{2}$

Hence area = 8



62. **(3)**

Sol.



Area of parallogram = $\left|\frac{(3-0)(1-0)}{\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}}\right| = 3$

63. **(2)**

Sol. $12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0$

$\Delta = 0$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$12.2.k + 2.\left(-\frac{5}{2}\right)\left(\frac{11}{2}\right)(-5) - 12\left(\frac{25}{4}\right) - 2\left(\frac{121}{4}\right) - k(25) = 0$

$\Rightarrow \quad k = 2$

64. **(2)**

Sol. $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$

$(x + \sqrt{2}y + p)(x + \sqrt{2}y + q) = 0$

$p + q = 4$

$pq = 1$

Distance between || lines is

$$\left|\frac{p-q}{\sqrt{3}}\right| = \frac{\sqrt{(p+q)^2 - 4pq}}{\sqrt{3}} = \frac{\sqrt{16-4}}{\sqrt{3}} = 2$$

65. **(3)**

Sol. Pair $6x^2 - xy + 4cy^2 = 0$ has its one line $3x + 4y = 0$

Putting $y = \frac{-3x}{4}$, $6x^2 + \frac{3x^2}{4} + 4c\,\frac{9x^2}{16} = 0$

$\Rightarrow 24x^2 + 3x^2 + 9cx^2 = 0$

66. **(4)**

Sol. Slope of $PQ = \frac{1}{1-k}$

Hence equation of line $\perp$ to PQ line

$y - \frac{7}{2} = (k-1)\left(x - \frac{(1+k)}{2}\right)$

Put $x = 0$

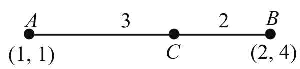
$y = \frac{7}{2} + \frac{(1-k)\ (1+k)}{2} = -4$

$7 + (1 - k^2) = -8 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4.$

Hence possible answer = –4.

67. **(6)**

Sol.



$\therefore C\left(\frac{8}{5}, \frac{14}{5}\right)$

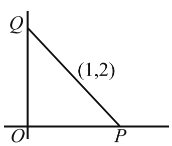
Line $2x + y = k$ passes $C\left(\frac{8}{5}, \frac{14}{5}\right)$

$\frac{2\times 8}{5} + \frac{14}{5} = k$

$k = 6$

68. **(2)**

Sol.



$(y - 2) = m(x - 1)$

$OP = 1 - \frac{2}{m}$

$OQ = 2 - m$

Area of $\Delta POQ = \frac{1}{2}(OP)(OQ)$

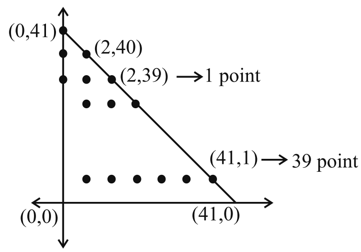
$= \frac{1}{2}\left(1 - \frac{2}{m}\right)(2 - m)$

$= \frac{1}{2}\left[2 - m - \frac{4}{m} + 2\right]$

$= \frac{1}{2}\left[4 - \left(m + \frac{4}{m}\right)\right]$

69. **(780)**

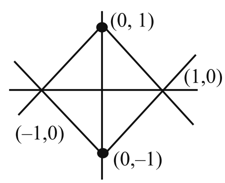
Sol.



$1 + 2 + \ldots\ldots 39 = \frac{39}{2}(39 + 1) = 780$

70. **(2)**

Sol.



$y = |x| - 1$

$y = -|x| + 1$

Region is clearly square with vertices at the

point (1,0), (0,1), (–1,0), (0,–1). So,

its area = $\sqrt{2}\times\sqrt{2} = 2$.

CIRCLES

Single Option Correct Type Questions (01 to 60)

1. **(3)**

Sol: $q = 2, p = 3$

Now, $3x^2 + 3y^2 - 12x + 30y + 12 = 0$

2. **(4)**

Sol: Centre (2, 3)

midpoint of intercept on x-axis is (2, 0) and on y-axis (0, 3)

equation of line

$$\frac{x}{2} + \frac{y}{3} = 1$$

3. **(2)**

Sol: Equation of circle $(x-0)(x-a) + (y-1)(y-b) = 0$

it cuts x-axis put $y = 0 \Rightarrow \quad x^2 - ax + b = 0$

4. **(1)**

Sol: Point on the line $x + y + 13 = 0$ nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$ is foot of $\perp$ from centre

$$\frac{x+2}{1} = \frac{y+3}{1} = -\left(\frac{-2-3+13}{1^2+1^2}\right) = -4$$

$x = -6 \qquad y = -7$

5. **(1)**

Sol: Centre (6, 2), $r = \sqrt{10}$

$OC = \sqrt{36+4} = 2\sqrt{10}$

slope line $OC = 1/3$

Let point be (x, y)

$$= \frac{x-6}{\frac{3}{\sqrt{10}}} = \frac{y-2}{\frac{1}{\sqrt{10}}} = \sqrt{10}$$

$\Rightarrow x = 6 + 3 = 9$

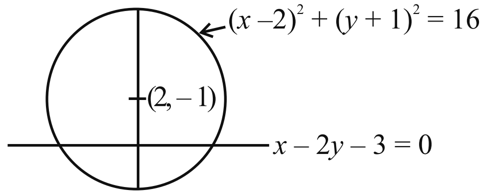
$y = 2 + 1 = 3$

6. **(2)**

Sol: Required diameter is $\perp$ to given line.

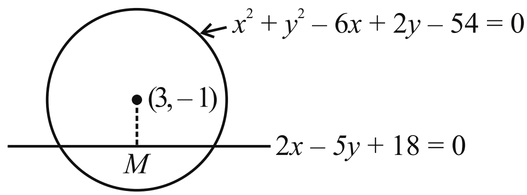
Hence $y + 1 = -2(x - 2)$

$\Rightarrow 2x + y - 3 = 0$



7. **(1)**

Sol: Required point is foot of $\perp$

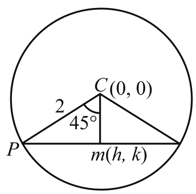


$$\frac{x-3}{2} = \frac{y+1}{-5} = -\left(\frac{6+5+8}{4+25}\right) = -1$$

$x = 1, y = 4$

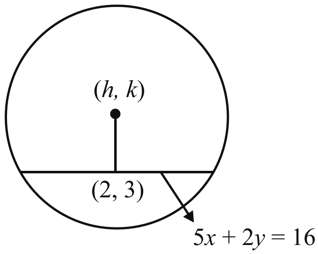
8. **(3)**

Sol: $\cos 45° = \frac{cm}{cp} = \frac{\sqrt{h^2+k^2}}{2}$



9. **(1)**

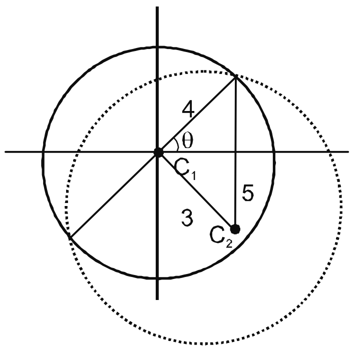
Sol:



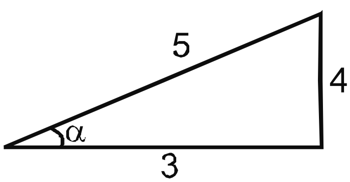
$$\frac{k-3}{h-2} = \frac{2}{5} \Rightarrow \quad 2x - 5y + 11 = 0$$

10. **(2)**

Sol:



slope of C_1C_2 is $\tan\alpha = -\frac{4}{3}$



By using parametric coordinates C_2 ($\pm 3\cos\alpha$, $\pm 3\sin\alpha$)

C_2 (± 3 (–3/5), ± 3 (4/5))

C_2 ($\pm$ 9/5, 12/5)

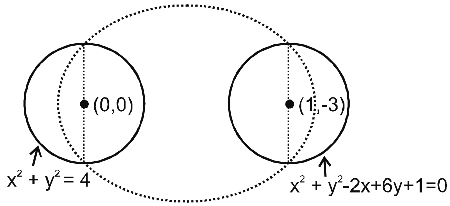
11. **(1)**

Sol: Let required circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Hence common chord with $x^2 + y^2 - 4 = 0$

is $2gx + 2fy + c + 4 = 0$

This is diameter of circle $x^2 + y^2 = 4$ hence $c = -4$.



Now again common chord with other circle

$2x(g + 1) + 2y(f - 3) + (c - 1) = 0$

This is diameter of $x^2 + y^2 - 2x + 6y + 1 = 0$

$2(g + 1) - 6(f - 3) - 5 = 0$

$2g - 6f + 15 = 0$

locus $2x - 6y - 15 = 0$ which is st. line.

12. **(1)**

Sol: Common chord of given circle

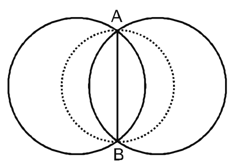
$2x + 3y - 1 = 0$

family of circle passing through point of intersection of given circle

$(x^2 + y^2 + 2x + 3y - 5) + \lambda(x^2 + y^2 - 4) = 0$

$(\lambda + 1)x^2 + (\lambda + 1)y^2 + 2x + 3y - (4\lambda + 5) = 0$

$$x^2 + y^2 + \frac{2x}{\lambda+1} + \frac{3}{\lambda+1}y - \frac{(4\lambda+5)}{\lambda+1} = 0$$



centre $\left(-\frac{1}{\lambda+1}, \frac{-3}{2(\lambda+1)}\right)$

13. **(3)**

Sol: $S + \lambda L = 0$

$(x - 1)(x - 3) + (y - 1)(y - 3) + \lambda(x - y) = 0$

$x^2 + y^2 - x(4 - \lambda) - y(4 + \lambda) + 6 = 0$

Centre $\left(\frac{4-\lambda}{2}, \frac{4+\lambda}{2}\right)$

$\therefore \quad \frac{4+\lambda}{2} = 0$

$\lambda = -4$

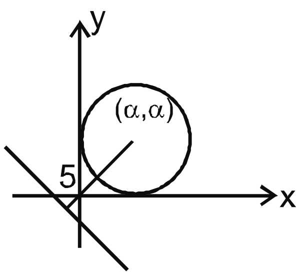
$\therefore\ x^2 + y^2 - 8x + 6 = 0$

14. **(2)**

Sol: Let centre be (α, α) & $\alpha > 0$

$\alpha > 0$

$\Rightarrow 5 = \left|\dfrac{3\alpha + 4\alpha + 11}{5}\right| \qquad \Rightarrow 7\alpha + 11 = \pm 25$



$7\alpha = 14, 7\alpha = -36$

$\alpha = 2, \alpha = \dfrac{-36}{7}$

so circle

$(x-2)^2 + (y-2)^2 = 4$

15. **(1)**

Sol: $x^2 + y^2 - 10x + \lambda(2x - y) = 0$ ....(i)

$x^2 + y^2 + 2x(\lambda - 5) - \lambda y = 0$

Centre $(-(\lambda - 5), \lambda/2)$

Using on $y = 2x$

$\dfrac{\lambda}{2} = -2(\lambda - 5)$

$\dfrac{5\lambda}{2} = 10$

Putting $\lambda = 4$

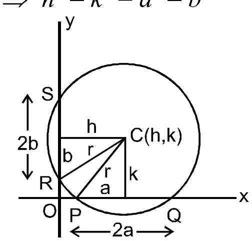
$x^2 + y^2 - 2x - 4y = 0$

16. **(2)**

Sol: $h^2 + b^2 = r^2$

$k^2 + a^2 = r^2$

$\Rightarrow h^2 - k^2 = a^2 - b^2$



$\therefore$ locus is $x^2 - y^2 = a^2 - b^2$

17. **(2)**

Sol: Let equation be $x^2 + y^2 + 2gx + 2fy + g^2 = 0$

Which passes through $(1, -2)$ & $(3, -4)$

$\Rightarrow g = -3, 5$ and $f = 2, 10$

18. **(3)**

Sol: Point $\left(t, \dfrac{1}{t}\right)$ lies on $x^2 + y^2 = 16$

$t^2 + \dfrac{1}{t^2} = 16$

$\Rightarrow \quad t^4 - 16t^2 + 1 = 0$ ....(i)

If roots are t_1, t_2, t_3, t_4 then

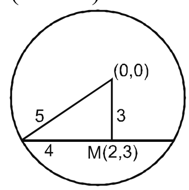
$t_1t_2t_3t_4 = 1$ ....(ii)

19. **(2)**

Sol: Let slope of required line is m

$y - 3 = m(x - 2)$

$\Rightarrow mx - y + (3 - 2m) = 0$



length of $\perp$ from origin

$= 3$

$\Rightarrow 9 + 4m^2 - 12m = 9 + 9m^2$

$\Rightarrow 5m^2 + 12m = 0 \Rightarrow m = 0, -\dfrac{12}{5}$

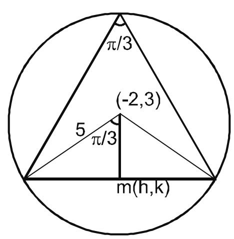
Hence lines are $y - 3 = 0 \Rightarrow y = 3$

and $y - 3 = -\dfrac{12}{5}(x - 2) \Rightarrow 5y - 15 = -12x + 24$

$\Rightarrow 12x + 5y = 39.$

20. **(2)**

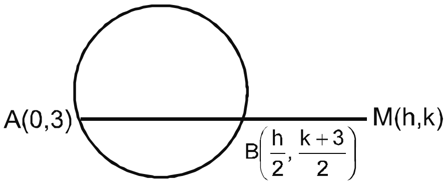
Sol: $\cos \pi/3 = \dfrac{\sqrt{(h+2)^2 + (k-3)^2}}{5}$



Locus $(x+2)^2 + (y-3)^2 = 6.25$

21. **(2)**

Sol:



B lies on circle

$$\left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0$$

$$\Rightarrow \frac{h^2}{4} + 2h + \frac{(k-3)^2}{4} = 0$$

Hence locus of (h, k) $x^2 + 8x + (y - 3)^2 = 0$

22. **(1)**

Sol: Two fixed pts. are point of intersection of

$x^2 + y^2 - 2x - 2 = 0$ & $y = 0$

$x^2 - 2x - 2 = 0$

$(x - 1)^2 - 3 = 0$

$x - 1 = -\sqrt{3}$, $\quad x - 1 = -\sqrt{3}$

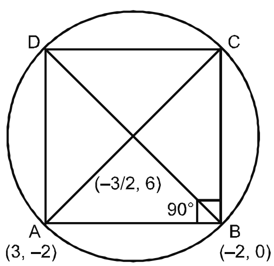
$(1+\sqrt{3},\ 0)$, $\quad (1-\sqrt{3},\ 0)$

23. **(4)**

Sol: Statement-1 is false as 3 points are collinear statement-2 is true.

24. **(1)**

Sol: The centre $\left(-\frac{3}{2}, -6\right)$ is mid-poins of diameter AC and BD



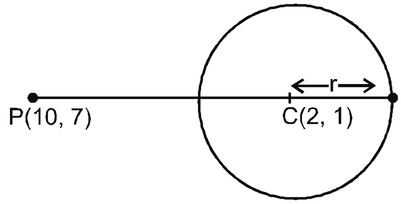
$\therefore$ C is $(p, q) = (-6, -10)$

$\therefore$ D is $(r, s) = (-1, -12)$

25. **(2)**

Sol: $S_1 \equiv 10^2 + 7^2 - 4 \times 10 - 2 \times 7 - 20 = 75 > 0$

Point (10, 7) lies out side the circle $x^2 + y^2 - 4x - 2y - 20 = 0$



greatest distance = $CP + r$

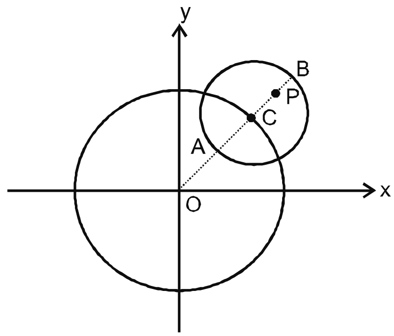
$= 10 + 5$

$= 15$ Unit.

26. **(1)**

Sol: For any point $P(x, y)$ in the circle

$OA \le OP \le OB$



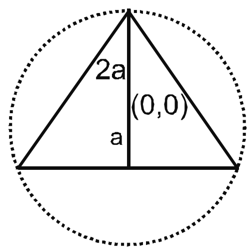
$$5 - 3 \le \sqrt{x^2 + y^2} \le 5 + 3$$

$4 \le x^2 + y^2 \le 64$

27. **(2)**

Sol: Since its equilateral triangle

Hence circum-centre is also centroid



which divides median in 2 : 1, therefore its radius is $2a$.

$x^2 + y^2 = 4a^2$.

28. **(1)**

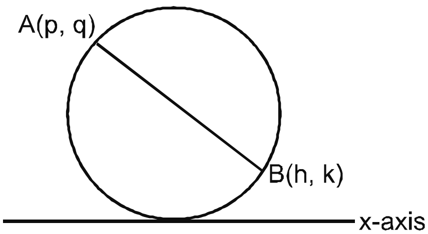
Sol: S_1: $(x - 1)^2 + (y - 3)^2 = r^2$ C_1 (1, 3), $r_1 = r$

S_2: $x^2 + y^2 - 8x + 2y + 8 = 0$ $C_2\,(4, -1)$, $r_2 = 3$
circles intersect $|r_1 - r_2| < C_1 C_2 < r_1 + r_2$
$|r - 3| < 5 < r + 3$
$\Rightarrow |r - 3| < 5 \Rightarrow -5 < r - 3 < 5 \Rightarrow -2 < r < 8$
$5 < r + 3 \Rightarrow r > 2$
After intersection $2 < r < 8$.

29. **(1)**

Sol: equation of circle
$(x - p)(x - h) + (y - q)(y - k) = 0$
$\Rightarrow x^2 + y^2 - x(h + p) - y(q + k) + (ph + qk) = 0$

This circle touches x-axis $g^2 = c$



$$\Rightarrow \left(\frac{h+p}{2}\right)^2 = ph + qk$$

Locus of (h, k) is $(x - p)^2 = 4qy$.

30. **(1)**

Sol: Point of intersection of $2x + 3y + 1 = 0$
$3x - y - 4 = 0$ is $(1, -1)$
and circumference of circle $= 2\pi r = 10\pi$ $\Rightarrow r = 5$
Hence equation of circle $(x - 1)^2 + (y + 1)^2 = 25$
$\Rightarrow x^2 + y^2 - 2x + 2y - 23 = 0$.

31. **(1)**

Sol: By family of circle $x^2 + y^2 - 2x + \lambda(x - y) = 0$
centre of this circle $\left(\frac{2-\lambda}{2}, \frac{\lambda}{2}\right)$
lies on $y = x \Rightarrow \frac{2-\lambda}{2} = \frac{\lambda}{2} \Rightarrow \lambda = 1$
Hence $x^2 + y^2 - x - y = 0$.

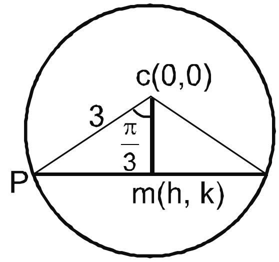
32. **(3)**

Sol: Let $S_1: x^2 + y^2 + 2ax + cy + a = 0$
$S_1: x^2 + y^2 - 3ax + dy - 1 = 0$
common chord $S_1 - S_2 = 0 \Rightarrow 5ax + y(c - d) + (a + 1) = 0$
given line is $5x + by - a = 0$
compare both $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$
$a = \frac{c-d}{b} = -1 - \frac{1}{a}$
From (i) & (iii) $a^2 + a + 1 = 0 \Rightarrow a = \omega, \omega^2$ no real a.

33. **(3)**

Sol: $\cos\frac{\pi}{3} = \frac{cm}{cp} = \frac{\sqrt{h^2+k^2}}{3}$

Locus of (h, k) is $x^2 + y^2 = \frac{9}{4}$.

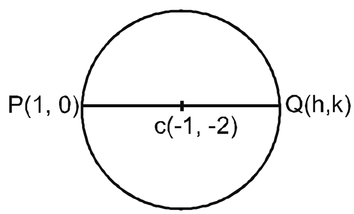


34. **(3)**

Sol: Point of intersection of lines
$3x - 4y - 7 = 0$
$2x - 3y - 5 = 0$ is $(1, -1)$
Area of circle $= \pi r^2 = 49\pi \Rightarrow r = 7$
Hence equation of circle $(x - 1)^2 + (y + 1)^2 = 7^2$
$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

35. **(3)**

Sol:



$\frac{h+1}{2} = -1 \Rightarrow h = -3$
$\frac{k+0}{2} = -2 \Rightarrow k = -4$ Hence $Q(-3, -4)$.

36. **(1)**

Sol: $S_1 - S_2 = 0$

37. **(2)**

Sol: Equation of circumcircle is $x^2 + y^2 - (5/2)x + 1 = 0$

38. **(1)**

Sol: $r = \sqrt{4+16+5} = 5$

$\left|\frac{6-16-m}{5}\right| < 5$

$\Rightarrow -25 < m + 10 < 25$

$\Rightarrow -35 < m < 15$

Hence correct option is (1)

39. **(2)**

Sol: $x^2 + y^2 = ax$ .... (1)

$\Rightarrow$ centre $c_1\left(\frac{a}{2}, 0\right)$ and radius $r_1 = \left|\frac{a}{2}\right|$

$x^2 + y^2 = c^2$ ....(2)

$\Rightarrow$ centre c_2 (0, 0) and radius $r_2 = c$

both touch each other *iff*

$|c_1c_2| = |r_1 - r_2|$

$\frac{a^2}{4} = c - \left|\frac{a}{2}\right|$

$\Rightarrow \frac{a^2}{4} = \frac{a^2}{4} - |a|\,c + c^2$

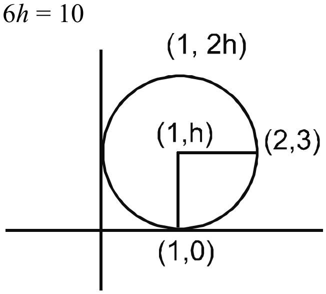
$\Rightarrow |a| = c$

40. **(1)**

Sol: $h^2 = (1-2)^2 + (h-3)^2$

$0 = 1 - 6h + 9$

$6h = 10$



$h = \frac{5}{3}$

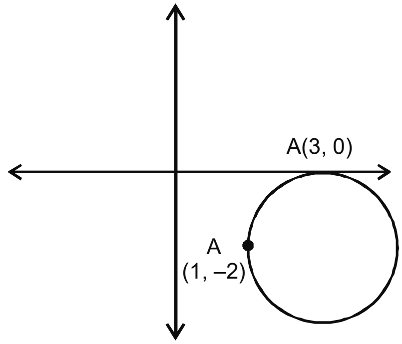
Now diameter is $2h = \frac{10}{3}$

41. **(3)**

Sol: Let the equation of circle be

$(x-3)^2 + (y-0)^2 + \lambda y = 0$

As it passes through (1, – 2)



$\therefore$ $(1-3)^2 + (-2)^2 + \lambda(-2) = 0$

$\Rightarrow$ $\lambda = 4$

$\therefore$ equation of circle is

$(x-3)^2 + y^2 - 8 = 0$

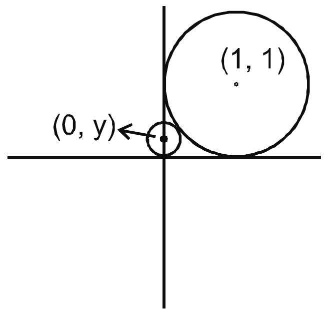
so (5, – 2) satisfies equation of circle

42. **(2)**

Sol: c_1 (1, 1) $r_1 = 1$

c_2 (0, y) $r_2 = |y|$

$c_1c_2 = r_1 + r_2$



$\sqrt{(1-0)^2 + (1-y)^2} = 1 + |y|$

$2 - 2y + y^2 = y^2 + 2|y| + 1$

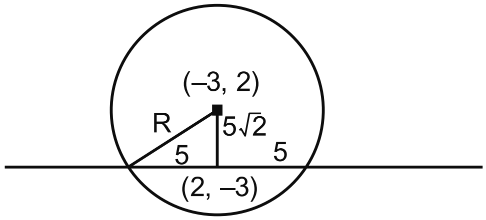
$4|y| = 1$

$|y| = \frac{1}{4}$

$y = \frac{1}{4}$

43. (1)

Sol:



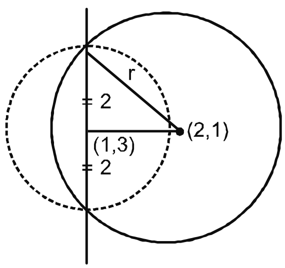
$\sqrt{(-3-2)^2+(2+3)^2} = 5\sqrt{2}$

Now, $r^2 = (5)^2 + (5\sqrt{2})^2$

$\Rightarrow\ r = 5\sqrt{3}$

44. (1)

Sol: Clearly from the figure the radius of bigger circle



$r^2 = 2^2 + \{(2-1)^2 + (1-3)^2\}$

$r^2 = 9$ or $r = 3$

45. (2)

Sol: $(ax^2 + by^2 + c)\,(x^2 - 5xy + 6y^2) = 0$

$\Rightarrow x = 3y$ or $x = 2y$ or $ax^2 + by^2 + c = 0$

If $a = b$ and c is of opposite sign, then it will represent a circle

Hence (2) is correct option.

Consider L_1: $2x + 3y + p - 3 = 0$

L_2: $2x + 3y + p + 3 = 0$

where p is a real number, and C: $x^2 + y^2 + 6x - 10y + 30 = 0$

46. (4)

Sol: Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

as it passes through (–1, 0) & (0, 2)

$1 - 2g + c = 0$

and $4 + 4f + c = 0$

also $f^2 = c$

$\Rightarrow f = -2,\ c = 4;\ g = \dfrac{5}{2}$

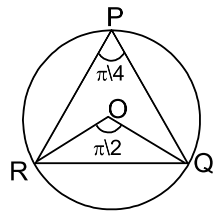
equation of circle is

$x^2 + y^2 + 5x - 4y + 4 = 0$

which passes through (–4, 0)

47. (2)

Sol: $m_{OQ} = \dfrac{4}{3}$



$m_{OR} = \dfrac{-3}{4}$

$\therefore\ m_{OQ}\,.m_{OQ} = -1$

$\Rightarrow OP \perp OQ\ \ \therefore\ \angle RPQ = \dfrac{\pi}{4}$

48. (3)

Sol: $\therefore$ radius is minimum as possible as

$\therefore$ Equation of circle $(x-1)(x-0) + (y-1)(y-0) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

$\therefore\ g = 1, f = 1, c = 0$

$\therefore\ g + f + c = 2$

49. (2)

Sol: $(\alpha - 1, \alpha + 1)$ lies inside the circle

$\therefore\ (\alpha-1)^2 + (\alpha+1)^2 - (\alpha-1) - (\alpha+1) - 6 < 0$

$\Rightarrow (\alpha - 2)(\alpha + 1) < 0 \Rightarrow -1 < \alpha < 2$ ... (1)

Also $C\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ & $(\alpha-1, \alpha+1)$ lies same side of line $x + y - 2 = 0$

50. (3)

Sol: Clearly $g^2 - c < 0$ & $f^2 - c < 0$

$\Rightarrow 9 < \lambda$ & $25 < \lambda$

Also point (1, 4) lies inside the circle

$\therefore\ 1 + 16 - 6 - 40 + \lambda < 0 \Rightarrow \lambda < 29$

$\therefore\ \lambda \in (25, 29)$

$\therefore$ maximum integral value λ is 28

51. **(1)**

Sol: Given circle $x^2 + y^2 + \lambda x + (1 - \lambda)y + 5 = 0$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{\lambda^2}{4} + \left(\frac{1-\lambda}{2}\right)^2 - 5} \le 5$$

$$\Rightarrow \lambda^2 + 1 + \lambda^2 - 2\lambda - 20 \le 100$$

$$\Rightarrow 2\lambda^2 - 2\lambda - 119 \le 0$$

$$\Rightarrow -7.2 \le \lambda \le 8.2$$

52. **(3)**

Sol: Circle passing through (1,0), (0,0) & (0,1)

$x^2 + y^2 - x - y = 0$

$(2k, 3k)$ lies on it if $4k^2 + 9k^2 - 2k - 3k = 0$

$\Rightarrow 13k^2 - 5k = 0$

$\Rightarrow k = 0$ or $\frac{5}{13}$

53. **(2)**

Sol: A(1,2), B(α,β) are end of diameter

$\therefore 2\alpha + \beta = 5 \Rightarrow \beta = 5 - 2\alpha$ .... (1)

Let $C(h, k)$ is centre then $h = \frac{\alpha+1}{2} \Rightarrow \alpha = 2h - 1$

$k = \frac{2+\beta}{2} \Rightarrow k = \frac{2+5-2\alpha}{2}$

$\Rightarrow 2k = 7 - 2\alpha$

$\Rightarrow 2k = 7 - 2(2h - 1)$ by (1)

$\Rightarrow 2y = 7 - 4x + 2 \Rightarrow 4x + 2y - 9 = 0$

54. **(2)**

Sol: We know that $PA.PB = PT^2$

$PT =$

$\sqrt{100 + 49 - 40 - 14 - 20} = \sqrt{149 - 74} = \sqrt{75}$

55. **(3)**

Sol: Here common chord is $S_1 - S_2 = 0$

$\Rightarrow 3x + 4y + 3 = 0$

$\therefore$ Length $= 2\sqrt{10 - \frac{144}{25}} = \frac{2\sqrt{106}}{5}$

$\therefore \alpha = 2, \beta = 5 \therefore \alpha + \beta = 7$

56. **(1)**

Sol: Any circle is $x^2 + y^2 - 9 + \lambda(x + y - 1) = 0$

$\Rightarrow x^2 + y^2 + \lambda x + \lambda y - 9 - \lambda = 0$

for smallest circle chord $x + y = 1$ circle will be diameter

$$\therefore \frac{-\lambda}{2} - \frac{\lambda}{2} = 1 \Rightarrow \lambda = -1$$

$\therefore$ equation of smallest circle is $x^2 + y^2 - x - y - 8 = 0$

57. **(2)**

Sol: Let $y = mx$ be a chord

$\therefore x^2(1 + m^2) - x(3 + 4m) - 4 = 0$

$x_1 + x_2 = \frac{3+4m}{1+m^2}$ and $x_1x_2 = \frac{-4}{1+m^2}$

(0,0) divides chord in the ratio 1: 4

$\therefore x_2 = -4x_1$

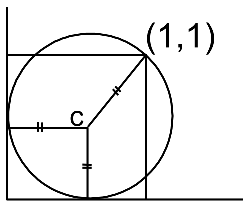
$\therefore$ lines the $y = 0$ or $y = \frac{-24}{7}x \Rightarrow 24x + 7y = 0$

58. **(1)**

Sol: $(r - 1)^2 + (r - 1)^2 = r^2$

$\Rightarrow r^2 - 4r + 2 = 0$

$$\Rightarrow r = \frac{4 + \sqrt{16-8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$



59. **(3)**

Sol: $\therefore$ Chord subtends an angle 45° at major segment it will subtends 90° at the centre (0,0)

60. **(1)**

Sol: $S + \lambda S' = 0$

$\Rightarrow x^2(\sin^2\theta + \lambda\cos^2\theta) + y^2(\cos^2\theta + \lambda\sin^2\theta) + 2xy(h + \lambda h') + x(32 + 16\lambda) + y(16 + 32\lambda) + 19(1 + \lambda) = 0$

it will represent a circle it

$\sin^2\theta + \lambda\cos^2\theta = \cos^2\theta + \lambda\sin^2\theta$ & $h + \lambda h' = 0$

$\lambda = 1$

$\therefore h + h' = 0$

Integer Type Questions (61 to 70)

61. (3)

Sol: Equation of circle C is $(x-2)^2+(y-1)^2=r^2$

62. (727)

Sol: $\ell = 2\sqrt{g^2-c} = 2\sqrt{16-16}=0$

$m = 2\sqrt{f^2-c} = 2\sqrt{25-16}=6$

$y=-x$ intersects circle at (8, –8) & (1, –1)

$l^2+10m^2+26n^2 = 0+360+2548=2908$

63. (8)

Sol: From equation of circle it is clear that circle passes
through origin. Let AB is chord of the circle.
$A \equiv (p, q)$. C is mid-point and
Co-ordinate of C is $(h, 0)$
Then coordinates of B are $(-p+2h, -q)$.
and B lies on the circle
$x^2+y^2=px+qy$, we have

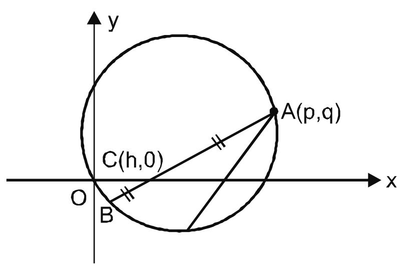
$(-p+2h)^2+(-q)^2 = p(-p+2h)+q(-q)$

$\Rightarrow p^2+4h^2-4ph+q^2 = -p^2+2ph-q^2$

$\Rightarrow 2p^2+2q^2-6ph+4h^2=0$

$\Rightarrow 2h^2-3ph+p^2+q^2=0$ ....(1)

There are given two distinct chords which are bisected at x-axis then, there will be two distinct
values of h satisfying (1).
So discriminant of this quadratic equation must
$\Rightarrow D>0$



$\Rightarrow (-3p)^2 - 4.2\,(p^2+q^2) > 0$

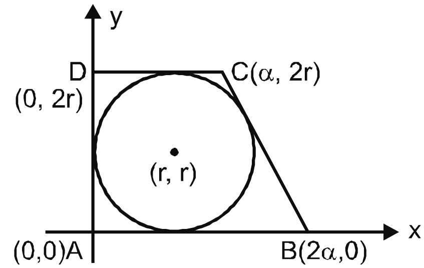
$\Rightarrow 9p^2-8p^2-8q^2>0$

$\Rightarrow p^2-8q^2>0$

$\Rightarrow p^2>8q^2$

64. (2)

Sol: $18 = \frac{1}{2}(3\alpha)(2r) \quad \alpha r = 6$



Line, $y = -\frac{2r}{\alpha}(x-2\alpha)$ is tangent to circle

$(x-r)^2+(y-r)^2=r^2$

$2\alpha = 3r$ and $\alpha r = 6$

$r=2$

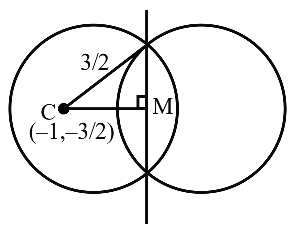
65. (2)

Sol: Equation of common chord $2x+1=0$

$C\left(-1, \frac{-3}{2}\right),\ r^2 = \frac{9}{4},\ CM^2 = \left|\frac{2(-1)+1}{2}\right|^2$

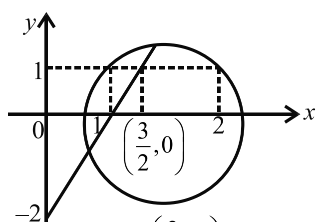
$AB = 2\sqrt{r^2-CM^2} = 2\sqrt{\frac{9}{4}-\frac{1}{4}}$

$= 2\sqrt{2},\ \left[2\sqrt{2}\right]=2$



66. (3)

Sol:



$k \in \left(\frac{3}{2}, 2\right)$

67. (6)

Sol: Let $S : x^2 + y^2 - 6x - 10y + \lambda = 0$

$$\Rightarrow \begin{cases} S_1 < 0 \\ g^2 < c \\ f^2 < c \end{cases} \Rightarrow \begin{cases} 4+16-12-40+\lambda < 0 \\ 9 < \lambda \\ 25 < \lambda \end{cases}$$

$\Rightarrow \quad 25 < \lambda < 32$

68. (6)

Sol: Chord of contact of $(\alpha, 3 - \alpha)$ w.r.t. $x^2 + y^2 = 9$ is

$\alpha x + (3 - \alpha)y = 9$

$\alpha(x - y) + 3(y - 3) = 0$

$\Rightarrow$ fixed point is (3, 3)

69. (28)

Sol: Consider family

$$(x-3)^2 + (y-5)^2 + \lambda(2x - y - 1) = 0$$

$$x^2 + y^2 + 2(\lambda - 3)x - (10+\lambda)y + 34 - \lambda = 0$$

Its centre is $\left(3-\lambda, 5+\frac{\lambda}{2}\right)$, if it lies on $x + y - 5 = 0$, then $\lambda = 6$

$$x^2 + y^2 + 6x - 16y + 28 = 0$$

70. (816)

Sol: Normal are

$$y + 2x = \sqrt{11} + 7\sqrt{7} \qquad \ldots(1)$$

$$2y + x = 2\sqrt{11} + 6\sqrt{7} \qquad \ldots(2)$$

On solving the above equations, we get centre of circle as $\left(\frac{8\sqrt{7}}{3}, \sqrt{11} + \frac{5\sqrt{7}}{3}\right)$.

Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius = perpendicular distance of tangent from centre $= 4\sqrt{\frac{7}{5}}$

$\therefore \quad (5h - 8k)^2 = 5r^2 = 816$

CONIC SECTIONS

Single Option Correct Type Questions (01 to 63)

1. **(4)**

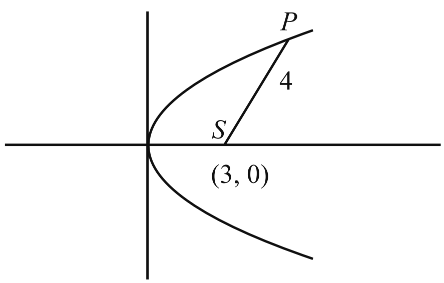
Sol: $(x-2)^2 + (y-3)^2 = \left|\dfrac{3x-4y+7}{5}\right|^2$

$\therefore$ focus is (2, 3) & directrix is $3x - 4y + 7 = 0$

latus rectum = 2 × $\perp_r$ distance from focus to directrix = $2 \times \dfrac{1}{5}$ = 2/5

2. **(2)**

Sol:



Let the point P is $(3t^2, 6t)$

and $PS = 3 + 3t^2 = 4$

$t^2 = 1/3$

$t = \pm\dfrac{1}{\sqrt{3}}$

$\therefore$ Points are

$(1, 2\sqrt{3})$ & $(1, -2\sqrt{3})$

3. **(1)**

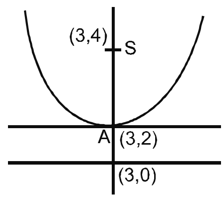
Sol: $y^2 - 12x - 4y + 4 = 0$

$y^2 - 4y = 12x - 4$

$(y-2)^2 = 12x$

$Y^2 = 12X$

focus: $X = A$, $Y = 0$



$x = 3, y = 2$

$A(3, 2)$

equ. of directrix is $y = 0$, $PS = PM$

$y = 0$, $PS = PM$

$\sqrt{(x-3)^2 + (y-4)^2} = |y|$

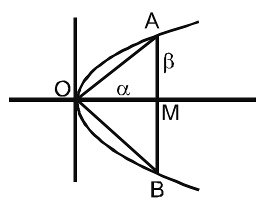
by squaring, we will get $(x-3)^2 + (y-4)^2 = y^2$

$(x-3)^2 + (y-4)^2 = y^2$

$x^2 - 6x - 8y + 25 = 0$

4. **(1)**

Sol:



$\angle AOM = 30°$ as angle $\angle AOB = 60°$

$\tan 30° = \dfrac{\beta}{\alpha}$

$\alpha = \beta\sqrt{3}$

$\therefore A$ is $(\beta\sqrt{3}, \beta)$

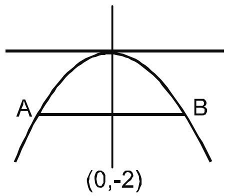
Now A will satisfy equation of parabola $y^2 = 4x$

$\beta^2 = 4\cdot\ \beta\sqrt{3} \Rightarrow \beta = \ 4\sqrt{3} \ \Rightarrow \beta \neq 0$

$\therefore \quad AB = 8\sqrt{3}$

5. **(3)**

Sol:



$x^2 = -8y$

$\therefore\ a = 2$

focus is (0, –2)

Clearly for A, B both, $y = -2$

$\therefore\ x^2 = -8(-2) = 16$

$\therefore\ x = \pm 4$

$\therefore\ A$ is (–4, –2), B is (4, –2)

6. **(1)**

Sol: Parabola is $9\left(x^2 - \frac{2}{3}x\right) = -36y - 9$

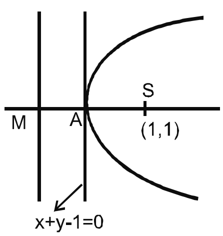
$\Rightarrow 9\left(x - \frac{1}{3}\right)^2 = -36y - 8$

$\Rightarrow \left(x - \frac{1}{3}\right)^2 = -4\left(y + \frac{2}{9}\right)$

clearly, vertex is $\left(\frac{1}{3}, -\frac{2}{9}\right)$

7. **(4)**

Sol:



Point A is $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\therefore\ M$ is (0, 0)

$\therefore$ Eq. of Directrix is $x + y = 0$

$\therefore$ Eq. of parabola is $(x-1)^2 + (y-1)^2$

$= \left(\frac{x+y}{\sqrt{2}}\right)^2$

Length of latus rectum = 2($\perp r$ distance from focus to the directrix) $= 2\left|\frac{1+1}{\sqrt{2}}\right|. = 2\sqrt{2}$

8. **(2)**

Sol: $x^2 - 2 = -2\cos t,$ $\qquad y = 4\cos^2\frac{t}{2}$

$\cos t = \frac{x^2-2}{-2},$ $\qquad y = 4\cos^2\frac{t}{2}$

$y = 2\left(2\cos^2\frac{t}{2}\right)$ $\qquad y = 2(1 + \cos t)$

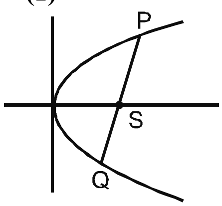
$y = 2\left(1 + \frac{x^2-2}{-2}\right)$

$y = 2 + 2 - x^2$

$y = 4 - x^2$

9. **(1)**

Sol:



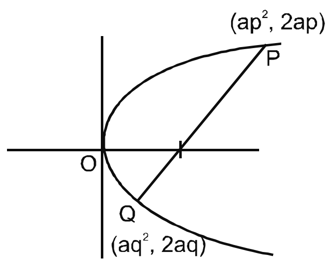
From the property $\frac{1}{PS} + \frac{1}{QS} = \frac{1}{a}$

$\frac{1}{3} + \frac{1}{2} = \frac{1}{a}$

$a = \frac{6}{5}$ $\qquad \therefore$ Latus rectum $= 4a = \frac{24}{5}$

10. **(3)**

Sol:



slope of $= \dfrac{2a(p-q)}{a(p-q)(p+q)} = 1$

$\therefore\ p + q = 2$

11. **(4)**

Sol: $4x^2 + 9y^2 + 8x + 36y + 4 = 0$

$\Rightarrow 4\,(x^2 + 2x + 1) + 9\,[y^2 + 4y + 4] = 36$

$4\,(x+1)^2 + 9\,(y+2)^2 = 36$

$\Rightarrow \dfrac{(x+1)^2}{9} + \dfrac{(y+2)^2}{4} = 1$

$\Rightarrow e = \sqrt{1-\dfrac{4}{9}} = \dfrac{\sqrt{5}}{3}$

12. **(3)**

Sol: $2 \times \dfrac{a}{e} = 3 \times 2ae \Rightarrow e^2 = \dfrac{1}{3} \Rightarrow e = \dfrac{1}{\sqrt{3}}$

13. **(4)**

Sol: Line $\dfrac{x}{7}+\dfrac{y}{2}=1$ meet x-axis at, $y = 0$

$\Rightarrow x = 7$

line $\dfrac{x}{3}-\dfrac{y}{5}=1$ meet y-axis at $x = 0 \Rightarrow y = -5$

$\therefore$ ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ passes through (7, 0) and (0, – 5)

Hence $\dfrac{49}{a^2}+0=1 \Rightarrow \quad a^2 = 49$

$0 + \dfrac{25}{b^2}=1 \Rightarrow b^2 = 25$

$\therefore \dfrac{x^2}{49}+\dfrac{y^2}{25}=1 \qquad \Rightarrow e = \sqrt{1-\dfrac{25}{49}}$

14. **(4)**

Sol: Distance between focii = $2ae$

$= \sqrt{(2-4)^2+(2-2)^2} = 2$

$2a = 10$

$\therefore e = \dfrac{2}{10} = \dfrac{1}{5}$

$\therefore\ b^2 = a^2(1 - e^2)$

$\therefore\ b^2 = 25\left(1-\dfrac{1}{25}\right) = 24$

Centre $= \left(\dfrac{2+4}{2}, \dfrac{2+2}{2}\right) = (3, 2)$

$\dfrac{(x-3)^2}{25} + \dfrac{(y-2)^2}{24} = 1$

15. **(3)**

Sol: $(3x)^2 - 2.3x + (1)^2 + (2y)^2 + 2.2y + (1)^2 = 1$

$\Rightarrow (3x - 1)^2 + (2y + 1)^2 = 1 \Rightarrow 9\left(x-\dfrac{1}{3}\right)^2 + 4\left(y+\dfrac{1}{2}\right)^2 = 1$

$\therefore \dfrac{\left(x-\frac{1}{3}\right)^2}{\frac{1}{9}} + \dfrac{\left(y+\frac{1}{2}\right)^2}{\frac{1}{4}} = 1$

Length of axes $2a = 2 \times \dfrac{1}{3} = \dfrac{2}{3}$

$2b = 2 \times \dfrac{1}{2} = 1$

16. **(2)**

Sol: $\dfrac{x^2}{r-2}+\dfrac{y^2}{5-r} = 1$ For ellipse $r - 2 > 0$ and $5 - r > 0 \Rightarrow 2 < r < 5$

17. **(2)**

Sol: $2ae = 10,\ \dfrac{a}{e}-ae=15$

$ae = 5 \qquad \dfrac{5}{e^2}-5=15 \Rightarrow \dfrac{5}{e^2}=20 \Rightarrow e = \dfrac{1}{2}$

18. **(2)**

Sol: Equation of axis: $x- y + k = 0$

$3 - 4 + k = 0$

$k = 1$

$x - y + 1 = 0$

point of intersections of axis and directrix

$x- y + 1 = 0$

$x + y -1 = 0$

$x = 0, y =1$

Now, $\frac{AS}{AM} = \frac{1}{2}$

For internal division For external division

$\therefore x = \frac{1\times 0+2\times 3}{3} = 2\ x = \frac{1\times 0-2\times 3}{1-2} = 6$

$y = \frac{1\times 1+2\times 4}{3} = 3\ y = \frac{1\times 1-2\times 4}{1-2} = 7$

19. **(3)**

Sol: If e_1 & e_2 are eccentricities of two conjugate hyperbolas

then $\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$

$\therefore e_1 = \sec\alpha$ & $e_2 = \operatorname{cosec}\alpha$

20. **(3)**

Sol: $\frac{2b^2}{a} = 8$ ... (1)

and $2b = \frac{2ae}{2}$ ...(2)

and $e^2 = 1 + \frac{b^2}{a^2}$ ... (3)

by (1), (2), (3) $e = \frac{2}{\sqrt{3}}$

21. **(1)**

Sol: $2b = 5$

$\Rightarrow b = \frac{5}{2}$

$2ae = 13$

$\therefore\ ae = \frac{13}{2}$

$\therefore\ b^2 = a^2e^2 - a^2$

$\Rightarrow \frac{25}{4} = \frac{169}{4} - a^2$

$\Rightarrow a^2 = \frac{144}{4} = 36$

$a = 6$

$\therefore$ Equation of Hyperbola is

$\frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$

22. **(3)**

Sol: $2a = 7$

$\therefore\ a = \frac{7}{2}$

$\therefore$ Let equation of hyperbola is

$\frac{x^2}{\frac{49}{4}} - \frac{y^2}{b^2} = 1$

It passes through (5, –2)

$\Rightarrow \frac{25\times 4}{49} - \frac{4}{b^2} = 1 \Rightarrow \frac{100-49}{49} = \frac{4}{b^2}$

$\Rightarrow \frac{51}{49} = \frac{4}{b^2}$

$\therefore\ b^2 = \frac{196}{51} \Rightarrow \frac{4x^2}{49} - \frac{51y^2}{196} = 1$

23. **(2)**

Sol: Centre of hyperbola $\equiv (5, 0)$

$\therefore\ 2a = 10$

$\therefore\ a = 5$

$\therefore\ ae = 13$

$b^2 = a^2e^2 - a^2$

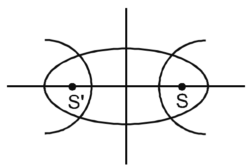
$b^2 = 169 - 25$

$\therefore\ b^2 = 144$

$\therefore\ \frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$

24. **(2)**

Sol:



ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Hyperbola, $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

$\therefore e_1^2 = 1-\frac{b^2}{a^2}$, $e_2^2 = 1+\frac{B^2}{A^2}$

and $2ae_1 = 2Ae_2$

Also, $b = B$

So, $\frac{b}{ae_1} = \frac{B}{Ae_2}$

$\therefore e_1^2 = 1 - \frac{B^2}{A^2}\frac{e_1^2}{e_2^2}$

$= 1 - \frac{(e_2^2-1)e_1^2}{e_2^2}$

$e_1^2 e_2^2 = e_2^2 - e_1^2 e_2^2 + e_1^2$

$\Rightarrow e_1^{-2} + e_2^{-2} = 2$

25. **(1)**

Sol: $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$

$e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2} \Rightarrow \frac{y^2}{b^2}-\frac{x^2}{a^2} = 1$, $(e')^2$

$= 1 + \frac{a^2}{b^2} = \frac{b^2+a^2}{b^2}$

$\frac{1}{e^2}+\frac{1}{(e')^2} = \frac{a^2}{a^2+b^2}+\frac{b^2}{b^2+a^2} = \frac{a^2+b^2}{a^2+b^2} = 1$

So the point lie on $x^2 + y^2 = 1$

26. **(4)**

Sol: Centre of ellipse = (0, 0)

Centre of hyperbola = (0, 0)

$\therefore$ Eccentricity of ellipse $= \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

$\therefore$ focii of ellipse $\equiv (\pm ae, 0) = (\pm 3, 0)$

$\therefore$ Eccentricity of hyperbola $= \sqrt{1+\frac{16}{25}} = \frac{\sqrt{41}}{5}$

foci of hyperbola $= (\pm ae, 0) = (\pm\sqrt{41}, 0)$

Vertex of ellipse and hyperbola $= (\pm 5, 0)$

27. **(4)**

Sol: Distance between foci $= \sqrt{19^2+5^2} = \sqrt{386}$

Now by $PS + S'P = 2a$ (for ellipse) (take point P at origin) we get $a = 19$

$\therefore 2ae = \sqrt{386} \Rightarrow e = \frac{\sqrt{386}}{38}$

If conic is hyperbola

$|PS - PS'| = 2a \Rightarrow a = 6$

by $2ae' = \sqrt{386}$ $\quad e' = \frac{\sqrt{386}}{12}$

28. **(3)**

Sol: Equation of chord joining given points

$\frac{x}{a}\cos\left(\frac{\theta-\phi}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right)$

$= \cos\left(\frac{\theta+\phi}{2}\right)$

If $(ae, 0)$ satisfies it

$\frac{1}{e} = \frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\cos\left(\frac{\theta+\phi}{2}\right)}$

Now by componendo dividendo

$\frac{1-e}{1+e} = \tan\frac{\theta}{2}\tan\frac{\phi}{2}$

again if $(-ae, 0)$ satisfies it

$-e\cos\left(\frac{\theta-\phi}{2}\right) = \cos\left(\frac{\theta+\phi}{2}\right)$

$\Rightarrow \frac{1+e}{1-e} = \tan\frac{\theta}{2}\tan\frac{\phi}{2}$

29. **(4)**

Sol: $\frac{16x^2}{225} = 1 \Rightarrow x = \pm\frac{15}{4}$

Hence intersection points are $P\left(\frac{15}{4},\frac{15}{4}\right)$ and $Q\left(-\frac{15}{4},-\frac{15}{4}\right)$

$2a = PQ = 2\sqrt{2}\times\frac{15}{4} = \frac{15}{\sqrt{2}} \Rightarrow a = \frac{15}{2\sqrt{2}}$

given $b = \frac{5}{2\sqrt{2}}$

$e = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$

30. **(4)**

Sol: $e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

End points of latus rectum $\left(3, \frac{16}{5}\right)$

put in $y^2 = 4x \Rightarrow \frac{256}{25} = 4a(3)$

$\Rightarrow a = \frac{64}{75}$

31. **(1)**

Sol: Given equation

$x^2 + 2y - 3x + 5 = 0$

$x^2 - 3x + 5 = -2y$

$x^2 - 3x + \frac{9}{4} + 5 - \frac{9}{4} = -2y$

$\left(x - \frac{3}{2}\right)^2 = -2y - \frac{11}{4}$

$\left(x - \frac{3}{2}\right)^2 = -2\left(y + \frac{11}{8}\right)$

$X^2 = -4AY$

Hence $\equiv \left(\frac{3}{2}, -\frac{11}{8}\right)$

Axis $x - \frac{3}{2} = 0 \quad \Rightarrow x = \frac{3}{2}$

Focus $x - \frac{3}{2} = 0, y + \frac{11}{8} = -\frac{1}{2}$

$y = -\frac{15}{8}$

$\therefore \left(\frac{3}{2}, -\frac{15}{8}\right)$

32. **(3)**

Sol: Distance of focal chord from (0, 0) is p

equation of chord ; $2x - (t_1 + t_2)\, y + 2a\, t_1\, t_2 = 0$

$2x - (t_1 + t_2)\, y - 2a = 0$ ...... (i)

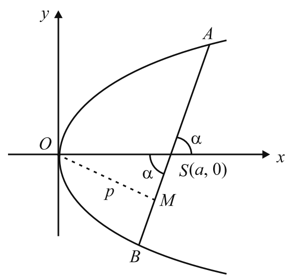
so perpendicular length from (0, 0)

$\left|\frac{2a}{\sqrt{4 + \left(t_1 - \frac{1}{t_1}\right)^2}}\right| = p \Rightarrow \left|\frac{2a}{\left(t_1 + \frac{1}{t_1}\right)}\right| = p$

$\Rightarrow \left(t_1 + \frac{1}{t_1}\right) = \frac{2a}{p}$

Now length of focal chord is $= a\left(t_1 + \frac{1}{t_1}\right)^2$

$= a\,\frac{4a^2}{p^2} = \frac{4a^3}{p^2}$



33. **(1)**

Sol: $lx + my = -n$

$\frac{\ell x + my}{-n} = 1$

Homogenizing, we get $y^2 = 4ax\left(\frac{\ell x + my}{-n}\right)$

or $4alx^2 + ny^2 + 4amxy = 0$

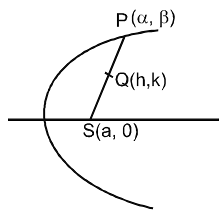
If the lines of above pair are at right angles, we must have

coeff. of x^2 + coeff. of $y^2 = 0$

$\therefore\ 4al + n = 0$

34. **(4)**

Sol:



$h = \frac{a+\alpha}{2}, k = \frac{\beta}{2}$

$\Rightarrow \alpha = 2h - a, \beta = 2k$

(α, β) satisfies the parabola

$\therefore \beta^2 = 4a\alpha$

$4k^2 = 4a(2h - a)$

$y^2 = a(2x - a)$

$y^2 = 2a\left(x - \frac{a}{2}\right)$

35. (1)

Sol: $\frac{a}{e} - ae = 8$

$a\left[2 - \frac{1}{2}\right] = 8$

$\frac{3}{2}a = 8 \Rightarrow a = \frac{16}{3}$

$\because b^2 = a^2(1 - e^2)$

$\therefore b^2 = \left(\frac{16}{3}\right)^2\left(1 - \frac{1}{4}\right)$

$\Rightarrow b^2 = \frac{64}{3} \Rightarrow b = \frac{8}{\sqrt{3}}$

36. (3)

Sol: $\frac{x^2}{r^2 - r - 6} + \frac{y^2}{r^2 - 6r + 5} = 1$

(i) $r^2 - r - 6 > 0$

$(r - 3)(r + 2) > 0$

$(-\infty, -2) \cup (3, \infty)$

(ii) $r^2 - 6r + 5 > 0$

$(r - 1)(r - 5) > 0$

$(-\infty, 1) \cup (5, \infty)$

(iii) Since in ellipse $e^2 = 1 - \frac{b^2}{a^2}$

$e < 1$

$\therefore 0 < 1 - \frac{r^2 - 6r + 5}{r^2 - r - 6} < 1$

$r \in (5, \infty)$

$\therefore$ for above equation to be ellipse

$\therefore r \in (5, \infty)$

37. (2)

Sol: $\frac{x^2}{16} + \frac{y^2}{25} = 1$

$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$be = \frac{3}{5} \times 5 = 3 \quad \Rightarrow \frac{2a^2}{b} = \frac{2 \times 16}{5} = \frac{32}{5}$

38. (4)

Sol: Doing partial differentiation

$10x - 2y + 8 = 0 \Rightarrow 5x - y + 4 = 0$ ....(1)

$10y - 2x + 8 = 0 \Rightarrow 5y - x + 4 = 0$ ....(2)

solving (1) & (2) centre is $(-1, -1)$

39. (3)

Sol: Eccentricity of ellipse

$e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

let the equation of hyperbola be

$\frac{x^2}{\ell^2} - \frac{y^2}{m^2} = 1$

distance between foci

$2l \cdot e_h = 2 \times 5 \times \frac{4}{5}$

$\Rightarrow l = 2$

$m^2 = l^2(4 - 1)$

$\Rightarrow m^2 = 12$

$\therefore$ Equation of hyperbola is

$\frac{x^2}{4} - \frac{y^2}{12} = 1 \Rightarrow \quad 3x^2 - y^2 - 12 = 0$

40. (1)

Sol: $7x + 13y - 87 = 0$

$5x - 8y + 7 = 0$

On solving we get (5,4)

Now let hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\because$ Passes (5, 4) $\therefore \frac{25}{a^2} - \frac{16}{b^2} = 1$ ....(i)

Also, $\frac{2b^2}{a} = \frac{32\sqrt{2}}{5}$ ....(ii)

By (i), (ii) we get $a^2 = \frac{25}{2}$, $b^2 = 16$.

41. (3)

Sol: $\dfrac{x^2}{5} - \dfrac{y^2}{5\cos^2\alpha} = 1 \therefore e^2{}_H = 1 + \cos^2\alpha$

& $\dfrac{x^2}{25\cos^2\alpha} + \dfrac{y^2}{25} = 1 \qquad \therefore e_E^2 = 1 - \cos^2\alpha$

$e_H^2 = 3e_E^2$ given

$\therefore 1 + \cos^2\alpha = 3 - 3\cos^2\alpha$

$\therefore 4\cos^2\alpha = 2$

$\therefore \cos^2\alpha = \dfrac{1}{2} \therefore$ one of the value of α is $\dfrac{\pi}{4}$

42. (1)

Sol: $\dfrac{x^2}{b^2} - \dfrac{y^2}{a^2} = 1$ passes through ($\pm ae$, 0)

$(\pm ae, 0) \ \dfrac{x^2}{b^2} - \dfrac{y^2}{a^2} = 1$

$\therefore \dfrac{a^2e^2}{b^2} = 1$

$\therefore e^2 = \dfrac{b^2}{a^2}$

$1 - \dfrac{b^2}{a^2} = \dfrac{b^2}{a^2}$

$\therefore \dfrac{b^2}{a^2} = \dfrac{1}{2}$

$\therefore e_H = \sqrt{1 + \dfrac{a^2}{b^2}} = \sqrt{3}$

43. (3)

Sol: $\dfrac{(x-1)^2}{9} - \dfrac{(y-2)^2}{16} = 1$

$e = \sqrt{1 + \dfrac{16}{9}} = \dfrac{5}{3}$

$\therefore S\,(1 + 5, 2) \equiv (6, 2)$

44. (1)

Sol: Let , $A\left(ct_1, \dfrac{c}{t_1}\right)$, $B\left(ct_2, \dfrac{c}{t_2}\right)$ $C\left(ct_3, \dfrac{c}{t_3}\right)$

then orthocentre be

$H\left(\dfrac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$ which lies on $xy = c^2$

45. (4)

Sol: STATEMENT-1 is false

STATEMENT-2 true (A standard result)

46. (4)

Sol: The coordinate of P are (1, 0). A general point Q on $y^2 = 8x$ is $(2t^2, 4t)$. Midpoint of PQ is (h, k), so

$2h = 2t^2 + 1$ ......(i)

$2k = 4t$ ......(ii)

$\Rightarrow t = \dfrac{k}{2}$

On putting the values of t from equation (ii) in equation (i) we get

$2h = \dfrac{2k^2}{4} + 1$

$4h = k^2 + 2$

Hence, the locus of (h, k) is $y^2 - 4x + 2 = 0$

47. (4)

Sol: $B \equiv (0, b)$

$F = (ae, 0)$

$F' = (-ae, 0)$

$\Rightarrow m(BF) = \dfrac{b-0}{0-ae} = -\dfrac{b}{ae}$

$\therefore m(BF') = \dfrac{b-o}{0+ae} = \dfrac{b}{ae}$

$\therefore \angle FBF' = 90^\circ$

$\Rightarrow -\dfrac{b}{ae} \times \dfrac{b}{ae} = -1$

$\therefore b^2 = a^2e^2$

but $b^2 = a^2 - a^2e^2$

$\Rightarrow 2a^2e^2 = a^2$

$\therefore e = \dfrac{1}{\sqrt{2}}$

48. **(1)**

Sol: $e = \frac{1}{2}$

$\frac{a}{e} - ae = 4$

$a\left[2 - \frac{1}{2}\right] = 4$

$a.\frac{3}{2} = 4$

$a = \frac{8}{3}$

49. **(4)**

Sol: The given equation of the parabola $y = \frac{a^3 \times x^2}{3} + \frac{a^2 x}{2} - 2a$

$y + 2a = \frac{a^3}{3}\left(x^2 + \frac{3}{2a}x\right)$

$y + 2a = \frac{a^3}{3}\left(x + \frac{3}{4a}\right)^2 - \frac{9}{16a^2} \times \frac{a^3}{3}$

$y + 2a - \frac{3a}{16} = \frac{a^3}{3}\left(x + \frac{3}{4a}\right)^2$

$\left(y + \frac{35a}{16}\right) = \frac{a^3}{3}\left(x + \frac{3}{4a}\right)^2$

Thus, the vertices of the parabola is $\left(-\frac{3}{4a}, \frac{-35a}{16}\right)$

Let $h = -\frac{3}{4a}$ and $k = -\frac{35a}{16}$

Now, $hk = \frac{105}{64}$

Thus locus of vertices of a parabola $xy = \frac{105}{64}$

50. **(4)**

Sol: $2ae = 6 \quad \Rightarrow ae = 3$

$2b = 8 \Rightarrow b = 4$

now $b^2 = a^2 - a^2e^2$

$16 = a^2 - 9$

$a^2 = 25$

$a = 5$

$\therefore e = \frac{3}{5}$

51. **(4)**

Sol: $e = \sqrt{1 + \tan^2\alpha} = |\sec\alpha|$

directrix $x = \frac{\cos\alpha}{|\sec\alpha|} = \cos\alpha\,|\cos\alpha|$

abscissae of foci $ae = \pm 1$

abscissae of vertices $a = \pm \cos\alpha$.

52. **(2)**

Sol: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{9}{a^2} + \frac{1}{b^2} = 1$ .... (1)

case - 1 when $a > b$

$b^2 = a^2(1 - e^2)$

$b^2 = a^2(1 - 2/5)$

$5b^2 = 3a^2$ .... (2)

(1) & (2)

$\frac{9 \times 3}{5b^2} + \frac{1}{b^2} = 1$

$\Rightarrow b^2 = \frac{32}{5}$

$\therefore a^2 = \frac{32}{3}$

$\therefore \frac{3x^2}{32} + \frac{5y^2}{32} = 1$

$\Rightarrow 3x^2 + 5y^2 - 32 = 0$

case - 2

$a < b$

$a^2 = b^2(1 - e^2)$

$a^2 = b^2(1 - 2/5)$

$5a^2 = 3b^2$ .... (2)

(1) & (2)

$\frac{9 \times 5}{3b^2} + \frac{1}{b^2} = 1$

$\frac{45 + 3}{3} = b^2$

$b^2 = \frac{48}{3} = 16$

$\therefore\ 5a^2 = 48$

$a^2 = \frac{48}{5}$

$\frac{5x^2}{48} + \frac{y^2}{16} = 1$

$\Rightarrow\ 5x^2 + 3y^2 - 48 = 0$

53. (2)

Sol: $ae = 2$

$e = 2$

$\therefore\ a = 1$

$b^2 = a^2 (e^2 - 1)$

$b^2 = 1\ (4 - 1)$

$b^2 = 3$

$\frac{x^2}{1} - \frac{y^2}{3} = 1$

$3x^2 - y^2 = 3$

54. (4)

Sol: $\Rightarrow$ Length of semi minor axis is = 2

Length of semi major axis is 4

then equation of ellipse is

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

$x^2 + 4y^2 = 16$

55. (1)

Sol: $a = 4,\ b = 3,\ e = \sqrt{1 - \frac{9}{16}} \Rightarrow \frac{\sqrt{7}}{4}$

Focii is $(\pm\, ae,\ 0) \Rightarrow (\pm\sqrt{7}\ ,\ 0)$

$r = \sqrt{(ae)^2 + b^2}$

$\sqrt{7+9} \quad = 4$

Now equation of circle is

$(x - 0)^2 + (y - 3)^2 = 16$

$x^2 + y^2 - 6y - 7 = 0$

56. (4)

Sol: $P = \left(t, \frac{t^2}{8}\right)$

$h = \frac{t}{4}$

$k = \frac{t^2}{32}$

$32k = 16\ h^2$

$\Rightarrow 2y = x^2$.

57. (3)

Sol: Equation of directrix is $x + y = 0$

Hence equation of the parabola is

$\left|\frac{x+y}{\sqrt{2}}\right| = \sqrt{(x-2)^2 + (y-2)^2}$

Hence equation of parabola is

$(x - y)^2 = 8(x + y - 2)$

58. (1)

Sol: For ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1 \quad a^2 = 4\ \Rightarrow a = 2,\ b^2 = 3$

$\Rightarrow e^2 = \frac{a^2 - b^2}{a^2} = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

$\therefore$ hyperbola is con-focal with ellipse

$ae = AE$, where $2A = 2\sin\theta\ \Rightarrow 1 = AE$

$\Rightarrow 1 = (\sin\theta)\ E$

where E = eccentricity of hyperbola

$\therefore\ B^2 = A^2\ (E^2 - 1)\ \Rightarrow\ B = \cos\theta$

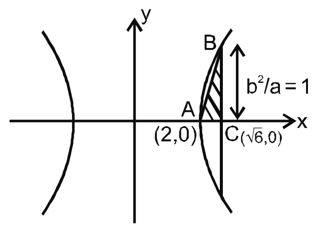
So equation of hyperbola is $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

59. (2)

Sol: $x^2 - 2y^2 - 2\sqrt{2}\,x - 4\sqrt{2}\,y - 6 = 0$

$\Rightarrow \frac{(x-\sqrt{2})^2}{2^2} - \frac{(y-\sqrt{2})^2}{(\sqrt{2})^2} = 1$



Let $x - \sqrt{2} = X$ and $y - \sqrt{2} = Y$

$\frac{X^2}{2^2} - \frac{Y^2}{\sqrt{2}^2} = 1$

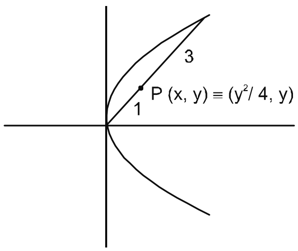
$a = 2,\ b = \sqrt{2} \quad \Rightarrow e = \sqrt{\frac{3}{2}}$

Area of $\Delta ABC = \frac{1}{2}\ (\sqrt{6} - 2)\ .\ 1 = \sqrt{\frac{3}{2}} - 1$

$\Delta ABC = \frac{1}{2}\ (\sqrt{6} - 2)\ .\ 1 = \sqrt{\frac{3}{2}} - 1$

60. **(3)**

Sol:



$\Rightarrow P\left(\frac{y^2}{16}, \frac{y}{4}\right)$

then locus of P is $x = y^2$

61. **(3)**

Sol: Let required ellipse is

E_2: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It passes through (0, 4)

$0 + \frac{16}{b^2} = 1 \Rightarrow \quad b^2 = 16$

It also passes through (±3, ±2)

$\frac{9}{a^2} + \frac{4}{b^2} = 1$

$\frac{9}{a^2} + \frac{1}{4} = 1$

$\frac{9}{a^2} = \frac{3}{4} \Rightarrow a^2 = b^2\ (1 - e^2)$

$\frac{12}{16} = 1 - e^2$

$e^2 = 1 - \frac{12}{16} = \frac{4}{16} = \frac{1}{4}$

$e = \frac{1}{2}$

62. **(4)**

Sol: Given equation is

$16\ (x - 1)^2 - 3\ (y + 2)^2 = 48$

$\Rightarrow \frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1$

Hence length of transverse axis $2a = 2\sqrt{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{32}{\sqrt{3}}$

Eccentricity is $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{19}{3}}$

Equation of directrix is $x - 1 = \pm\frac{a}{e}$

$\Rightarrow x - 1 = \pm\frac{3}{\sqrt{19}} \Rightarrow x = 1 \pm \frac{3}{\sqrt{19}}$

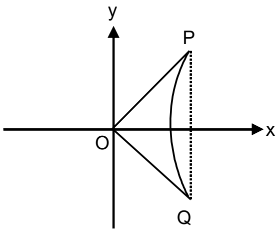
63. **(4)**

Sol: ΔOPQ is equilateral

Hence $OP = OQ = PQ$

$\Rightarrow a^2 \sec^2\theta + b^2\tan^2\theta = (2b\tan\theta)^2$

$\Rightarrow \sin^2\theta = \frac{a^2}{3b^2}$



Now $\sin^2\theta < 1$

$\Rightarrow \frac{a^2}{3b^2} < 1 \Rightarrow \frac{b^2}{a^2} > \frac{1}{3} \Rightarrow e^2 = 1 + \frac{b^2}{a^2} > \frac{4}{3}$

$\Rightarrow e > \frac{2}{\sqrt{3}}$

Integer Type Questions (64 to 74)

64. **(8)**

Sol: $4a = 16$

$\therefore a = 4$

as $y_1 = 2x_1$

$\therefore y_1^2 = 16x_1$ gives $y_1 = 8, x_1 = 4$

$\therefore$ point is (4, 8)

focal distance $= x_1 + a = 4 + 4 = 8$

65. **(5)**

Sol: $(2 - 2x)^2 = 4x \Rightarrow x^2 - 2x + 1 = x$

$x^2 - 3x + 1 = 0$ ⟨ x_1, x_2

$(x_1 - x_2)^2 = 9 - 4 = 5$

similarly $(y_1 - y_2)^2 = 20$

Length of chord $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{5 + 20} = 5$

66. **(8)**

Sol: Length of chord $= \frac{4}{m^2}\sqrt{a(a - mc)(1 + m^2)}$

$m = \tan 60° = \sqrt{3}$

Length of chord $= \frac{4}{3}\sqrt{3(3 - \sqrt{3} \times 0)(1 + 3)} = \frac{4}{3}$

$\sqrt{36} = 8$

67. **(12)**

Sol: Max. area $= \frac{1}{2} \times 2ae \times b = \frac{1}{2} \times 2 \times 3 \times 4 = 12$

68. **(1)**

Sol: $e = \sqrt{1 - \frac{5}{9}}$, $e' = \sqrt{1 + \frac{45/4}{45/5}}$

$e = \frac{2}{3}, e' = \frac{3}{2}$

$\therefore e e' = 1$

69. **(4)**

Sol: Point of intersection of parabola $y^2 = 4x$ and $x^2 + y^2 = 5$ are (1, 2) and (1, – 2)

$\Rightarrow$ length of common chord = 4

70. **(9)**

Sol: We have, $a = 3$ and $\frac{b^2}{a} = 4 \Rightarrow b^2 = 12$

Hence, the equation of the hyperbola is $\frac{x^2}{9} - \frac{y^2}{12} = 1 \Rightarrow 4x^2 - 3y^2 = 36$

71. **(2)**

Sol: $2a = 8, ae = 2 \Rightarrow b^2 = a^2 - a^2e^2 = 12$

Hence ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

$\frac{x^2}{16} + \frac{y^2}{12} = 1$

Point $(\alpha, 3)$ lies on it $\Rightarrow \alpha^2 = 16\left(1 - \frac{9}{12}\right) \Rightarrow \alpha = \pm 2$

$(\alpha, 3) \Rightarrow \alpha^2 = 16\left(1 - \frac{9}{12}\right) \Rightarrow \alpha = \pm 2$

72. **(6)**

Sol: $2a = 6$ and $2b = 8$

$|PS_1 - PS_2| = 2a = 6$

73. (4)

Sol: Focus is $S \equiv (2, 0)$. Points $P \equiv (0, 0)$ and $Q = (2t^2, 4t)$

$$\text{Area of } PQS = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 0 & 1 \\ 2t^2 & 4t & 1 \end{vmatrix}$$

$= \frac{1}{2}(8t) = 4t$ ......(i)

$Q(2t^2, 4t)$ satisfies circle

$4t^4 + 16t^2 - 4t^2 - 16t = 0$

$t^3 + 3t - 4 = 0$

$(t-1)(t^2 + t + 4) = 0$

put $t = 1$ in Area of PQS.

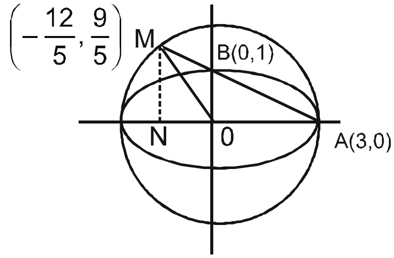
$\Rightarrow$ Area of PQS is 4

74. (27)

Sol: Equation of auxiliary circle is $x^2 + y^2 = 9$.... (1)

Equation of AM is $\frac{x}{3} + \frac{y}{1} = 1$ .... (2)

on solving (1) and (2), we get $M\left(-\frac{12}{5}, \frac{9}{5}\right)$



Now, area of $\Delta AOM = \frac{1}{2} . OA \times MN = \frac{27}{10}$ square unit

VECTORS &
3-D GEOMETRY

Single Option Correct Type Questions (01 to 57)

1. **(1)**

Sol. $\overline{BC} = \hat{i} + \hat{j}$

$\Rightarrow \overline{AB} = \hat{i} - \hat{j}$

$\overline{AB} + \overline{BC} = 2\hat{i}$

$\Rightarrow \overline{AC} = 2\hat{i}$

2. **(3)**

Sol. $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$,

$\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\overline{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\overline{BC} = 2\hat{i} - \hat{j} + \hat{k}$

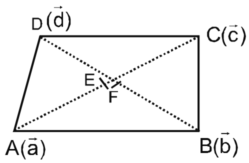
$\overline{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$\overline{AB}^2 = \overline{BC}^2 + \overline{CA}^2$

$\therefore$ Right angle D

3. **(3)**

Sol.



$\overrightarrow{OE} = \dfrac{\vec{a} + \vec{c}}{2}$

$\overrightarrow{OF} = \dfrac{\vec{b} + \vec{d}}{2}$

$\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD}$

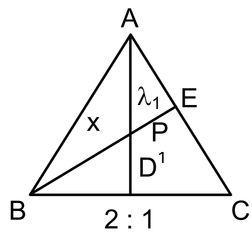
$= \vec{b} - \vec{a} + \vec{d} - \vec{a} + \vec{b} - \vec{c} + \vec{d} - \vec{c}$

$= 2[\vec{b} + \vec{d} - \vec{a} - \vec{c}]$

$= 4\left[\dfrac{\vec{b} + \vec{d}}{2} - \dfrac{(\vec{a} + \vec{c})}{2}\right] = 4\overrightarrow{EF}$

4. **(4)**

Sol.



$E(i - j + 4k)$

$D = \dfrac{-2\hat{i} - 2\hat{j} + 12\hat{k} + 5\hat{i} + 2\hat{j} + 4\hat{k}}{3}$

$= i + 0 + \dfrac{16\hat{k}}{3} = i + \dfrac{16\hat{k}}{3}$

$\vec{P} = \dfrac{\mu(\hat{i} - \hat{j} + 4\hat{k}) + 5\hat{i} + 2\hat{j} + 4\hat{k}}{\mu + 1}$

$\vec{P} = \dfrac{\lambda\left(\hat{i} + \dfrac{16\hat{k}}{3}\right) + 1\left(\dfrac{3\hat{i} - \hat{j} + 2\hat{k}}{1}\right)}{\lambda + 1}$

$\dfrac{\mu + 5}{4 + 1} = \dfrac{\lambda + 3}{\lambda + 1}$ ...... (1)

$\dfrac{\mu + 2}{\mu + 1} = \dfrac{-1}{\lambda + 1}$ ..... (2)

$$\frac{4+4\mu}{\mu+1}=\frac{\frac{16}{3}\lambda+2}{\lambda+1} \text{ .... (3)}$$

5l + 5 + μl + μ – lμ + 1 + 3μ + 3

4l + 5 – 3 = 2μ

$\mu = 2l + 1$

put value of μ in equation (2)

$$\frac{2-(2\lambda+1)}{2\lambda+2}=\frac{-1}{\lambda+1}$$

1 – 2l = –2

$$l = \frac{3}{2}$$

μ = 2l + 1 = 3 + 1 = 4

$BP : PE = \mu : 1 = 4 : 1$

5. **(2)**

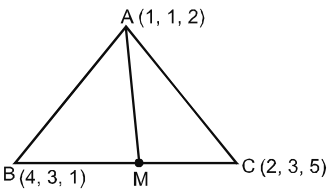
Sol. $x^2 + y^2 + y^2 + z^2 + z^2 + x^2 = 36$

$2(x^2 + y^2 + z^2) = 36$

$$\sqrt{x^2+y^2+z^2}=3\sqrt{2}$$

6. **(4)**

Sol.



$$AB = \sqrt{9+4+1}=\sqrt{14}$$

$$AC = \sqrt{1+4+9}=\sqrt{14}$$

M º (3, 3, 3)

$$\overrightarrow{AM} = 2\hat{i}+2\hat{j}+\hat{k}$$

7. **(3)**

Sol. Let point P is (p, q, r)

$PA^2 - PB^2 = 2k^2$

$\Rightarrow [(p-3)^2 + (q-4)^2 + (r-5)^2] -$

$[(p+1)^2 + (q-3)^2 + (r+7)^2] = 2k^2$

$\Rightarrow -6p - 2p - 8q + 6q - 10r - 14r$

$+ 9 + 25 + 25 - 1 - 9 - 49 = 2k^2$

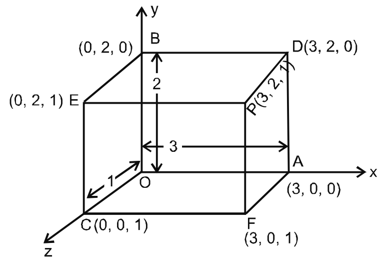
$\Rightarrow 8p + 2q + 24r + 9 + 2k^2 = 0$

hence locus is [put $p = x,\ q = y,\ r = z$]

$8x + 2y + 24z + 9 + 2k^2 = 0$

8. **(4)**

Sol.



dr's of $\overrightarrow{OP}$ are 3, 2, 1

dr's of $\overrightarrow{FB}$ are – 3, 2, – 1

dr's of $\overrightarrow{AE}$ are – 3, 2, 1

dr's of $\overrightarrow{CD}$ are 3, 2, – 1

$$\cos\theta_1=\left|\frac{-9+4-1}{14}\right|=\frac{3}{7},\cos\theta_2=\left|\frac{-9+4+1}{14}\right|=\frac{2}{7}$$

$$\cos q_3=\left|\frac{9+4-1}{14}\right|=\frac{6}{7}$$

So angles are $\cos^{-1}\frac{2}{7}$, $\cos^{-1}\frac{3}{7}$, $\cos^{-1}\frac{6}{7}$

9. **(1)**

Sol. diagonals are $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

$$\cos\theta=\frac{(\vec{a}+\vec{b}).(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}=\frac{1}{3}$$

$$\therefore\ \theta=\cos^{-1}\left(\frac{1}{3}\right)$$

10. **(3)**

Sol. $\bar{a} = \hat{i}$

Let $\bar{b} = x\hat{i} + y\hat{j}$

$$\tan 120^\circ = \frac{y}{x}$$

$$\therefore\ \frac{y}{x} = -\sqrt{3}\ \therefore y = -\sqrt{3}\,x$$

$$\therefore\ \bar{b} = x(\hat{i}-\sqrt{3}\ \hat{j})$$

$$\text{Unit vector } \bar{b} = \pm\frac{\hat{i}-\sqrt{3}\ \hat{j}}{2}$$

$\therefore \bar{b} = \dfrac{-\hat{i}+\sqrt{3}\ \hat{j}}{2}$

$\bar{a}+\bar{b} = \dfrac{\hat{i}+\sqrt{3}\ \hat{j}}{2}$

11. (4)

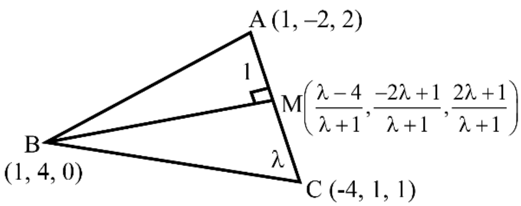
Sol. $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$

$$\left.\begin{array}{l}\vec{a}.\vec{b}+\vec{a}.\vec{c}=0\\ \vec{b}.\vec{c}+\vec{a}.\vec{b}=0\\ \vec{c}.\vec{a}+\vec{b}.\vec{c}=0\end{array}\right] \text{Þ}\ \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{a}.\vec{c}=0$$

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{9+16+25} = 5\sqrt{2}$

12. (1)

Sol.



$\overrightarrow{BM} \cdot \overrightarrow{CA} = 0$

$\Rightarrow \left(\dfrac{\lambda-4}{\lambda+1}-1\right)(1+4) + \left(\dfrac{-2\lambda+1}{\lambda+1}-4\right)$

$(-2-1) + \left(\dfrac{2\lambda+1}{\lambda+1}-0\right)(2-1) = 0$

$\left(\dfrac{(-5)(5)}{\lambda+1}\right) + \left(\dfrac{-6\lambda-3}{\lambda+1}\right)$

$(3) + \left(\dfrac{2\lambda+1}{\lambda+1}\right) = 0$

$-25 + 18l + 9 + 2l + 1 = 0$

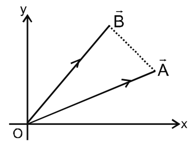
$\therefore \overrightarrow{BM} = \dfrac{-5}{\lambda+1}\hat{i} + \dfrac{-6\lambda-3}{\lambda+1}\hat{j} + \dfrac{2\lambda+1}{\lambda+1}\hat{k}$

$\overrightarrow{BM} = \dfrac{10}{7}(-2\hat{i} - 3\hat{j} + \hat{k})$

$\Rightarrow l = \dfrac{3}{4}$

13. (3)

Sol.



$\overline{OA}\times\overline{OB}$ = a fixed vector

$\Rightarrow |\overline{OA}\times\overline{OB}|$ = const. number

$\Rightarrow$ DOAB = const.

$\Rightarrow B$ is on the line || to base OA

14. (4)

Sol. $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ & $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times\vec{b} - \vec{a}\times\vec{c} = \vec{c}\times\vec{d} - \vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = (\vec{c}-\vec{b})\times\vec{d}$

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = 0$

15. (3)

Sol. Point $(2 + l, 2l - 1, -2l + 3)$

$(2, -1, 3)$ $d = 6$

$36 = l^2 + (2l)^2 + (-2l)^2$

$9l^2 = 36$

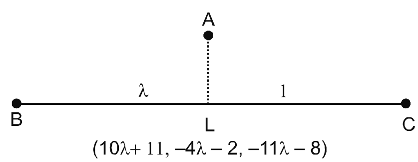
$l = 2, -2$

$l = 2$ $P(4, 3, -1)$

$l = -2$ $P(0, -5, 7)$

16. (3)

Sol.



DR's of line BC º $(10, -4, -11)$

DR's of line $AL = (10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$

$10(10\lambda + 9) + (4\lambda + 1)4 + (11\lambda + 13)11 = 0$

$\lambda = -1$

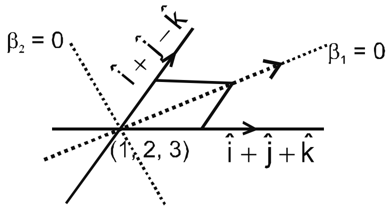
$\therefore L(1, 2, 3)$ $AL = \sqrt{14}$

17. (1)

Sol. Dr's of bisector

$\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}} + \dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}} = \lambda(\hat{i}+\hat{j})$

Hence Dr's are 1, 1, 0 $(\lambda \in R)$



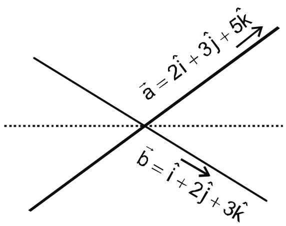
Equation of bisector

$$\frac{x-1}{\lambda}=\frac{y-2}{\lambda}=\frac{z-3}{0}$$

$$\frac{x-1}{2}=\frac{y-2}{2};\ z-3=0$$

18. **(3)**

Sol.



$$\frac{x}{2}=\frac{y}{3}=\frac{z}{5} \qquad \textbf{... (i)}$$

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \qquad \textbf{... (ii)}$$

$$\hat{a}+\hat{b}=\frac{2\hat{i}+3\hat{j}+5\hat{k}}{\sqrt{38}}+\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$$

$\Rightarrow$ (A) and (B) will be incorrect

Let the *dr*'*s* of line $\perp$ to (1) and (2) be a, b, c

$\Rightarrow 2a + 3b + 5c = 0$ ... (iii)

and $a + 2b + 3c = 0$ ... (iv)

$$\therefore \frac{a}{9-10}=\frac{b}{5-6}=\frac{c}{4-3}$$

$$\Rightarrow \frac{a}{-1}=\frac{b}{-1}=\frac{c}{1}$$

$$\Rightarrow \frac{a}{1}=\frac{b}{1}=\frac{c}{-1}$$

$\therefore$ equation of line passing through (0, 0, 0) and is $\perp r$ to the lines (1) and (2) is

$$\frac{x}{1}=\frac{y}{1}=\frac{z}{-1}$$

19. **(4)**

Sol. $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}=r$ ... (1)

$$\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4} \qquad \text{... (2)}$$

$$\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h} \qquad \text{... (3)}$$

$\because P(r, 2r, 3r)$ lies on (2)

$$\therefore \frac{r-1}{3}=\frac{2r-2}{-1}=\frac{3r-3}{4}\Rightarrow r=1$$

$\therefore$ point of intersection of (1) and (2) be (1,2,3).

(1, 2, 3) will also satisfy (3) as the lines are concurrent

$$\therefore \frac{1+k}{3}=\frac{1}{2}=\frac{1}{h}\Rightarrow h=2;\ k=\frac{1}{2}\ \text{Ans.}$$

20. **(2)**

Sol. Before rotation $\vec{a}=2p\hat{i}+\hat{j}$

after rotation $\vec{a}=(p+1)\hat{i}'+\hat{j}'$

Since length of vector remains unaltered

$$\sqrt{4p^2+1}=\sqrt{(p+1)^2+1}$$

$4p^2 = (p + 1)^2 \Rightarrow p + 1 = \pm\ 2p$

$p = 1$ or $-\frac{1}{3}$

21. **(2)**

Sol. *Dr*'*s* of diagonal $BD = a, -a, a$ or 1,–1,1

Dr'*s* of diagonal $AF = -a, a, a$ or –1, 1, 1

Angle between above diagonals

$$\cos\theta=\left|\frac{-1-1+1}{\sqrt{3}\sqrt{3}}\right|=\frac{1}{3}$$

22. **(1)**

Sol. $|\vec{e}_1-\vec{e}_2|^2<1$

$$\Rightarrow \vec{e}_1^{\,2}+\vec{e}_2^{\,2}-2\vec{e}_1.\vec{e}_2<1$$

$\Rightarrow 1 + 1 - 2\cos(2\theta) < 1$

$\Rightarrow$ 2cos2q > 1 $\Rightarrow$ cos2q > $\frac{1}{2}$

$2q \in \left[0, \frac{\pi}{3}\right) \Rightarrow \theta \in \left[0, \frac{\pi}{6}\right)$

23. **(1)**

Sol. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ given expression is

$x[y\hat{k} - z\hat{j}] + y[z\hat{i} - x\hat{k}] + z[x\hat{j} - y\hat{i}] = \vec{0}$

24. **(1)**

Sol. For option (ii)

$3\hat{i} + 3\hat{j} = 2\hat{j} + 6\hat{j} + \lambda(\hat{i} - 3\hat{j})$

$\lambda = 1$

option (iii)

$\hat{i} + 9\hat{j} = 2\hat{i} + 6\hat{j} + \lambda(\hat{i} - 3\hat{j})$

$\lambda = -1$

25. **(4)**

Sol. Equation of line

$\vec{r} = 3\hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ ......(i)

$15 = |\lambda| \times 3$

$\lambda = \pm 5$ put equation (i)

26. **(3)**

Sol. $(\vec{r} - \vec{a}) \times \vec{b} = 0$

$\vec{r} = \vec{a} + \mu\vec{b} = \vec{a} + \vec{b} = 3\hat{i} + \hat{j} - \hat{k}$

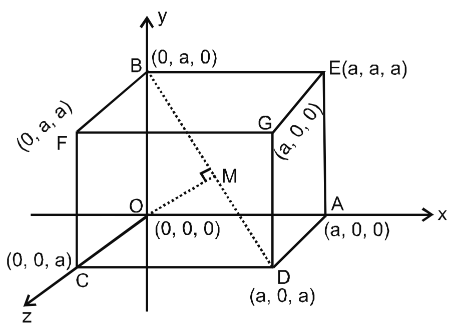
$\lambda = m = 1$

$(\vec{r} - \vec{b}) \times \vec{a} = 0$

$\therefore \vec{r} = \vec{a} + \lambda\vec{b}$ $\lambda = m = 1$

27. **(4)**

Sol.



Dr's of *BD* º *a*, – *a*, *a*

BD º *a*, – *a*, *a*

Equation of line *BD* is

$\frac{x-a}{a} = \frac{y-0}{-a} = \frac{z-a}{a} = \lambda$

Let $M = \begin{cases} x = a\lambda + a \\ y = -a\lambda \\ z = a\lambda + a \end{cases}$

OM is $\perp^{ar}$ to *BD*

$\Rightarrow a(a\lambda + a) + (-a)(-a\lambda) + a(a\lambda + a) = 0$

$\Rightarrow \lambda + 1 + \lambda + \lambda + 1 = 0 \Rightarrow \lambda = -2/3$

$M\left(\frac{a}{3}, \frac{2a}{3}, \frac{a}{3}\right)$

$OM = \sqrt{\frac{2}{3}}\, a$

28. **(1)**

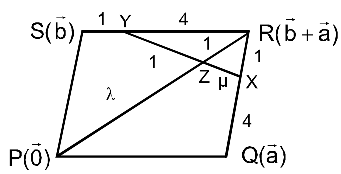
Sol. $\cos\theta = 1 - 1 - 1 = -1 < 0$

acute angle bisector

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + t\,(\hat{j} - \hat{k})$

29. **(3)**

Sol.



PVs of vertex *P*, *Q*, *R*, *S* are (Let) $\vec{0}, \vec{a}, \vec{b} + \vec{a}, \vec{b}$

using section rule *PVs* of

$X \equiv \frac{4(\vec{b} + \vec{a}) + \vec{a}}{5}$ and $Y \equiv \frac{(\vec{b} + \vec{a}) + 4\vec{b}}{5}$ b

again Let $\frac{PZ}{ZR} = \lambda$ and $\frac{XZ}{YZ} = \mu$

PVs of point *Z* may be given as

$\frac{\lambda(\vec{b} + \vec{a}) + \vec{0}}{\lambda + 1}$ & also as

$$\frac{\mu\left(\vec{b}+\frac{\vec{a}}{5}\right)+1\left(\vec{a}+\frac{4\vec{b}}{5}\right)}{\mu+1}$$

Equating both answers and coefficient of $\vec{a}\ \&\ \vec{b}$

(they are representing non collinear vectors $\overrightarrow{PQ}\ \&\ \overrightarrow{PS}$)

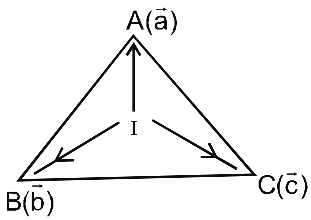
$$\frac{\lambda}{\lambda+1}=\frac{\mu+\left(\frac{1}{5}\right)}{\mu+1} \quad \text{and}$$

$$\frac{\lambda}{\lambda+1}=\frac{\left(\frac{4\mu}{5}\right)+1}{\mu+1}$$

Solving these equations gives $\lambda=\frac{21}{4}$

30. **(2)**

Sol.



$$I=\frac{a\vec{a}+b\vec{b}+c\vec{c}}{a+b+c}$$

31. **(1)**

Sol. Statement-1

$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2 \Rightarrow 1-\cos^2\alpha+1-\cos^2\beta+1-\cos^2\gamma=1$

$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ (True)

Statement-2

$l^2+m^2+n^2=1$ (True)

$l=\cos\alpha,\ m=\cos\beta,\ n=\cos\gamma$

$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

Statement - 2 explains statement – 1

32. **(2)**

Sol. Both the statements are independently true

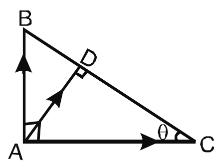
$$S.D.=\left|\frac{\begin{vmatrix}-1 & 6 & -7\\ -4 & 3 & 2\\ -4 & 1 & 1\end{vmatrix}}{\sqrt{\Sigma(mn'-m'n)^2}}\right|$$

$$=\left|\frac{-1(1)-6(4)-7(8)}{\sqrt{1^2+(4)^2+(8)^2}}\right|=\frac{81}{9}=9$$

33. **(3)**

Sol. Let $|\overrightarrow{BC}|=l$

$\therefore$ In DABC



$l=\sqrt{AB^2+AC^2}$ and $\tan\theta=\frac{AB}{AC}$

$\Rightarrow$ sinq $=\frac{AB}{\ell}$ and $\cos\theta=\frac{AC}{\ell}$

$\therefore$ Resultant vector

$=\frac{1}{AB}\hat{i}+\frac{1}{AC}\hat{j}=\left(\frac{1}{\ell\sin\theta}\hat{i}+\frac{1}{\ell\cos\theta}\hat{j}\right)=$ k $\overrightarrow{AD}$

Now $AD=AC\sin\theta=l\cos\theta\sin\theta$

$=\frac{AB.AC}{\ell}$ ... (i)

Magnitude of resultant vector

$$\sqrt{\frac{1}{\ell^2}\left(\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}\right)}$$

$=\frac{\ell}{(AB)(AC)}=\frac{1}{AD}$ [from Eq (i)]

34. **(4)**

Sol. $\overrightarrow{AC}\cdot\overrightarrow{BC}=0$

$\Rightarrow \left\{(a-2)\hat{i}-2\hat{j}\right\}\cdot\{(a-1)\hat{i}+6\hat{k}\}=0$

$\Rightarrow (a-2)(a-1)=0$

35. **(4)**

Sol. Given equations can be rewritten as $\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$

And $\frac{x-b'}{a'}=\frac{y-0}{1}=\frac{z-d'}{c'}$

These lines will be perpendicular, if $aa' + cc' + 1 = 0$

36. **(4)**

Sol. $|2\hat{u}\times 3\hat{v}|=6\sin\theta=1$ Þ sinq = $\frac{1}{6}$

as $\theta\in\left(0,\frac{\pi}{2}\right)$

So only one value of θ is possible.

37. **(4)**

Sol. Since, a line makes an angle of $\frac{\pi}{4}$ with positive directions of each of x and y-axis, therefore

$\alpha=\frac{\pi}{4}, \beta=\frac{\pi}{4}$

We know, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\therefore \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\gamma=1$

$\Rightarrow \frac{1}{2}+\frac{1}{2}+\cos^2\gamma=1$

$\Rightarrow \cos^2\gamma=0 \Rightarrow \gamma=\frac{\pi}{2}$

38. **(4)**

Sol. $\cos\theta=\frac{\vec{a}.\vec{c}}{|\vec{a}||\vec{c}|}=\frac{-56|\vec{b}|^2}{56|\vec{b}|^2}$

$\Rightarrow \cos\theta=-1$

$\Rightarrow \theta=\pi$

39. **(3)**

Sol. Equation of line passing through (5, 1, a) and (3, b, 1) is

$\frac{x-3}{5-3}=\frac{y-b}{1-b}=\frac{z-1}{a-1}$ ....(i)

Point $\left(0,\frac{17}{2},-\frac{13}{2}\right)$ satisfies equation (i), we get

$-\frac{3}{2}=\frac{\frac{17}{2}-b}{1-b}=\frac{-\frac{13}{2}-1}{a-1}$

$\Rightarrow a-1=\frac{\left(-\frac{15}{2}\right)}{\left(-\frac{3}{2}\right)}=5$

$\Rightarrow a=6$

Also, $-3(1-b)=2\left(\frac{17}{2}-b\right)$

$\Rightarrow 3b-3=17-2b$

$\Rightarrow 5b=20 \qquad \Rightarrow b=4$

40. **(1)**

Sol. Given, $\frac{x-1}{k}=\frac{y-2}{2}\ \frac{z-3}{3}$ ......(i)

and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ ....(i)

Since, lines intersect at a point. Then shortest distance between them is zero.

$\therefore \begin{vmatrix} k & 2 & 3 \\ 3 & k & 2 \\ 1 & 1 & -2 \end{vmatrix}=0$

$\Rightarrow k(-2k-2)-2(-6-2)+3(3-k)=0$

$\Rightarrow -2k^2-5k+25=0$

$\Rightarrow 2k^2+5k-25=0$

$\Rightarrow 2k^2+10k-5k-25=0$

$\Rightarrow 2k(k+5)-5(k+5)=0$

$\Rightarrow k=\frac{5}{2},-5$

Hence integer value of k is -5.

41. **(3)**

Sol. $lr = 6, mr = -3, nr = 2$

$\therefore r^2 (l^2 + m^2 + n^2) = 36 + 9 + 4 = 49$

$\Rightarrow r = 7$

$< l, m, n > \equiv < \frac{6}{7}, -\frac{3}{7}, \frac{2}{7} >$

42. **(4)**

Sol. Since $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times \vec{c} = \vec{0}$

$\Rightarrow (\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} + \vec{a} \times \vec{c} = \vec{0}$

Since $\vec{a} \times \vec{c} = -2\hat{i} - \hat{j} - \hat{k}$

$\Rightarrow 3(\hat{j} - \hat{k}) - 2\vec{b} - 2\hat{i} - \hat{j} - \hat{k} = \vec{0}$

$\Rightarrow \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

Hence correct option is (4)

43. **(4)**

Sol. $\because \vec{a}, \vec{b}, \vec{c}$ are mutually orthogonal

$\therefore \vec{a}.\vec{c} = 0 \Rightarrow \lambda - 1 + 2m = 0$ ....(i)

and

$\vec{b}.\vec{c} = 0 \Rightarrow 2\lambda + 4 + m = 0$ ....(ii)

solving (i) and (ii), we get $\lambda = -3$ and $m = 2$

$\lambda = -3 = 2$

Hence correct option is (4)

44. **(2)**

Sol. $l = \frac{1}{\sqrt{2}}, \quad m = -\frac{1}{2}$

$l^2 + m^2 + n^2 = 1 \Rightarrow n^2 = \frac{1}{4} \Rightarrow n = \pm \frac{1}{2}$

$\cos\theta = \frac{1}{2}, \theta = 60^o$

Hence correct option is (2)

45. **(4)**

Sol. $\vec{a}.\vec{b} \neq 0, \vec{b} \times \vec{c} = \vec{b} \times \vec{d}, \vec{a}.\vec{d} = 0$

$(\vec{b} \times \vec{c}) \times \vec{a} = (\vec{b} \times \vec{d}) \times \vec{a}$

$(\vec{b}.\vec{a})\vec{c} - (\vec{c}.\vec{a})\vec{b} = (\vec{b}.\vec{a})\vec{d} - (\vec{d}.\vec{a})\vec{b}$

$\vec{d} = \vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

46. **(3)**

Sol. $\vec{a} + 3\vec{b} = \lambda\vec{c}$ ....(1)

$\vec{b} + 2\vec{c} = \mu\vec{a}$ ....(2)

(1) – 3(2) gives $(1 + 3\mu)\ \vec{a} - (l + 6)\ \vec{c} = 0$

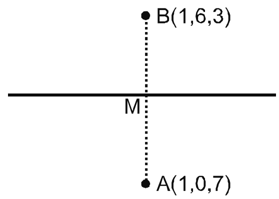
As $\vec{a}$ *and* $\vec{c}$ are non collinear

$\therefore 1 + 3\mu = 0$ and $l + 6 = 0$

From (1) $\vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$

47. **(2)**

Sol.



Mid- point of AB º $M(1,3,5)$

M lies on line

Direction ratios of AB is < 0, 6, – 4 >

Direction ratios of given line is < 1, 2, 3 >

As AB is perpendicular to line

$\therefore 0.1 + 6.2 - 4.3 = 0$

48. **(3)**

Sol. Let foot of perpendicular is $(2\alpha, 3\alpha + 2, 4\alpha + 3)$

$\Rightarrow D$' ratio of the perpendicular line $< 2\alpha - 3, 3\alpha + 3, 4\alpha - 8 >$

And D' ratio of the line < 2, 3, 4 >

$\Rightarrow 2(2\alpha - 3) + 3(3\alpha + 3) + 4(4\alpha - 8) = 0$

$\Rightarrow 29\alpha - 29 = 0$

$\Rightarrow \alpha = 1$

$\Rightarrow$ feet of perpendicular is (2, 5, 7)

$\Rightarrow$ length of perpendicular is $\sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

49. **(3)**

Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

$\vec{a}$ (1, – 1, 1); $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{b}$ (2, 3, 4)

$\vec{c}$ (3, k, 0); $\vec{r} = \vec{c} + \mu\vec{d}$

$\vec{d}$ (1, 2, 1)

These lines will intersect if lines are coplanar

$\vec{a} - \vec{c}$, $\hat{b}$ & $\vec{d}$ are coplanar

$\therefore [\vec{a} - \vec{c}, \vec{b}, \vec{d}] = 0$

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-5) - (k+1)(-2) - 1(1) = 0$

$\Rightarrow 2(k+1) = 11$

$\Rightarrow k = \dfrac{9}{2}$

50. **(3)**

Sol. $\vec{c} = \hat{a} + 2\hat{b}$

$\vec{d} = 5\hat{a} - 4\hat{b}$

$\vec{c}.\vec{d} = 0$

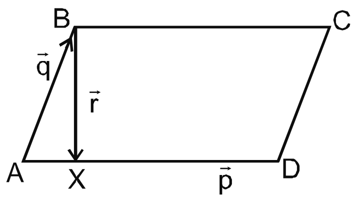
$\Rightarrow (\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0$

$\Rightarrow 5 + 6\hat{a}.\hat{b} - 8 = 0$

$\Rightarrow \hat{a}.\hat{b} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

51. **(2)**

Sol. $\overline{AX} = \dfrac{\vec{p}.\vec{q}}{|\vec{p}|}\dfrac{\vec{p}}{|\vec{p}|} = \dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$



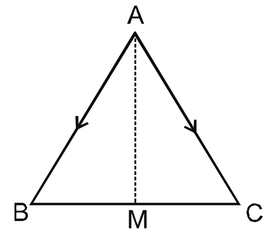
$\overline{BX} = \overline{BA} + \overline{AX} = -\vec{q} + \dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

52. **(3)**

Sol. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

$\Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$\Rightarrow \overrightarrow{BM} = \dfrac{\overrightarrow{AC} - \overrightarrow{AB}}{2}$



$\Rightarrow \overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA} = 0$

$\Rightarrow \overrightarrow{AB} + \dfrac{\overrightarrow{AC} - \overrightarrow{AB}}{2} = \overrightarrow{AM}$

$\Rightarrow \overrightarrow{AM} = \dfrac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow |\overrightarrow{AM}| = \sqrt{33}$

53. **(3)**

Sol. $\ell + m + n = 0$ ....(1)

$\ell^2 = m^2 + n^2$ ....(2)

$\Rightarrow \ell^2 - m^2 - (-l - m)^2 = 0$

$\Rightarrow 2m(m + l) = 0$

$m = 0$ or $l = -$ m

so direction ratios are – 1, 0, 1 and – 1, 1, 0

$$\cos\theta = \left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$$

$\Rightarrow \cos\theta = \left|\dfrac{1+0+0}{\sqrt{2}\quad\sqrt{2}}\right| = \dfrac{1}{2}$

$\therefore \theta = \dfrac{\pi}{3}$

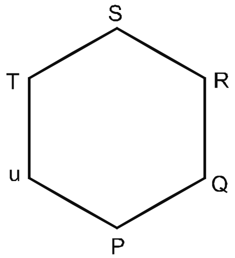
54. **(3)**

Sol. Statement - 1

$\because \overrightarrow{RS} + \overrightarrow{ST} = \overrightarrow{RT}$

and $\overrightarrow{RT}$ is not parallel to $\overrightarrow{PQ}$

So $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq 0$



Statement - 2
while PQ & RS are also non parallel

So, $\overrightarrow{PQ} \times \overrightarrow{RS} \neq 0,\ \overrightarrow{PQ} \times \overrightarrow{ST} = 0$

55. **(2)**

Sol. $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\vec{a}+\vec{b}=-\vec{c}$

For a triangle

$\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a} \neq 0$

56. **(3)**

Sol. Line is

$$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{0} = a \qquad ......(1)$$

$Q(\alpha, \alpha, 1)$

Direction ratio of PQ are

$\lambda - \alpha, \lambda - \alpha, \lambda - 1$

Since PQ is perpendicular to (1)

$\therefore \lambda - \alpha + \lambda - \alpha + 0 = 0$

$\lambda = \alpha$

$\therefore$ Direction ratio of PQ are

$0, 0, \lambda - 1$

Another line is

$$\frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0} = b \qquad ......(2)$$

$\therefore R(-\beta, \beta, -1)$

$\therefore$ Direction ratio of PR are

$\lambda + \beta, \lambda - \beta, \lambda + 1$

Since PQ is perpendicular to (ii)

$\therefore -\lambda - \beta + \lambda - \beta = 0$

$\beta = 0$

$\therefore R(0, 0, -1)$

and Direction ratio of PQ are $\lambda, \lambda, \lambda + 1$

Since $PQ \perp PR$

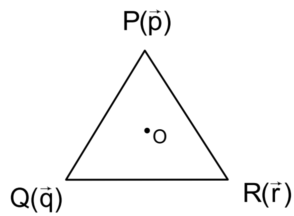
$\therefore 0 + 0 + \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \Rightarrow B, C$

For l = 1 the point is on the line so it will be rejected.

$\Rightarrow \lambda = -1.$

57. **(2)**

Sol.



$\vec{p}.\vec{q} + \vec{r}.\vec{s} = \vec{r}.\vec{p} + \vec{q}.\vec{s} = \vec{q}.\vec{r} + \vec{p}.\vec{s}$

$\Rightarrow \vec{p}.(\vec{q}-\vec{r}) - \vec{s}.(\vec{q}-\vec{r}) = 0 \Rightarrow \overrightarrow{PS}.\overrightarrow{QR} = 0$

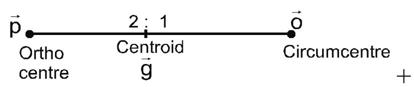
Similarly $\overrightarrow{PQ}.\overrightarrow{SR} = 0$

$\Rightarrow S$ is orthocentre of the triangle

Integer Type Questions (58to65)

58. **(3)**

Sol.



$$\vec{g} = \frac{2.\vec{0}+1.\vec{p}}{2+1}$$

$\Rightarrow \vec{p} = 3\,\vec{g} \qquad \therefore k = 3$

59. (5)

Sol. $(\vec{a}+P\vec{b}).\ \vec{c}\ =0 \Rightarrow P=-\dfrac{\vec{a}.\vec{c}}{\vec{b}.\vec{c}}=5$

60. (5)

Sol. $\vec{u}+\vec{v}+\vec{w}=0$

$\Rightarrow |\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2+2(\vec{u}.\vec{v})+2(\vec{v}.\vec{w})+2(\vec{w}.\vec{u})=0$

$\Rightarrow 9+16+25+2\ [\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}]=0$

$\Rightarrow \sqrt{|\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}|}=5$

61. (7)

Sol. $\lambda=\dfrac{\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}}{\dfrac{\vec{a}.\vec{b}}{|\vec{a}|}}=\dfrac{|\vec{a}|}{|\vec{b}|}=\dfrac{7}{3}$

62. (2)

Sol. $\lambda(\ \vec{b}\times\vec{a}\ )+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

$\{\vec{a}+\vec{b}+\vec{c}=\vec{0}\ \therefore \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\}$

$\Rightarrow \lambda\vec{b}\times\vec{a}\ +\vec{a}\times\vec{b}\ +\ \vec{a}\times\vec{b}=\vec{0}$

$\Rightarrow \lambda=2$

63. (16)

Sol. $\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c}\end{vmatrix}=[\vec{a}\,\vec{b}\,\vec{c}]^2=4^2$

$=16$

64. (2)

Sol. $\vec{a}=2\hat{i}+\hat{j}-2\hat{k},\ \vec{b}=\hat{i}+\hat{j},|\vec{c}-\vec{a}|=3$

$|(\vec{a}\times\vec{b})\times\vec{c}|=3,\ \ \vec{c}\wedge\vec{a}\times\vec{b}=\dfrac{\pi}{6}$

Now $|\vec{a}\times\vec{b}||\vec{c}|\ \sin30°=3,$

$|\vec{a}\times\vec{b}||\vec{c}|=6$

$\Rightarrow |\vec{a}||\vec{b}||\vec{c}|\sin\theta=6,\ \theta=\vec{a}\wedge\vec{b}$

$|\vec{a}|=3,\ |\vec{b}|=\sqrt{2}\ \theta=\cos^{-1}\left(\dfrac{2+1}{3\sqrt{2}}\right)=\dfrac{\pi}{4}$

$|\vec{c}|=\dfrac{6}{3\sqrt{2}}.\sqrt{2}=2$

$|\vec{c}-\vec{a}|=3$

Squaring, we get $|\vec{c}|^2-2\vec{a}.\vec{c}+|\vec{a}|^2=9$

$\Rightarrow \vec{a}.\vec{c}=\dfrac{|\vec{c}|^2}{2}=2$

65. (4)

Sol. Let $\vec{c}=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{a}\times\vec{c}=\vec{c}\times\vec{b}$

$\Rightarrow (\vec{a}+\vec{b})\times\vec{c}=\vec{0}$

$\Rightarrow (\vec{a}+\vec{b})\parallel\vec{c}$

Let $(\vec{a}+\vec{b})=\lambda\vec{c}$

$\Rightarrow |\vec{a}+\vec{b}|=|\lambda|\ |\vec{c}|$

$\Rightarrow \sqrt{29}=|\lambda|.\sqrt{29}$

$\Rightarrow \lambda=\pm 1$

$\therefore\ \vec{a}+\vec{b}=\pm\ (2\hat{i}+3\hat{j}+4\hat{k})$

Now $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$

$=\pm(-14+6+12)=\pm 4$

LIMITS

Single Option Correct Type Questions (01 to 65)

1. **(4)**

Sol. $\lim_{x\to 2}\left\{\frac{x}{2}\right\}$

R.H.L. = $\lim_{h\to 0}\left\{\frac{2+h}{2}\right\} = \lim_{h\to 0}\left\{1+\frac{h}{2}\right\}$

$= \lim_{h\to 0}\left\{\frac{h}{2}\right\} = \lim_{h\to 0}\frac{h}{2} = 0$

L.H.L. = $\lim_{h\to 0}\left\{\frac{2-h}{2}\right\} = \lim_{h\to 0}\left\{1-\frac{h}{2}\right\}$

$= \lim_{h\to 0}\left\{1-\frac{h}{2}\right\} = 1$

L.H.L. ≠ R.H.L.

so $\lim_{x\to 0}\left\{\frac{x}{2}\right\}$ does not exist.

2. **(4)**

Sol. $\lim_{x\to\pi}$ sgn [tan x]

L.H.L. = $\lim_{x\to\pi^-}$ sgn [tan x]

= $\lim_{h\to 0}$ sgn [tan ($\pi - h$)]

= $\lim_{h\to 0}$ sgn (– ve) = –1

R.H.L. = $\lim_{x\to\pi^+}$ sgn [tan x]

= $\lim_{h\to 0}$ sgn [tan($\pi + h$)]

= $\lim_{h\to 0}$ sgn (+ ve) = 0

L.H.L. ≠ R.H.L.

so limit does not exist

3. **(3)**

Sol. $\lim_{x\to 1}$ $(1 - x + [x - 1] + [1 - x])$

L.H.L.= $\lim_{x\to 1^-}$ $(1 - x + [x - 1] + [1 - x])$

= $\lim_{h\to 0}$ $(1 - (1-h) + [1 - h - 1] + [1 - 1 + h])$

= $\lim_{h\to 0}$ $(h + [-h] + [h])$

= 0 – 1 + 0 = – 1

R.H.L.= $\lim_{x\to 1^+}$ $(1 - x + [x - 1] + [1 - x])$

= $\lim_{h\to 0}$ $(1 - (1+ h) + [1 + h - 1] + [1 - (1 + h)]$

= $\lim_{h\to 0}$ $(- h + [h] + [-h])$

= 0 + 0 – 1 = – 1

L.H.L. = R.H.L. = – 1

so $\lim_{x\to 1}$ $(1 - x + [x - 1] + [1 - x])$ = – 1

4. **(3)**

Sol. $f(x) = \frac{|x+\pi|}{\sin x}$

$f(-\pi^+) = \lim_{h\to 0}\frac{|-\pi+h+\pi|}{\sin(-\pi+h)}$

$= \lim_{h\to 0}\frac{|h|}{\sin h} = -1$

$f(-\pi^-) = \lim_{h\to 0}\frac{|-\pi-h+\pi|}{\sin(-\pi-h)}$

$= \lim_{h\to 0}\frac{|h|}{\sin h} = 1$

$f(-\pi^+) \neq f(-\pi^-)$

so $f(x)$ does not exist

5. (4)

Sol. $\lim_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

$=\lim_{x\to a}\dfrac{a+2x-3x}{3a+x-4x}\cdot\dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$\lim_{x\to a}\dfrac{(a-x)}{3(a-x)}\cdot\dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$=\dfrac{1}{3}\cdot\dfrac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}=\dfrac{2}{3\sqrt{3}}$

6. (2)

Sol. $\lim_{x\to 0}\dfrac{\sqrt{2}\left(1-\cos\dfrac{x}{2}\right)}{4\sin^2\dfrac{x}{2}\cos^2\dfrac{x}{2}}$

$=\lim_{x\to 0}\dfrac{2\sqrt{2}\sin^2\left(\dfrac{x}{4}\right)}{16\sin^2\dfrac{x}{4}.\cos^2\dfrac{x}{4}\cos^2\dfrac{x}{2}}=\dfrac{\sqrt{2}}{8}$

7. (4)

Sol. $\lim_{x\to a}\dfrac{(x-b)-(a-b)}{(x^2-a^2)(\sqrt{x-b}+\sqrt{a-b})}$

8. (2)

Sol. $\lim_{x\to 0^+}\dfrac{\sqrt{x}}{\left(\dfrac{e^x-1}{x}\right)^{3/2}}\cdot\dfrac{\tan x}{x\sqrt{x}}=1$

9. (2)

Sol. $\lim_{x\to 0}\dfrac{(4^x-1)^3}{\sin\left(\dfrac{x}{p}\right)\ell n\left(1+\dfrac{x^2}{3}\right)}$

$=\lim_{x\to 0}\dfrac{x^3\left(\dfrac{4^x-1}{x}\right)^3}{\left(\dfrac{x}{p}\right)\left[\dfrac{\sin\left(\dfrac{x}{p}\right)}{\dfrac{x}{p}}\right]\ell n\left(1+\dfrac{x^2}{3}\right)}$

$=3p\lim_{x\to 0}\dfrac{\left(\dfrac{4^x-1}{x}\right)^3}{\left[\dfrac{\sin\left(\dfrac{x}{p}\right)}{\dfrac{x}{p}}\right]}\cdot\dfrac{\left(\dfrac{x^2}{3}\right)}{\ell n\left(1+\dfrac{x^2}{3}\right)}=3p(\ell n4)^3$

10. (3)

Sol. $\lim_{x\to 0}\dfrac{1}{(\sqrt{1+x^2}(1+\sqrt{1-x^2})\left(\dfrac{\sin^{-1}x}{x}\right)^3}$

$=\dfrac{1}{1\cdot 2\cdot 1}=\dfrac{1}{2}$

11. (4)

Sol. $\lim_{x\to 0}\dfrac{\alpha\left(\dfrac{e^{\alpha x}-1}{\alpha x}\right)-\left(\dfrac{e^{\beta x}-1}{\beta x}\right)\beta}{\alpha\left(\dfrac{\sin\alpha x}{\alpha x}\right)-\beta\left(\dfrac{\sin\beta x}{\beta x}\right)}$

$=\dfrac{\alpha-\beta}{\alpha-\beta}=1$

12. (3)

Sol. $\lim_{x\to 0}\dfrac{(\cos 2x-\cos 4x).\cos x.\cos 3x}{(\cos x-\cos 3x).\cos 2x.\cos 4x}$

$=\lim_{x\to 0}\dfrac{2\sin 3x.\sin x.\cos x.\cos 3x}{2\sin 2x.\sin x.\cos 2x.\cos 4x}$

$=\lim_{x\to 0}\dfrac{3}{2}\cdot\dfrac{\sin(3x)}{3x}\cdot\dfrac{2x}{\sin 2x}\cdot\dfrac{\cos x\cos 3x}{\cos 2x.\cos 4x}=\dfrac{3}{2}$

13. (2)

Sol. $\lim_{x\to 2}\dfrac{(x+6)^{1/3}-2}{2-x}$ put $x = 2 + h$

$=\lim_{h\to 0}\dfrac{(h+8)^{1/3}-2}{-h}=\lim_{h\to 0}\dfrac{2\left\{\left(1+\dfrac{h}{8}\right)^{\frac{1}{3}}-1\right\}}{-h}$

$$= \lim_{h\to 0} \frac{2\left\{1+\frac{1}{3}\cdot\frac{h}{8}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{h}{8}\right)^2+\ldots-1\right\}}{-h}$$

$$= -\frac{1}{12}$$

14. (1)

Sol. $$\lim_{x\to\infty} \frac{x\left(\sqrt{3-\frac{1}{x^2}}-\sqrt{2-\frac{1}{x^2}}\right)}{x(4+3/x)} = \frac{\sqrt{3}-\sqrt{2}}{4}$$

15. (3)

Sol. $$\lim_{n\to\infty} \frac{n^2(n+1)^2}{4(1-n^4)}$$

$$= \lim_{n\to\infty} = \frac{n^4\left(1+\frac{1}{n}\right)^2}{n^4(4)\left(\frac{1}{n^4}-1\right)} - \frac{1}{4}$$

$$= -\frac{-1}{4}$$

16. (1)

Sol. $$\lim_{n\to\infty} = \frac{1(1-5^n)}{(1-5)(1+5^n)\,(1-5^n)} = 0$$

17. (1)

Sol. $$\lim_{n\to\infty} \frac{(n+2)!(n+4)}{(n+4)(n+3)(n+2)!} = 0$$

18. (3)

Sol. $$\lim_{n\to\infty}\left(\sqrt{(x+a)(x+b)}-x\right)$$

$$= \lim_{n\to\infty} \frac{(x+a)(x+b)-x^2}{\sqrt{(x+a)(x+b)}+x}$$

$$= \lim_{n\to\infty} \frac{(a+b)x+ab}{\left[\sqrt{1+\frac{(a+b)}{x}+\frac{ab}{x^2}}+1\right]}$$

$$= \frac{a+b}{1+1} = \frac{a+b}{2}$$

19. (1)

Sol.

$$\lim_{x\to\infty} \frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+\ldots\ldots+\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left(1+\left(\frac{10}{x}\right)^{10}\right)}$$

= 1 + 1 +.......+ 100 terms

= 100

20. (2)

Sol.

$$\lim_{x\to\infty} \frac{1+\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}\ldots\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)-\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\ldots\right)}{x^3}$$

$$= \lim_{x\to\infty} \frac{\frac{-3x^3}{3!}-\left(\frac{1}{4!}+\frac{1}{4!}\right)x^4+\ldots..}{x^3} = -\frac{3}{6} = -\frac{1}{2}$$

21. (3)

Sol. By L.H. Rule

$$= \lim_{x\to 1} \frac{\frac{1}{x}-1}{-2+2x}$$

again by L.H. rule

$$= \lim_{x\to 1} -\frac{1}{2x^2} = -1/2$$

22. (4)

Sol. $$\lim_{h\to 0} \frac{1}{2h}\left(\left(1+\frac{h}{8}\right)^{-\frac{1}{3}}-1\right)$$

$$= \lim_{h\to 0} \frac{1}{2h}\left[1-\frac{h}{24}+\frac{\frac{1}{3}\left(\frac{1}{3}+1\right)}{2!}\left(\frac{h}{8}\right)^2 \ldots\ldots\ldots\infty-1\right]$$

$$= -\frac{1}{48}$$

23. (2)

Sol. $\lim\limits_{x\to 0}\dfrac{ax\left(1+x+\dfrac{x^2}{2!}+...\right)-b\left(x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-...\right)+cx\left(1-x+\dfrac{x^2}{2!}-\dfrac{x^3}{3!}+....\right)}{x^2\left(x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}...........\right)}$

$=2$

$\lim\limits_{x\to 0}\dfrac{x(a-b-c)=x^2\left(a+\dfrac{b}{2}-c\right)+x^3\left(\dfrac{a}{2}-\dfrac{b}{3}+\dfrac{c}{2}\right)+x^4\text{ and term containing higher power}}{x^3-\dfrac{x^5}{3!}+\dfrac{x^7}{5!}-.......}$

$=2$

then $a - b + c = 0$ .....(1)

$2a + b - 2c = 0$ .....(2)

$3a - 2b + 3c = 12$ .....(3)

On solving (1), (2), (3)

we get $a = 3,\ b = 12,\ c = 9$

24. (2)

Sol. $\lim\limits_{x\to 1}\dfrac{\ell nx}{\cos\left(\dfrac{\pi}{2x}\right)}$

By L.H. rule

$=\lim\limits_{x\to 1}+1\dfrac{1}{x.\sin\left(\dfrac{\pi}{2x}\right).\dfrac{\pi}{2x^2}}$

$=\lim\limits_{x\to 1}\dfrac{2x}{\pi\sin\left(\dfrac{\pi}{2x}\right)}=\dfrac{2}{\pi}$

25. (2)

Sol. $\lim\limits_{x\to a}\dfrac{a^x-x^a}{x^x-a^a}=-1$

$\Rightarrow \lim\limits_{x\to a}\dfrac{a^x\ell na-ax^{a-1}}{x^x\ell nex}=-1$

By L.H. rule

$\Rightarrow \dfrac{a^x\ell na-a\cdot a^{a-1}}{a^a\ell nea}=-1$

$\Rightarrow \ell n(a/e) = -\ \ell n\ ea$

$\Rightarrow \dfrac{a}{e}=\dfrac{1}{ea}\quad \Rightarrow a = 1$

26. (1)

Sol. $\lim\limits_{x\to 0}\dfrac{(a+x)^2\sin(a+x)-a^2\sin a}{x}$

$\left(\dfrac{0}{0}\text{form}\right)$

using L' Hospital rule

$=\lim\limits_{x\to 0}\dfrac{2(a-x)\sin(a+x)+(a+x)^2\cos(a+x)}{1}$

$= 2a\sin a + a^2\cos a$

27. (1)

Sol. $\lim\limits_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{5}{x}}$

$=\lim\limits_{h\to 0}\left(1+\tan^2\sqrt{0+h}\right)^{\frac{5}{0+h}}$

$=\lim\limits_{h\to 0}\left(1+\tan^2\sqrt{h}\right)^{\frac{5}{h}}$ (1^∞ form)

$e^{\lim\limits_{h\to 0}\frac{5\tan^2\sqrt{h}}{h}}=e^{\lim\limits_{h\to 0}5\left(\frac{\tan\sqrt{h}}{\sqrt{h}}\right)^2}=e^5$

28. (3)

Sol. α, β are the roots of the equation $ax^2 + bx + c = 0$

$\because\ ax^2 + bx + c = a(x-\alpha)\ (x-\beta)$

$=\lim\limits_{x\to\alpha}\left(1+ax^2+bx+c\right)^{\frac{1}{x-\alpha}}$

$= e^{\lim\limits_{x\to\alpha}\frac{ax^2+bx+c}{x-\alpha}}$

$= e^{\lim\limits_{x\to\alpha}\frac{a(x-\alpha)(x-\beta)}{(x-\alpha)}}$

$= e^{a(\alpha-\beta)}$

29. (3)

Sol. $\lim\limits_{x\to\infty}\left(\dfrac{x^2-2x+1}{x^2-4x+2}\right)^x$

$= e^{\lim\limits_{x\to\infty}\left(\frac{x^2-2x+1-x^2+4x-2}{x^2-4x+2}\right)x}$

$$= e^{\lim_{x\to 0}\left(\frac{2x-1)x}{x^2-4x+2}\right)}$$

$$= e^{\lim_{x\to\infty}\left(\frac{2-\frac{1}{x}}{1-\frac{4}{x}+\frac{2}{x^2}}\right)} = e^2$$

30. (4)

Sol. $y = \lim_{x\to 0^+}(\operatorname{cosec} x)^{1/\ell nx}$

$$\ell ny = \lim_{x\to 0^+} \cdot \frac{1}{\ell nx}\ell n(\operatorname{cosec}(x))$$

$$= -\lim_{x\to 0^+}\frac{\ell n(\sin x)}{\ell nx}$$

$$= -\lim_{x\to 0^+}\frac{1}{\sin x}\cdot\frac{\cos x}{1/x}$$

$$= -\lim_{x\to 0^+}\cos x\cdot\frac{1}{\frac{\sin x}{x}}$$

$= -1 \quad y = 1/e$

31. (4)

Sol. $\lim_{n\to\infty}\frac{[1\cdot 2x]+[2\cdot 3x]+......+[n\cdot(n+1)x]}{n^3}$

$(1\cdot 2)x - 1 < [1\cdot 2x] \le (1\cdot 2)x$

$(2\cdot 3)x - 1 < [2\cdot 3x] \le (2\cdot 3)x$

$n(n+1)\,x - 1 < [n\,(n+1)x] \le n(n+1)x$

so $(1\cdot 2)x + (2\cdot 3)x + \ldots\, n\,(n+1)x - n$

$< [1\cdot 2\,x] + [2\cdot 3\,x] + \ldots.\ [n(n+1)x]$

$\le (1\cdot 2)\,x + (2\cdot 3)\,x + \ldots.\ n(n+1)x$

$x\ .\ (\Sigma n^2 + \Sigma n) - n \le [1\cdot 2x] + [2\cdot 3x] +......[n(n+1)x] \le x\,(\Sigma n^2 + \Sigma n)$

$$\lim_{n\to\infty}\frac{x.\left[\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]-n}{n^3}$$

$$< \lim_{n\to\infty}\frac{[1\cdot 2x]+[2\cdot 3x]+.......+[n(n+1)x]}{n^3}$$

$$\le \lim_{n\to\infty}\frac{x\left[\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]}{n^3}$$

$$\lim_{n\to\infty} x\left[\frac{1.\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}+\left(\frac{\frac{1}{n}+\frac{1}{n^2}}{2}\right)\right]-\frac{1}{h^2}$$

$$< \lim_{n\to\infty}\frac{[1\cdot 2x]+[2\cdot 3x]+.......+[n(n+1)x]}{n^3}$$

$$\le$$

$$\lim_{n\to\infty} x\left[\frac{1.\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}+\left(\frac{\frac{1}{n}+\frac{1}{n^2}}{2}\right)\right]$$

$$\frac{x}{3} < \lim_{n\to\infty}\frac{[1\cdot 2x]+[2\cdot 3x]+....+[n(n+1)x]}{n^3} \le \frac{x}{3}$$

so $\lim_{n\to\infty}\frac{[1\cdot 2x]+[2\cdot 3x]+....+[n(n+1)x]}{n^3} = \frac{x}{3}$

32. (2)

Sol. $\lim_{x\to 0}\frac{\sin(6x^2)}{\ell n\cos\ (2x^2-x)}$

$$= \lim_{x\to 0}\frac{\sin\ 6x^2}{6x^2}\cdot\frac{6x^2}{\ell n(\cos(2x^2-x))}$$

$$= 1.\ \lim_{x\to 0}\frac{6x^2}{\ell n(\cos(2x^2-x))}\ \left(\frac{0}{0}\ form\right)$$

$$= \lim_{x\to 0}\frac{12x}{\frac{(-\sin(2x^2-x))\ (4x-1)}{\cos\ (2x^2-x)}}$$

$$= \lim_{x\to 0}\ \frac{-12\ \cos(2x^2-x)}{4x-1}\ .\ \lim_{x\to 0}\frac{x}{\sin(2x^2-x)}$$

$$= \left(\frac{-12}{-1}\right).\ \lim_{x\to 0}\frac{1}{(\cos(2x^2-x))(4x-1)} = 12.$$

$$\left(\frac{1}{-1}\right) = -12$$

33. (2)

Sol. $\lim_{x\to 0^+}\frac{\cos^{-1}(1-x)}{\sqrt{x}}$

$= \lim_{h\to 0}\frac{\cos^{-1}[1-(0+h)]}{\sqrt{0+h}}$

$= \lim_{h\to 0}\frac{\cos^{-1}(1-h)}{\sqrt{h}}$

Let 1 – h = cos θ

$\sin\theta = \sqrt{1-(1-h)^2}$

$\therefore\quad \theta = \sin^{-1}\sqrt{2h-h^2}$

$= \lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{h}}$

$= \lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{2h-h^2}}\cdot\frac{\sqrt{2h-h^2}}{\sqrt{h}}$

$= 1\times\sqrt{2} = \sqrt{2}$

34. **(2)**

Sol. $\lim_{n\to\infty} n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{\pi}{4n}\right)$

$= \lim_{n\to\infty}\cos\left(\frac{\pi}{4n}\right)\lim_{n\to\infty}\frac{\sin\left(\frac{\pi}{4n}\right)}{\left(\frac{\pi}{4n}\right)}\times\frac{\pi}{4}$

$= 1\,.\,1\,.\,\frac{\pi}{4} = \frac{\pi}{4}$

35. **(2)**

Sol. $\lim_{x\to 1}\frac{\left(\sum_{k=1}^{100}x^k\right)-100}{x-1}$

$= \lim_{x\to 1}\left[\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+.....+\frac{x^{100}-1}{x-1}\right]$

$= 1+2+3+.....+100 = \frac{(100).(101)}{2}$

$= 5050$

36. **(1)**

Sol. Case - 1 when $h\in Q$

L.H.L. $= \lim_{h\to 0^+} f(0-h)$

$= \lim_{h\to 0^+}(-h)$

$= 0$

R.H.L. $= \lim_{h\to 0^+} f(0+h)$

$= \lim_{h\to 0^+}(+h)$

$= 0$

Case-2 : When $h\in Q^C$

L.H.L. $= \lim_{h\to 0^+} f(0-h)$

$= \lim_{h\to 0^+} h$

$= 0$

R.H.L. $= \lim_{h\to 0^+} f(0+h)$

$= \lim_{h\to 0^+}(-h)$

$= 0$

Hence $\lim_{x\to 0} f(x) = 0$

37. **(3)**

Sol. $\sin h < h < \tan h \qquad h\in\left(0,\frac{\pi}{2}\right)$

$\frac{h}{\sinh} > 1 \Rightarrow \qquad \frac{-h}{\sin h} < -1$

R.L. $= \lim_{h\to 0}\left[\frac{\pi/2+h-\pi/2}{\cos(\pi/2+h)}\right] = \lim_{h\to 0}\left[\frac{-h}{\sinh}\right]$

$= -2$

L.L. = R.L. = –2

38. **(2)**

Sol. $\lim_{x\to a^-}\lim_{x\to a^-}\left(\frac{|x|^3}{a}-\left[\frac{x}{a}\right]^3\right)$

$x = a - h$

$= \lim_{h\to 0}\left(\frac{|a-h|^3}{a}-\left[\frac{a-h}{a}\right]^3\right)$

$= \lim_{h\to 0}\left(\frac{|a|^3}{a}-\left[1-\frac{h}{a}\right]^3\right)$

$= -a^2-1$

39. **(1)**

Sol. $\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}(bx-\sin x)} = 1$

$\Rightarrow \lim\limits_{x\to 0} \frac{1}{\sqrt{a+x}} \lim\limits_{x\to 0} \frac{x^3}{bx - \sin x} = 1$

$\Rightarrow \frac{1}{\sqrt{a}} . \lim\limits_{x\to 0} \frac{x^3}{bx - \left[x - \frac{x^3}{3\,!} + \frac{x^5}{5\,!} - ......\right]} = 1$

$\Rightarrow \frac{1}{\sqrt{a}} . \lim\limits_{x\to 0} \frac{x^3}{(b-1)\ x + \frac{x^3}{3\,!} - \frac{x^5}{5\,!} + ...} = 1$

If limit exists, then $b - 1 = 0 \Rightarrow b = 1$

so $\frac{1}{\sqrt{a}} . \lim\limits_{x\to 0} \frac{x^3}{x^3\left[\frac{1}{6} - \frac{x^2}{120} + ...\right]} = 1$

$\Rightarrow \frac{1}{\sqrt{a}} \times \frac{1}{\frac{1}{6}} = 1 \quad \Rightarrow a = 36$,

so $a = 36, b = 1$

40. (4)

Sol. $\lim\limits_{x\to 0} e^{\frac{x\ell n(1+b^2)}{x}} = 1 + b^2 = 2b \sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{2}\left(b + \frac{1}{b}\right)$

We know b + $\frac{1}{b} \geq 2$

$\Rightarrow \sin^2\theta \geq 1$ but $\sin^2\theta \leq 1$

$\Rightarrow \sin^2\theta = 1 \quad \Rightarrow \theta = \pm\frac{\pi}{2}$

41. (1)

Sol. For $\lim\limits_{x\to 0} \frac{1 + a\cos x}{x^2}$

for $\left(\frac{0}{0}\right)$ form

$1 + a = 0 \Rightarrow a = -1$

for $\lim\limits_{x\to 0} \frac{b\sin x}{x^3} = \lim\limits_{x\to 0} \frac{b}{x^2}$

so $b = 0$

42. (4)

Sol. $f(x) = \frac{x^2 - 9x + 20}{x - [x]}$

$\lim\limits_{x\to 5^-} \frac{x^2 - 9x + 20}{x - [x]} = \frac{25 - 45 + 20}{1} = 0$

$\lim\limits_{x\to 5^+} \frac{x^2 - 9x + 20}{x - [x]}$

$= \lim\limits_{h\to 0} \frac{(5+h)^2 - 9(5+h) + 20}{5 + h - [5+h]}$

$= \lim\limits_{h\to 0} \frac{25 + 10h + h^2 - 45 - 9h + 20}{h}$

$= \lim\limits_{h\to 0} \frac{h^2 + h}{h} = \lim\limits_{h\to 0} \frac{h(h+1)}{h} = 1$

$\because \lim\limits_{x\to 5^-} f(x) \neq \lim\limits_{x\to 5^+} f(x)$

so $\lim\limits_{x\to 5} f(x)$ does not exist

43. (2)

Sol. By L.H. rule

$= \lim\limits_{x\to 1/\sqrt{2}} \frac{1 + \frac{\sin\ (\sin^{-1} x)}{\sqrt{1-x^2}}}{0 - \frac{\sec^2(\sin^{-1} x)}{\sqrt{1-x^2}}} = -\frac{1}{\sqrt{2}}$

44. (4)

Sol. $\lim\limits_{x\to\infty} (x + e^x)^{2/x}$

$\ell n y = \frac{2}{x} \lim\limits_{x\to\infty} \ell n(x + e^x)$

$= \lim\limits_{x\to\infty} \frac{2}{x}\left[\ell n\left(1 + \frac{x}{e^x}\right) + \ell n e^x\right]$

$= \lim\limits_{x\to\infty} \left[2 + 2.\frac{\ell n\left(1 + \frac{x}{e^x}\right)}{x/e^x}.\frac{1}{e^x}\right]$

$= \lim\limits_{x\to\infty} 2 + 2\cdot 1\cdot 0 \qquad \Rightarrow y = e^2$

45. **(2)**

Sol. $\lim\limits_{x\to\infty} \dfrac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}}-\left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}$, $n \in N$

$\lim\limits_{x\to\infty} \dfrac{(2)^{\frac{x^n}{e^x}}-(3)^{\frac{x^n}{e^x}}}{\frac{x^n}{e^x}}$

when $x\to\infty$, $\dfrac{x^n}{e^x}\to 0$

Put $\dfrac{x^n}{e^x}=t$

$\lim\limits_{t\to 0}\left(\dfrac{2^t-3^t}{t}\right) = \ell n^2 - \ell n^3 = \ell n\left(\dfrac{2}{3}\right)$

46. **(1)**

Sol. $\lim\limits_{x\to\infty}\dfrac{x\ell n\left(1+\frac{\ell nx}{x}\right)}{\ell nx}$

$= \lim\limits_{x\to\infty}\dfrac{\ell n\left(1+\frac{\ell nx}{x}\right)}{\left(\frac{\ell nx}{x}\right)}$ $\left(\because \lim\limits_{x\to\infty}\dfrac{\ell nx}{x} = 0\right.$ and

$\lim\limits_{f(x)\to 0}\dfrac{\ell n(1+f(x))}{f(x)} = 1)$

$= 1$

47. **(2)**

Sol. $e^{\lim\limits_{x\to 0}\left(\frac{1^x+2^x+....+n^x}{n}-1\right)\cdot\frac{1}{x}}$

$= e^{\lim\limits_{x\to 0}\frac{1}{n}\left(\left(\frac{1^x-1}{x}\right)+\left(\frac{2^x-1}{x}\right)+.......+\left(\frac{n^x-1}{x}\right)\right)}$

$= e^{\frac{1}{n}(\ell n1+\ell n2+......+\ell nn)}$

$= e^{\frac{1}{n}(\ell n(1.2.3.....n))} = (n!)^{1/n}$

48. **(2)**

Sol. $= e^{\lim\limits_{x\to 1}\left(\frac{\ell n5x}{\ell n5}-1\right)\cdot\frac{\ell n5}{\ell nx}}$

$= e^{\lim\limits_{x\to 1}\left(\frac{\ell n5x-\ell n5}{\ell n5}\right)\left(\frac{\ell n5}{\ell nx}\right)} = e^{\lim\limits_{x\to 1}\frac{\ell n\left(\frac{5x}{5}\right)}{\ell nx}} = e$

49. **(3)**

Sol. $\sin\theta < \theta < \tan\theta$, $\theta \in \left(0, \dfrac{\pi}{2}\right)$

$\dfrac{\sin\theta}{\theta} < 1 < \dfrac{\tan\theta}{\theta}$

$\dfrac{n\sin\theta}{\theta} < n < \dfrac{n\tan\theta}{\theta}$

$\left(\left[\dfrac{n\sin\theta}{\theta}\right]+\left[\dfrac{n\tan\theta}{\theta}\right]\right)$; $n\in N$

L.H.L. $= \lim\limits_{\theta\to 0^-}\left(\left[\dfrac{n\sin\theta}{\theta}\right]+\left[\dfrac{n\tan\theta}{\theta}\right]\right) = n$

$-1+n = 2n-1$

R.H.L. $= \lim\limits_{\theta\to 0^+}\left(\left[\dfrac{n\sin\theta}{\theta}\right]+\left[\dfrac{n\tan\theta}{\theta}\right]\right)$

$= n-1+n = 2n-1$

$\therefore$ L.H.L. = R.H.L $= 2n-1$

50. **(4)**

Sol. Statement-1 : $\lim\limits_{x\to 0}\left(\dfrac{1+\tan x}{1-\tan x}\right)^{\frac{1}{x}} = \lim\limits_{x\to 0}$

$\left(1+\dfrac{2\tan x}{1-\tan x}\right)^{\frac{1}{x}} = \lim\limits_{x\to 0}\exp\left[\dfrac{2\tan x}{x(1-\tan x)}\right] = e^2$

Hence statement-1 is false and statement-2 is true.

51. **(3)**

Sol. Statement-1 is true

Statement-2 is false. Consider

$\lim\limits_{x\to\infty}\dfrac{2x^2+3x+1}{3x^4+5} = 0 \neq \dfrac{2}{3}$

52. **(3)**

Sol. (P) $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3} = \lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)\left(\dfrac{1-\cos x}{x^2}\right)$

$= \dfrac{1}{2}$

(Q) $\lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}} =$

$\lim_{x\to\infty}\sqrt{\frac{1-\frac{1}{x}\sin x}{1+\frac{1}{x}\cos^2 x}} = \sqrt{\frac{1-0}{1+0}} \qquad = 1$

(R) $\lim_{x\to 1}\frac{-\sqrt{25-x^2}+\sqrt{24}}{x-1}$

$= \lim_{x\to 1}\frac{(x^2-1)}{(x-1)(\sqrt{25-x^2}+\sqrt{24})}$

$= \lim_{x\to 1}\frac{(x+1)}{(\sqrt{25-x^2}+\sqrt{24})} = \frac{2}{2\sqrt{24}}$

$= \frac{1}{\sqrt{24}} = \frac{1}{2\sqrt{6}}$

(S) $\left(\frac{\tan x}{x}\right)^{1/x^2} = e^{\lim_{x\to 0}\left(\frac{\tan x}{x}-1\right)\cdot\frac{1}{x^2}}$

$= e^{\lim_{x\to 0}\left(\frac{\tan x - x}{x^3}\right)}$

$= e^{\lim_{x\to 0}\left(\frac{x+\frac{x^3}{3}+.....-x}{x^3}\right)}$

$= e^{1/3}$

53. **(4)**

Sol. $ax^2 + bx + c = 0$

$a(x - \alpha)\ (x - \beta) = 0$

$\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$

$= \lim_{x\to\alpha}\frac{2\sin^2\left(\frac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$

$= \lim_{x\to\alpha}\frac{2\sin^2\left(\frac{a}{2}\ (x-\alpha)\ (x-\beta)\right)}{(x-\alpha)^2}$

$= \lim_{x\to\alpha}\left(\frac{\sin\left(\frac{a(x-\alpha)\ (x-\beta)}{2}\right)}{\frac{a(x-\alpha)\ (x-\beta)}{2}}\right)^2 \cdot \frac{2a^2(x-\beta)^2}{4}$

$= (1)\ \frac{2a^2(\alpha-\beta)^2}{4} = \frac{a^2(\alpha-\beta)^2}{2}$

54. **(4)**

Sol. $\lim_{x\to 0}\frac{f(3x)}{f(x)} = 1$

$f(x) < f(2x) < f(3x)$ Divide by $f(x)$

$1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$

using sandwitch theorem

$\Rightarrow \lim_{x\to\infty}\frac{f(2x)}{f(x)} = 1$

55. **(1)**

Sol. Let $x = 2 + h$

$\lim_{h\to 0}\frac{\sqrt{1-\cos 2h}}{h} = \lim_{h\to 0}\frac{|\sin h|}{h}$

RHL = 1, LHL = –1

$\therefore$ does not exist

56. **(4)**

Sol. $I = \lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}\frac{(3+\cos x)}{1}.\frac{x}{\tan 4x}$

$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2}.\frac{3+\cos x}{1}.\frac{x}{\tan 4x}$

$= 2\cdot 4\cdot \frac{1}{4} = 2$

57. **(2)**

Sol. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}\ \left(\frac{0}{0} form\right)$

$= \lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$= \frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x} \times \frac{\pi\sin^2 x}{x^2} = \pi.$

58. (2)

Sol. $P = e^{\lim\limits_{x\to 0^+}\left(1+\tan^2\sqrt{x}-1\right)\frac{1}{2x}} = e^{\lim\limits_{x\to 0^+}\frac{\left(\tan\sqrt{x}\right)^2}{2(\sqrt{x})^2}} = e^{\frac{1}{2}}$

$\log P = \log\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$= \frac{1}{2}$

59. (2)

Sol. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cot x - \cot x}{(\pi-2x)^3}$

$\lim\limits_{h\to 0}\frac{1}{8}\frac{\cos\left(\frac{\pi}{2}-h\right)\left[1-\sin\left(\frac{\pi}{2}-h\right)\right]}{\sin\left(\frac{\pi}{2}-h\right)\left(\frac{\pi}{2}-\frac{\pi}{2}+h\right)^3} = \frac{1}{16}$

60. (1)

Sol. $\lim\limits_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+......+\left[\frac{15}{x}\right]\right)$

$\lim\limits_{x\to 0^+}\left(x\left[\frac{1}{x}\right]+x\left[\frac{2}{x}\right]+......+x\left[\frac{15}{x}\right]\right)$

$= 1 + 2 + 3 + \ldots\ldots\ldots + 15$

$= \frac{15}{2}(15+1) = 120$

61. (3)

Sol.

$\ell im_{x\to 0} \frac{(\cos x - 1)\ (\cos x - e^x)}{x^n}$

$\ell im_{x\to 0} \frac{\left[\left(1-\frac{x^2}{2\ !}+\frac{x^4}{4\ !}-.....\right)-1\right]\left[\left(1-\frac{x^2}{2\ !}+\frac{x^4}{4\ !}-.....\right)-\left(1+x+\frac{x^2}{2\ !}+\frac{x^3}{3\ !}....\right)\right]}{x^n}$

$\ell im_{x\to 0} \frac{x^3\left(-\frac{1}{2\ !}+\frac{x^2}{4\ !}-.........\right)\left(-1-x\left(\frac{1}{2\ !}+\frac{1}{2\ !}\right)-.........\right)}{x^n}$

$\ell im_{x\to 0} \frac{\left(-\frac{1}{2\ !}+\frac{x^2}{4\ !}-.........\right)\left(-1-x\left(\frac{1}{2\ !}+\frac{1}{2\ !}\right)-.........\right)}{x^{n-3}}$

is finite and non-zero if $n = 3$

62. (3)

Sol. $\lim\limits_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ (1^∞ form) $= e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{f(1+x)-f(1)}{f(1)}\right)}$

$= e^{\frac{f'(1)}{f(1)}} = e^{6/3} = e^2$.

63. (4)

Sol. $\ell im_{x\to 0} \frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$

$\Rightarrow$

$\ell im_{x\to 0}\left(\frac{\sin nx}{nx}\right)(n)\left[(a-n)\ n - \frac{\tan x}{x}\right] = 0$

$\Rightarrow$ $(1)\ (n)\ [(a-n)\ (n) - 1] = 0$

$\Rightarrow$ $n(a-n) - 1 = 0$

$[\because n \neq 0]$

$\Rightarrow$ $a - n = \frac{1}{n}$

$\Rightarrow$ $a = n + \frac{1}{n}$

64. (3)

Sol. for $x > 0$

$\ell im_{x\to 0} (\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x}$

$\ell im_{x\to 0}(\sin x)^{\frac{1}{x}} + \ell im_{x\to 0}\left(\frac{1}{x}\right)^{\sin x}$

$= 0 + \ell im_{x\to 0}\left(\frac{1}{x}\right)^{\sin x} = e^{\ell im_{x\to 0}\ (\sin x).\ \ell n\left(\frac{1}{x}\right)}$

$$= e^{\lim_{x\to 0}\frac{\ell n(x)}{-\operatorname{cosec} x}} \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$= e^{\lim_{x\to 0}\frac{\frac{1}{x}}{(\operatorname{cosec} x \cot x)}}$$

$$= e^{\lim_{x\to 0}\frac{\sin^2 x}{x\cos x}} = e^{o} = 1$$

65. **(2)**

Sol.

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right) = 4$$

$$\lim_{x\to\infty}\left(\frac{x^2(1-a)+x(1-a-b)+(1-b)}{x+1}\right) = 4$$

Limit is finite

It exists when $1 - a = 0 \Rightarrow a = 1$

then $$\lim_{x\to\infty}\left(\frac{1-a-b+\frac{1-b}{x}}{1+\frac{1}{x}}\right) = 4$$

$\therefore\ 1 - a - b = 4 \quad \Rightarrow \quad b = -4$

Integer Type Questions (66 to 75)

66. **(0)**

Sol. L.H.L. $= \lim\limits_{x\to 0^-} f(x)$

$= \lim\limits_{h\to 0^-} 4(0-h) = 0$

R.H.L. $= \lim\limits_{h\to 0^+} f(0+h)$

$= \lim\limits_{h\to 0^+} 3(0+h)^2 = 0$

$\Rightarrow$ RHL = RHL = 0

67. **(3)**

Sol. $$\lim_{x\to 4}\left(\frac{(\sqrt{x})^3-(2)^3}{(\sqrt{x}-2(\sqrt{x}+2)}\right)$$

$$= \lim_{x\to 4}\left(\frac{(\sqrt{x}-2)(x+2\sqrt{x}+4)}{(\sqrt{x}-2)(\sqrt{x}+2)}\right)$$

$$= \lim_{x\to 4}\left(\frac{x+2\sqrt{x}+4}{\sqrt{x}+2}\right) = 3$$

68. **(1)**

Sol. $$\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x - x} = \lim_{x\to 0}\frac{e^x(e^{\tan x - x}-1)}{(\tan x - x)} = e^0.1 = 1$$

69. **(0)**

Sol. $$\lim_{n\to\infty}\frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}$$

$$= \lim_{n\to\infty}\frac{5\cdot 5^n+3^n-4^n}{5^n+2^n+27.9^n}$$

$$= \lim_{n\to\infty}\frac{5.\left(\frac{5}{9}\right)^n+\left(\frac{3}{9}\right)^n-\left(\frac{4}{9}\right)^n}{\left(\frac{5}{9}\right)^n+\left(\frac{2}{9}\right)^n+29}$$

$$= \frac{0+0-0}{0+0+27} = 0$$

70. **(1)**

Sol. $$\lim_{n\to\infty}\sqrt{\frac{3x-\sin x}{3x+\cos^2 x}}$$

$$= \lim_{y\to 0}\sqrt{\frac{\frac{3}{y}-\sin\frac{1}{y}}{\frac{3}{y}+\cos^2\frac{1}{y}}}$$

$$= \lim_{y\to 0}\sqrt{\frac{3-y\sin\frac{1}{y}}{3+y\cos^2\frac{1}{y}}} = \sqrt{\frac{3-0}{3+0}} = 1$$

71. **(2)**

Sol. $$\lim_{x\to 0}\frac{2\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+....\right) - 2\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+....\right)}{x\left(x-\frac{x^3}{3}!+.......\right)}$$

$$= \lim_{x\to 0}\frac{x^2\left(2+2\left(\frac{1}{4!}+\frac{1}{6!}\right)x^4+.....\right)}{x^2\left(1-\frac{x^2}{3!}+....\right)} = 2$$

72. (0)

Sol. $\lim\limits_{x\to\pi/2} \dfrac{\ell n \sin x}{\cot x}$

By L.H. rule

$= \lim\limits_{x\to\pi/2} -\dfrac{\cot x}{\operatorname{cosec}^2 x}$

$= 0$

73. (32)

Sol. L.H.L. = $\lim\limits_{h\to 0^-} f(2-h).g(2-h)$

$= \lim\limits_{h\to 0^-} ((2-h)^2+4).(2-h)^2$

$= 8.4$

$= 32$

R.H.L. = $\lim\limits_{h\to 0^+} (2+h+2).8$

$= 32$

$\Rightarrow \lim\limits_{x\to 2} = 32$

74. (1)

Sol. $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}}{\sqrt{x}\left[\sqrt{1+\sqrt{\dfrac{1}{x}+\sqrt{\dfrac{1}{x^3}}}}\right]} = 1$

75. (2)

Sol. $\lim\limits_{x\to 0} \dfrac{a+b\sin x-\cos x+ce^x}{x^3}$

$$\lim_{x\to 0}\frac{a+b\left[x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-....\right]-\left[1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-....\right]+c\left[1+\dfrac{x}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\dfrac{x^4}{4!}+....\right]}{x^3}$$

$$\lim_{x\to 0}\frac{(a-1+c)+x(b+c)+x^2\left(\dfrac{1}{2}+\dfrac{c}{2}\right)+x^3\left(\dfrac{-b+c}{6}\right)+x^4\left(\dfrac{-1}{4!}+\dfrac{c}{4!}\right)+...}{x^3}$$

limit exists so

$a+c-1=0$

$b+c=0$

$\dfrac{1}{2}+\dfrac{c}{2}=0 \qquad \Rightarrow c=-1$

so $b=1, \qquad a=2$

CONTINUITY, DIFFERENTIABILITY, MOD

Single Option Correct Type Questions (01 to 50)

1. **(2)**

Sol: $f(x) = (\tan x \ \cot\alpha)^{\frac{1}{x-\alpha}}$

$$\lim_{x\to\alpha} f(x) = e^{\frac{1}{(x-\alpha)}\log(\tan x\cot\alpha)}$$

$$= e^{\frac{\log[(\tan x\cot\alpha-1)+1]}{(\tan x\cot\alpha-1)}\frac{(\tan x\cot\alpha-1)}{(x-\alpha)}}$$

$$= e^{1\times\frac{(\sin x\cos\alpha-\cos x\sin\alpha)}{(x-\alpha)\cos x\sin\alpha}}$$

$$= e^{2\operatorname{cosec}2\alpha}$$

2. **(2)**

Sol: $f(x) = e^{\frac{1}{(x-\alpha)}\log\left(\frac{\sin x}{\sin\alpha}\right)}$

$$\lim_{x\to\alpha} f(x) = e^{\frac{1}{(x-\alpha)}\frac{\log\left[\left(\frac{\sin x}{\sin\alpha}-1\right)+1\right]}{\left(\frac{\sin x}{\sin\alpha}-1\right)}\left(\frac{\sin x}{\sin\alpha}-1\right)}$$

$$= e^{1.\frac{\sin x-\sin\alpha}{(x-\alpha)\sin\alpha}}$$

$$= e^{\frac{2\sin\left(\frac{x-\alpha}{2}\right).\cos\left(\frac{x+\alpha}{2}\right)}{(x-\alpha)\sin\alpha}}$$

$$= e^{\cot\alpha}$$

3. **(3)**

Sol: $f[g(x)] = \begin{cases}[g(x)]^2+2 & , \ x\ge 2\\ 1-g(x) & , \ x<2\end{cases}$

$$= \begin{cases}(2x)^2+2\ ; & 2x\ge 2 \ \ and \ \ x>1\\ (3-x)^2+2\ ; & 3-x\ge 2 \ \ and \ \ x\le 1\\ 1-2x\ ; & 2x<2 \ \ and \ \ x>1\\ 1-(3-x)\ ; & 3-x<2 \ \ and \ \ x\le 1\end{cases}$$

$$= \begin{cases}4x^2+2\ ; & x\ge 1 \ \ and \ \ x>1\\ x^2-6x+11\ ; & x\le 1 \ \ and \ \ x\le 1\\ 1-2x\ ; & x<1 \ \ and \ \ x>1\\ x-2\ ; & x>1 \ \ and \ \ x\le 1\end{cases}$$

$$f[g(x)] = \begin{cases}4x^2+2 & , \ x>1\\ x^2-6x+11 & , \ x\le 1\end{cases}$$

so $\lim\limits_{x\to 1} f[g(x)]$

$R.H.L. = \lim\limits_{x\to 1^+} (4x^2+2) = 6$

$L.H.L. = \lim\limits_{x\to 1^-} (x^2-6x+11) = 6$

$L.H.L. = R.H.L.$

so $\lim\limits_{x\to 1} f[g(x)] = 6$

4. **(3)**

Sol: Obviously

5. **(3)**

Sol: $t = \frac{1}{x-1} \Rightarrow x\ne 1$

$$y = \frac{1}{\left(\frac{1}{x-1}\right)^2+\left(\frac{1}{x-1}\right)-2}$$

$$= \frac{-(x-1)^2}{2(x-1)^2-(x-1)-1}$$

$= \dfrac{-(x-1)^2}{(2(x-1)+1)((x-1)-1)}$

$\Rightarrow x - 1 \neq \dfrac{-1}{2}$, 1

$\Rightarrow x \neq \dfrac{1}{2}, 2$

So discontinuous at $x = \dfrac{1}{2}$, 1, 2

6. **(2)**

Sol: $f(x) = \begin{cases} \dfrac{x(3e^{1/x}+4)}{2-e^{1/x}}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

$LHD\ (x = 0) = \lim\limits_{h\to 0} \dfrac{\dfrac{-h(3e^{-1/h}+4)}{2-e^{-1/h}} - 0}{-h} = 2$

$RHD\ (x = 0) = \lim\limits_{h\to 0} \dfrac{h\left(\dfrac{3e^{-1/h}+4}{2-e^{-1/h}}\right) - 0}{h} = -3$

not differentiable

$\lim\limits_{x\to 0} f(x) = 0 = \lim\limits_{x\to 0^+} f(x) = f(0)$

continuous

7. **(2)**

Sol: $f'(0^+) = \lim\limits_{h\to 0} \dfrac{h(\sqrt{h}-\sqrt{h+1})-0}{h} = -1$

& for $x < 0$, $f(x)$ is not defined

Hence $f(x)$ is differentiable at $x = 0$

8. **(3)**

Sol: Not differentiable at $x = 0$

9. **(2)**

Sol: $f(x) = \begin{cases} x\left(\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

$L.H.L. = R.H.L. = 0$

$L.H.D. = \lim\limits_{x\to 0^-} \dfrac{(-h)\left(\dfrac{e^{-1/h}-e^{1/h}}{e^{-1/h}+e^{-1/h}}\right) - 0}{-h} = -1$

$R.H.D. = \lim\limits_{x\to 0^+} \dfrac{h\dfrac{(e^{1/h}-e^{-1/h})}{e^{1/h}+e^{-1/h}} - 0}{h} = 1$

not differentiable at $x = 0$

10. **(1)**

Sol: $f(x) = \begin{cases} x^n \sin\dfrac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

at $x = 0$, $L.H.L. = (-h)^n \sin\left(\dfrac{-1}{h}\right)$

$R.H.L. = (h)^n \sin\left(\dfrac{1}{h}\right)$

$L.H.L. = R.H.L.$ if $n > 0$

$L.H.D. = \dfrac{(-h)^n \sin\left(\dfrac{-1}{h}\right) - 0}{-h}$

$= (-h)^{n-1} \sin\left(\dfrac{-1}{h}\right)$

$R.H.D. = h^{n-1} \sin\left(\dfrac{1}{h}\right)$

$L.H.D. = R.H.D.$ if $n - 1 > 0$

so $f(x)$ is continuous but not differentiable

for $n \in (0, 1]$

11. **(1)**

Sol: Except $x = 0$

$f'(x) = \begin{bmatrix} 1, & x \geq 0 \\ -1, & x < 0 \end{bmatrix}$

12. **(3)**

Sol: $f(x) = \cos^{-1}(\cos x)$

$f'(x) = -\dfrac{1}{\sqrt{1-\cos^2 x}}\ (-\sin x)$

$= +\dfrac{1}{|\sin x|}\sin x$

$= sgn\ (\sin x)$

13. **(2)**

Sol: $|f(x)|$ may not be differentiable at $f(x) = 0$ $|f|^2$ is differentiable everywhere

14. **(3)**

Sol: $f(x) = \sum_{k=0}^{n} a_k \, |x|^k$

$= a_0 + a_1 |x| + a_2|x|_2 + a_3|x|_3 + ...+ a_n|x|_n$

$f(0) = a_0$ we know that $\lim_{x\to 0} |x| = 0$

$\lim_{x\to 0} f(0) = a_0$

$f(x)$ is continous for $x = 0$

$|x|^n$ is differentiable if n ¹ 1, n Î N

$f(x)$ is not differentiable at $x = 0$, due to presence of $|x|$

If all $a_{2k+1} = 0, f(x)$ does not contains $|x|$

$f(x)$ is differentiable at $x = 0$

15. **(3)**

Sol: $f(x.\ y) = f(x) + f(y) \quad \forall x, y \in R^+$ and differentiable

$\Rightarrow f(x) = \log_a x$

also $f(e) = 1 \Rightarrow a = e$

$\Rightarrow f(x) = \log_e x$

$$\Rightarrow \lim_{x\to 0} \frac{f(x+1)}{2x} = \lim_{x\to 0} \frac{\ell n(1+x)}{2x} = \frac{1}{2}$$

16. **(1)**

Sol: $\lim_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^2} = a$$

$$\Rightarrow \frac{2}{x^2} \sin\left(\frac{\sin x + x}{2}\right) \sin\left(\frac{x - \sin x}{2}\right) = a$$

$$\Rightarrow a = \lim_{x\to 0} \ 2. \frac{\sin\left(\frac{\sin x + x}{2}\right)}{\frac{\sin x + x}{2}} . \frac{\sin\left(\frac{x - \sin x}{2}\right)}{\frac{x - \sin x}{2}}$$

$$\frac{1}{4}\left(\frac{\sin x + x}{x}\right)\left(\frac{x - \sin x}{x}\right)$$

$= 2.\ 1\ .\ 1.\ \frac{1}{4}\ (1 + 1)\ (1 - 1) = 0$

17. **(3)**

Sol: for contunity $f\left(\frac{\pi}{2}\right) =$

$$\lim_{n\to 0} \frac{\ell n\ [(1-h)^2 - 2(1-h) + 5]}{\ell n\ [1-4h]}$$

(here $h \to 0$ from both side)

$$f\left(\frac{\pi}{2}\right) = \lim_{n\to 0} \frac{1-\cosh}{4h^2} \frac{\ell n\ (\cosh)}{\ell n\ [1+4h^2]}$$

$$= \lim_{h\to 0} \frac{2}{16\times 16}\left(\frac{\sin^2 h/2}{h^2/2}\right) . \frac{4h^2}{\ell n\ (1+4h^2)}$$

$$. \frac{\ell n(1-2\sin^2 h/2)}{2\sin^2 h/2} . \frac{2\sin^2 h/2}{h^2/2}$$

$= \frac{1}{64}$. 1. 1.(–1) .1 $= -\frac{1}{64}$

18. **(1)**

Sol: (1) $f(x)$ is continuous no where

(2) $g(x)$ is continuous at $x = 1/2$

(3) $h(x)$ is continuous at $x = 0$

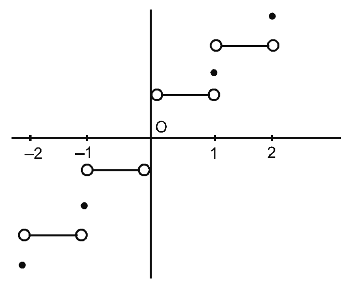
(4) $k(x)$ is continuous at $x = 0$

19. **(3)**

Sol: $f(x) = x + \{-x\} + [x]$

Since $\{-x\} = \begin{cases} 1 - \{x\}, & x \notin \text{I} \\ 0, & x \in \text{I} \end{cases}$

$= \begin{cases} x + 1 - \{x\} + [x], & x \notin \text{I} \\ x + [x], & x \in \text{I} \end{cases}$



$= \begin{cases} 1 + 2[x], & x \notin I \\ 2x, & x \in I \end{cases}$

Curve of $y = f(x)$

discontinuous at all integers in [–2, 2]

20. **(2)**

Sol: $f(x) = [x] + \sqrt{\{x\}}$

for $n \in I$

$f(n^+) = [n^+] + \sqrt{\{n^+\}} = n$

$f(n^-) = [n^-] + \sqrt{\{n^-\}} = n - 1 + 1 = n$

$\Rightarrow$ continuous for all $x \in I$

Hence continuous for all $x \in R$

21. **(3)**

Sol: $f(1^-) = f(1) \Rightarrow a - b = -1$

$$f'(1^-) = \lim_{h\to 0} \frac{a(1-h)^2 - b + 1}{-h}$$

$$= \lim_{h\to 0} \frac{(a-b+1)}{-h} + 2a = 2a$$

$$F'(1^+) = \lim_{h\to 0} \frac{\frac{-1}{1+h}+1}{h} = \lim_{h\to 0} \frac{h}{h(1+h)} = 1$$

$\Rightarrow a = \frac{1}{2}, b = \frac{3}{2}$

so $a + b = 2$

22. **(2)**

Sol: $f(0) = 0$

$$f'(0^+) = \lim_{h\to 0} \frac{\frac{h}{\sqrt{h+1}-\sqrt{h}} - 0}{h} = 1$$

$f(x)$ is not defined for $x < 0$

$\Rightarrow f(x)$ is differentiable at $x = 0$

23. **(2)**

Sol: $f(x) = \begin{cases} x\tan^{-1}\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

$L.H.L. = R.H.L. = 0$

continuous at $x = 0$

24. **(3)**

Sol: $f(x) = \begin{cases} \sin 2x & , 0 < x \le \frac{\pi}{6} \\ ax + b & , \frac{\pi}{6} < x < 1 \end{cases}$

$$L.H.L. = \lim_{x\to\frac{\pi}{6}} \sin 2x = \frac{\sqrt{3}}{2}$$

$$R.H.L. = \frac{a\pi}{6} + b$$

$$\frac{a\pi}{6} + b = \frac{\sqrt{3}}{2} \quad \text{...(1)}$$

$$L.H.D. = \lim_{x\to\frac{\pi}{6}} 2\cos 2x = 1$$

$R.H.D. = a$

$a = 1$ ...(2)

$$b = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

25. **(1)**

Sol: $f(1^-) = f(1) = 1$

$f(1^+) = a + b + c$

$\Rightarrow a + b + c = 1$

$f'(1^-) = 1$

$$f'(1^+) = \lim_{h\to 0} \frac{a(1+h)^2 + b(1+h) + c - 1}{h}$$

$= 2a + b$

$\Rightarrow 2a + b = 1$

so $(a, b, c) \equiv (a, 1 - 2a, a), a \in R, a \neq 0$

26. **(1)**

Sol: $L.H.L. = \lim_{x\to -1-h} f(x) = a(-1-h)^2 + b = a + b$

$R.H.L. = \lim_{x\to -1+h} f(x) = b - a + 4$

$a = 2$

27. **(2)**

Sol: $f(x) = [x]\,[\sin px], x \in (-1, 1)$

$= \begin{cases} 1 & , x \in (-1, 0) \\ 0 & , x \in [0, 1) \end{cases}$

$f(x)$ is continuous in $(-1, 0)$

28. **(2)**

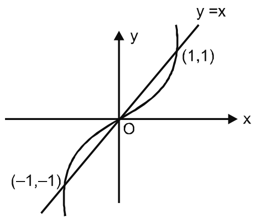
Sol: $f(x) = \begin{cases} a - x & , x < a - b \\ b & , a - b < x < b - a \\ a + x & , b - a < x \end{cases}$

so $f(x)$ cannot be differentiable at maximum two points.

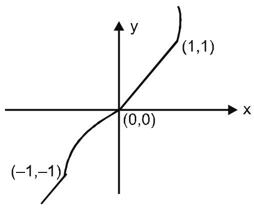
29. **(4)**

Sol: By use of graph

$f(x) = \max \{x, x3\}$



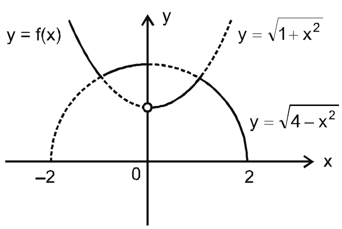
$$f(x) = \begin{cases} x^3, & x \in [-1,\ 0) \cup [1,\ \infty) \\ x\ , & x \in (-\infty,\ -1) \cup [0,\ 1) \end{cases}$$



From graph its clear that points {–1, 0, 1} are points of non differentiability due to kink points.

30. **(4)**

Sol:



31. **(1)**

Sol: Let $x = \tan\theta$ $\qquad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow f(x) = \sin^{-1}\left(\frac{1}{\sin 2\theta}\right)$

$\Rightarrow \sin 2\theta = \pm 1$

$\Rightarrow 2\theta = \pm\frac{\pi}{2}$

$\Rightarrow \theta = \pm\frac{\pi}{4}$

Hence $f(x)$ is defined only at $x = \pm 1$

so continuous but not differentiable at $x = \pm 1$

32. **(3)**

Sol: Let $h(x) = |x|$, then

$g(x) = |f(x)| = h(f(x))$

since, composition of two continuous functions is continuous, g is continuous if f is continuous. So answer is (3).

(1) is wrong answer.

Let $f(x) = x \Rightarrow g(x) = |x|$

Now, $f(x)$ is an onto function. since co-domain of x

is R and range of x is R. But $g(x)$ is not onto function. since range of $g(x)$ is $[0, \infty)$ but co-domain

is given R.

(2) Let $f(x) = x$ Þ $g(x) = |x|$. Now $f(x)$ is one-one function but $g(x)$ is many - one function. Hence (2) is wrong.

(4) Let $f(x) = x$ Þ $g(x) = |x|$. Now, $f(x)$ is differentiable for all $x \in R$ but $g(x) = |x|$ is not differentiable at $x = 0$ Hence (4) is wrong.

33. **(3)**

Sol: $\lim_{x\to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$

$= \lim_{x\to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x}$

$= \lim_{x\to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$

$= \frac{2f''(0) - 12f''(0) + 16f''(0)}{2} = 3f''(0) = 12$

34. **(3)**

Sol: $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$ ....**(1)**

$f(0) = 0,\ f'(0) = 3$

Put $x = 3x$ and $y = 0$

$f(x) = \frac{f(3x)}{3}$ ....**(2)**

$\lim_{h\to 0} f(x+h) = \lim_{h\to 0} f\left(\frac{3x+3h}{3}\right)$

$= \lim_{h\to 0} f \; \frac{f(3x)+f(3h)}{3} = \frac{f(3x)}{3} = f(x)$

Similarly we can prove $\lim_{h\to 0} f(x-h) = f(x)$

$\Rightarrow f(x)$ is continuous for all x in R

Given that $f'(0) = 3$

$\Rightarrow \lim_{h\to 0} \frac{f(h)}{h} = \lim_{h\to 0} \frac{f(-h)}{-h} = 3$

35. (3)

Sol: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{f(x)+f(h)+xh-f(x)}{h}$

$= \lim_{h\to 0} \frac{f(h)}{h} + x = 3 + x$

$\Rightarrow f(x) = 3x + \frac{x^2}{2} + C$

$\because f(0) = 0 \Rightarrow C = 0$

$\Rightarrow f(x) = 3x + \frac{x^2}{2}$

36. (4)

Sol: $f'(x) \; \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{f\left(\frac{3x+3h}{3}\right) - f\left(\frac{3x+0}{3}\right)}{h}$

$= \lim_{h\to 0} \frac{\frac{4-2(f(3x)+f(3h))}{3} - \frac{4-2(f(3x)+f(0))}{3}}{h}$

$= \lim_{h\to 0} \frac{-2}{3} \frac{(f(3h)-f(0))}{h} = -2f'(0)$

$\Rightarrow f(x) = -2f'(0)\,x + C$

put $y = 0$ in the given equation

$f\left(\frac{x}{3}\right) = \frac{4-2(f(x)+f(0))}{3}$

$\Rightarrow \frac{1}{3} f'\left(\frac{x}{3}\right) = \frac{-2}{3} f'(x)$

put $x = 0 \Rightarrow f'(0) = 0$ & $f(0) = \frac{4}{7} = C$

$\Rightarrow f(x) = \frac{4}{7}$

37. (1)

Sol: **Statement - 1**

$f(x) = \{\tan x\} - [\tan x]$

$f(x) = \tan x - 2\,[\tan x]$

$= \begin{cases} \tan x, & 0 \le x < \frac{\pi}{4} \\ \tan x - 2, & \frac{\pi}{4} \le x < \tan^{-1} 2 \end{cases}$

obviously at $x = \frac{\pi}{3}$ $f(x)$ is continuous. **(True)**

Statement-2

$y = f(x)$ & $y = g(x)$ both are continuous at $x = a$ then $y = f(x) \pm (g(x)$ will also be continuous at $x = a$ **(True)**

Statement-1 can be explained with the help of statement-2.

38. (2)

Sol: Since, $f(x) = \frac{x}{1+|x|}$

Let $f(x) = \frac{g(x)}{h(x)} = \frac{x}{1+|x|}$

It is clear that $g(x) = x$ and $h(x) = 1 + |x|$ are differentiable on $(-\infty, \infty)$ and $(-\infty, 0) \cup (0, \infty)$ respectively.

Thus, $f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$.

Now we have to check the differentiability at $x = 0$

$\therefore \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\frac{x}{1+|x|} - 0}{x}$

$= \lim_{x\to 0} \frac{1}{1+|x|} = 1$

Hence, $f(x)$ is differentiable on $(-\infty, \infty)$.

39. **(3)**

Sol: $f(x) = \min\{x+1, |x|+1\}$

$f(x) = x+1, \ \forall \ x \in R.$

$f(x) \geq 1, \ \forall \ x \in R.$

40. **(2)**

Sol: Now, $f'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$$= \lim_{h\to 0} \frac{f(1-h-1)\sin\left(\frac{1}{1-h-1}\right)-0}{-h}$$

$$= \lim_{h\to 0} \sin\left(-\frac{1}{h}\right) = -\lim_{h\to 0} \sin\frac{1}{h}$$

and $f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right)-0}{h}$$

$$= \lim_{h\to 0} \sin\frac{1}{h}$$

$\therefore \ f'(1^-) \neq f'(1^+)$

f is not differentiable at $x = 1$.

Again, now

$$f'(0) = \lim_{h\to 0} \frac{(0+h+1)\sin\left(\frac{1}{0+h+1}\right)-\sin 1}{-h}$$

$$= \lim_{h\to 0} \frac{\left[-\left\{(h+1)\cos\left(\frac{1}{h+1}\right)\times\left(\frac{1}{(h+1)^2}\right)\right\}+\sin\left(\frac{1}{h+1}\right)\right]}{-1}$$

(using L' hospital's rule)

$= \cos 1 - \sin 1$

and $f'(0^+)$

$$= \lim_{h\to 0} \frac{(0+h-1)\ \sin\left(\frac{1}{0+h-1}\right)-\sin\ 1}{h}$$

= (using L 'Hospital's rule)

$= \cos 1 - \sin 1 \Rightarrow f'(0^-) = f'(0^+)$

$\therefore$ f is differentiable at $x = 0$.

41. **(3)**

Sol: $gof(x) = \sin(x|x|)$

$$gof(x) = \begin{cases} \cos(x|x|)\left(|x|+x\ .\ \frac{x}{|x|}\right) & , \quad x \neq 0 \\ |x|\cos(x|x|) & , \quad x = 0 \end{cases}$$

$\therefore gof$ is differentiable at $x = 0$ and the derivatives is continuous

$\therefore$ statement-1 is true

Statement-2

$$\lim_{h\to 0^+} \frac{(gof)'(0+h)-(gof)'(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{\cos(h|h|)\left(|h|+h\frac{h}{|h|}\right)}{h}$$

$$= \frac{\cos(h^2)(h+h)}{h} = 2$$

$$\lim_{h\to 0^-} \frac{(gof)'(0-h)-(gof)'(0)}{h}$$

$$= \lim_{h\to 0^-} \frac{\cos(h|h|)\left(|h|+\frac{h^2}{|h|}\right)}{h}$$

$$= \lim_{h\to 0^-} \frac{\cos(-h^2)(-h-h)}{h} = -2$$

$\therefore$ not differentiable

$\therefore$ statement-2 is false

42. **(3)**

Sol: $f(0) = q$

$$f(0^+) = \lim_{x\to 0^+} \frac{(1+x)^{1/2}-1}{x}$$

$$= \lim_{x\to 0^+} \frac{1+\frac{1}{2}x+.....-1}{x} = \frac{1}{2}$$

$$f(0^-) = \lim_{x\to 0^-} \frac{\sin\ (p+1)x+\sin x}{x}$$

$$f(0^-) = \lim_{x\to 0^-} \frac{(\cos(p+1)x)(p+1)+(\cos x)}{1}$$

$= (p+1) + 1 = p + 2$

$\therefore\ p + 2 = q = \frac{1}{2}$

$\Rightarrow p = -\frac{3}{2}\ ,\ q = \frac{1}{2}$

43. **(3)**

Sol: $f(x) = \begin{cases} x\sin(1/x) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$ at $x = 0$

$LHL = \lim_{h\to 0^+} \left\{-h\sin\left(-\frac{1}{h}\right)\right\}$

= 0 × *a* finite quantity between – 1 and 1

$\text{RHL} = \lim_{h\to 0^+} h\sin\frac{1}{h} = 0$

$f(0) = 0$

$\therefore f(x)$ is continuous on R. $f(x), R$

$f2(x)$ is not continuous at $x = 0\ f2(x), x = 0$

44. **(3)**

Sol: $\lim_{x\to a} \frac{x^2 f(a) - a^2 f(x)}{x-a}$

$= \lim_{x\to a} \frac{2xf(a) - a^2 f'(x)}{1} = 2af(a) - a2f'(a)$

Alter $\lim_{x\to a} \frac{x^2 f(a) - a^2 f(x)}{x-a}$

$= \lim_{x\to a} \frac{x^2 f(a) - a^2 f(a) + a^2 f(a) - a^2 f(x)}{x-a}$

$= \lim_{x\to a} \frac{(x^2 - a^2) f(a) - a^2(f(x) - f(a))}{x-a}$

$= \lim_{x\to a} (x + a) f(a) - a^2 \left\{\frac{f(x) - f(a)}{(x-a)}\right\}$

$= 2af(a) - a^2 f'(a)$

45. **(1)**

Sol: Doubtful points are $x = n,\ n \in I$

$L.H.L = \lim_{x\to n^-} [x]\cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$

$R.H.L. = \lim_{x\to n^+} [x]\cos\left(\frac{2n-1}{2}\right)\pi = n\cos\left(\frac{2n-1}{2}\right)\pi = 0$

$f(n) = 0$

Hence continuous

46. **(1)**

Sol: $f(x) = |\ell n2 - \sin x|$

$f(f(x)) = |\ell n2 - \sin|\ell n2 - \sin x||$

In the vicinity of $x = 0$

$g(x) = ln2 - \sin(ln2 - \sin x)$

Hence $g(x)$ is differentiable at $x = 0$ as it is sum and composite of differentiable function

$g'(x) = \cos(ln2 - \sin x).\cos x$

$g'(0) = \cos(ln2)$

47. **(4)**

Sol: $f(x) = \begin{cases} -\frac{1}{2}(x+1) & , \quad x < -1 \\ \tan^{-1} x & , \quad -1 \le x \le 1 \\ \frac{1}{2}(x-1) & , \quad x > 1 \end{cases}$

at $x = -1$ $LHL = 0$

$RHL = -\frac{\pi}{4}$ so LHL ¹ RHL

therefore function is discontinuous at $x = -1$

therefore function is not differentiable at $x = -1$

At $x = 1$, $LHL = \frac{\pi}{4}$

$RHL = 0$

therefore LHL ¹ RHL so function is discontinuous at $x = 1$

therefore function is not differentiable at $x = 1$

$f(x)$ is discontinuous at $x = -1$ and $x = 1$.

48. (3)

Sol: $\lim\limits_{x\to 0}\dfrac{f(x^2)-f(x)}{f(x)-f(0)}$. Put $x = 0$ and we get $\dfrac{0}{0}$ form. Also because 'f' is strictly increasing and differentiable. Apply L-Hospital rule, we get

$$\lim_{x\to 0}\frac{2xf'(x^2)-f'(x)}{f'(x)}$$

$= -1$. Since 'f' is strictly increasing $f(x) \neq 0$ in an internal

49. (3)

Sol: Clearly 'p' $= \lim\limits_{h\to 0^+}\dfrac{f(1-h)-f(1)}{-h}$

$$=\frac{|h|-0}{-h}=-1$$

Now $\lim\limits_{x\to 1^+}\dfrac{(x-1)^n}{\log\cos^m(x-1)}=-1$

$$\lim_{x\to 1^+}\frac{(x-1)^n}{m\dfrac{\log\ [1+\cos(x-1)-1]}{\cos(x-1)-1}\times[\cos(x-1)-1]}$$

$= -1$

or, $\lim\limits_{x\to 1^+}\dfrac{(x-1)^n}{m}\ \dfrac{1}{-2\sin^2\dfrac{(x-1)}{2}}=-1$

Hence $n = 2$, $m = 2$.(3) is correct answer.

Aliter: LHD of $|x-1|$ at 1 is $-1 \equiv p$

$$\lim_{x\to 1^+} g(x)=\lim_{t\to 0^+}\frac{t^n}{\log\cos^m t}$$

$$=\frac{1}{m}\lim_{t\to 0^+} t^{n-2}\cdot\frac{\cos t-1}{\log\cos t}\cdot\frac{t^2}{\cos t-1}$$

$$=-\frac{2}{m}\lim_{t\to 0^+} t^{n-2}\ \left(\because \lim_{t\to 0}\frac{1-\cos t}{t^2}=\frac{1}{2}\right)$$

$$\therefore -\frac{2}{m}\lim_{t\to 0^+} t^{n-2}=-1 \Rightarrow n=2,\ m=2$$

50. (2)

Sol: (I) for derivability at $x = 0$

$L.H.D = f'(0^-) = \lim\limits_{h\to 0^+}\dfrac{f(0-h)-f(0)}{-h}$

$$=\lim_{h\to 0^+}\frac{h^2.\left|\cos\left(-\frac{\pi}{h}\right)\right|-0}{-h}$$

$$=\lim_{h\to 0^+} -h\ .\ \left|\cos\frac{\pi}{h}\right| = 0$$

$RHD\ f'(0^+) = \lim\limits_{h\to 0^+}\dfrac{f(0+h)-f(0)}{h}$

$$=\lim_{h\to 0^+}\frac{h^2.\left|\cos\left(\frac{\pi}{h}\right)\right|-0}{h}=0$$

So $f(x)$ is derivable at $x = 0$

(ii) check for derivability at $x = 2$

$RHD = f'(2^+) = \lim\limits_{h\to 0^+}\dfrac{f(2+h)-f(2)}{h}$

$$=\lim_{h\to 0^+}\frac{(2+h)^2.\left|\cos\left(\frac{\pi}{2+h}\right)\right|-0}{h}$$

$$=\lim_{h\to 0^+}\frac{(2+h)^2.\sin\left(\frac{\pi}{2}-\frac{\pi}{2+h}\right)}{h}$$

$$=\lim_{h\to 0^+}\frac{(2+h)^2.\sin\left(\frac{\pi h}{2(2+h)}\right)}{\left(\frac{\pi}{2(2+h)}\right)h}.\frac{\pi}{2(2+h)}$$

$$=(2)^2.\frac{\pi}{2(2)}=\pi$$

$LHD = \lim\limits_{h\to 0^+}\dfrac{f(2-h)-f(2)}{-h}$

$$=\lim_{h\to 0^+}\frac{(2-h)^2.\left|\cos\left(\frac{\pi}{2-h}\right)\right|-0}{-h}$$

$$=\lim_{h\to 0^+}\frac{(2-h)^2.\left(-\cos\left(\frac{\pi}{2-h}\right)\right)-0}{-h}$$

$$= \lim_{h\to 0^+} \frac{(2-h)^2.\sin\left(\frac{\pi}{2}-\frac{\pi}{2-h}\right)}{h}$$

$$\lim_{h\to 0^+} \frac{(2-h)^2.\sin\left(-\frac{\pi h}{2(2-h)}\right)}{\left(-\frac{\pi h}{2(2-h)}\right)}.\frac{-\pi}{2(2-h)} = -\pi$$

So $f(x)$ is not derivable at $x = 2$

Integer Type Questions (51 to 58)

51. (2)

Sol: $f(x) = \frac{1}{1-x}$ discontinuous at $x = 1$

$f(f(x)) = \frac{1-x}{-x}$ discontinuous at $x = 0$

$g(x) = f\left(f(f(x))\right) = x$

$g(x)$ is discontinuous at $x = 0$ & $x = 1$

52. (2)

Sol: $f(x + y) = f(x) + f(y) \Rightarrow f(x) = kx$

as $f(1) = 2$ Þ $k = 2$

$\Rightarrow f(x) = 2x \Rightarrow f'(x) = 2$

53. (0)

Sol: Since $|f(x) - f(y)| \le (x-y)^2$

$$\lim_{x\to y} \frac{|f(x)-f(y)|}{|x-y|} \le \lim_{x\to y} |x-y|$$

$\Rightarrow |f'(y)| \le 0$

$\Rightarrow f'(y) = 0$ $\Rightarrow f(y) = \text{constant}$

$\Rightarrow f(y) = 0$ $[\because f(0) = 0 \text{ given}] \Rightarrow f(1) = 0$

54. (5)

Sol: $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0}\frac{f(1+h)}{h} - \lim_{h\to 0}\frac{f(1)}{h} \because \lim_{h\to 0}\frac{f(1+h)}{h} = 5,$

so $\lim_{h\to 0}\frac{f(1)}{h}$ must be finite as $f'(1)$ exist and

$\lim_{h\to 0}\frac{f(1)}{h}$ can be finite only, if $f(1) = 0$ and

$\lim_{h\to 0}\frac{f(1)}{h} = 0 \therefore f'(1) = \lim_{h\to 0}\frac{f(1+h)}{h} = 5.$

55. (1)

Sol: $\lim_{x\to 0} \frac{e^{2x}-1-2x}{x(e^{2x}-1)}$

$= \lim_{x\to 0} \frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}$ (using L' Hospital's)

$= \lim_{x\to 0} \frac{4e^{2x}}{4e^{2x}+4xe^{2x}} = 1$ (using L' Hospital's)

$\therefore$ $f(x)$ is continuous at $x = 0$, then

$\lim_{x\to 0} f(x) = f(0) \Rightarrow 1 = f(0)$

56. (1)

Sol: $y = (1+x)(1+x^2)(1+x^4).......(1+x^{2n})$

Multiplying numerator and denominator by $(1-x)$

$\Rightarrow y = \left(\frac{1-x^{2n+1}}{(1-x)}\right)$, So $\left.\frac{dy}{dx}\right|_{x=0} = 1$

57. (2)

Sol: $f'(0) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 2 & 0 & 4 \end{vmatrix}$

$\Rightarrow$ $f'(0) = 4 - 2 = 2$

58. (2)

Sol: $y = \frac{x}{a+\frac{x}{b+y}}$

$\Rightarrow$ $y = \frac{x(b+y)}{ab+ay+x}$

$\Rightarrow yab + y^2a + xy = xb + xy$

$\Rightarrow yab + y^2a - xb = 0$

$\frac{dy}{dx} = -\left(\frac{-b}{ab+2ay}\right)$, $\frac{dy}{dx} = \frac{b}{a(b+2y)}$

INVERSE TRIGONOMETRIC FUNCTIONS

Single Option Correct Type Questions (01 to 60)

1. **(4)**

Sol. $\therefore -1 \le x \le 1$ ...(1)

$x \in R$ ...(2)

$x \le -1$ or $x \ge 1$ ...(3)

By (1) $\cap$ (2) $\cap$ (3)

$\Rightarrow x \in \{-1, 1\}$

2. **(4)**

Sol. $\text{cosec}^{-1}(\cos x)$ is define if

$\cos x \ge 1$ or $\cos x \le -1$

$\Rightarrow \cos x = \pm 1$ $\Rightarrow x = n\pi, n \in z$

3. **(3)**

Sol. Domain of $f(x)$ is $x \in \{-1, 1\}$

$$f(-1) = -\frac{\pi}{2} - \frac{\pi}{4} + \pi = \frac{\pi}{4}$$

$$f(1) = \frac{\pi}{2} + \frac{\pi}{4} + 0 = \frac{3\pi}{4}$$

4. **(3)**

Sol. $\sin\left(\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right) = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

5. **(2)**

Sol. $\cos\left[\frac{\pi}{6} + \frac{\pi}{4}\right] = \frac{\sqrt{3}-1}{2\sqrt{2}}$

6. **(4)**

Sol. $2\pi + \left(-\frac{17\pi}{15}\right) = \frac{13\pi}{15}$

7. **(3)**

Sol. $\sin^{-1}(\sin 10) = 3\pi - 10$

$\because$ $(\sin^{-1}(\sin\theta) = 3\pi - \theta$ if $\frac{5\pi}{2} < \theta < \frac{7\pi}{2}$)

8. **(2)**

Sol. $\cos^{-1}\sqrt{\cos^2\frac{x}{2}} = \cos^{-1}\left(\cos\frac{x}{2}\right) = \frac{x}{2}$

9. **(4)**

Sol. $\sin^{-1}\sin\left(\pi - \frac{2\pi}{15}\right) = \sin^{-1}\sin\frac{2\pi}{15} = \frac{2\pi}{15}$

10. **(2)**

Sol. By property if $x < 0$

$$\tan^{-1}\frac{1}{x} = \cot^{-1}x - \pi$$

$$\therefore \tan^{-1}x + \tan^{-1}\frac{1}{x} = \tan^{-1}x + \cot^{-1}x - \pi = \frac{\pi}{2} - \pi$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{-\pi}{2}$$

11. **(3)**

Sol. $x^2 - 7x + \frac{25}{2} = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x = 3, 4$

12. **(3)**

Sol. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{1}x\right), x \ne 0$

let $\theta = \frac{1}{2}\cos^{-1}x$, $2\theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$$= \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right)$$

$$= \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta} = 2\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$$

$$= = \frac{2}{\cos 2\theta} = \frac{2}{\cos\cos^{-1}x} = \frac{2}{x}$$

13. **(2)**

Sol. $1 - \cos^2\left(\cos^{-1}\frac{1}{2}\right) + 1 - \sin^2\left(\sin^{-1}\frac{1}{3}\right)$

$= 2 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2 = 2 - \frac{1}{4} - \frac{1}{9} = \frac{59}{36}$

14. **(3)**

Sol. $\cos\left(\tan^{-1}\left(\sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right)$

$= \cos\left(\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \cos\left(\cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right)$

$= \sqrt{\frac{1+x^2}{2+x^2}}$

15. **(1)**

Sol. $\frac{\left(\frac{3}{4}+\frac{2}{3}\right)}{\left(1-\frac{3}{4}\ \frac{2}{3}\right)} = \frac{17}{6}$

16. **(2)**

Sol. $\tan\left(\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right)\right)$

$\Rightarrow$ numerical value $= \frac{7}{17}$

17. **(3)**

Sol. $\sqrt{\frac{1+\frac{1}{8}}{2}} = \frac{3}{4}$

18. **(4)**

Sol. $\cos\tan^{-1}\sin\cot^{-1}\left(\frac{1}{2}\right)$

$= \cos\tan^{-1}\sin\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$= \cos\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$= \cos\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) = \frac{\sqrt{5}}{3}$

19. **(2)**

Sol. $\tan^{-1}\frac{a-b}{1+ab} = \tan^{-1}a - \tan^{-1}b$ ......(i)

$\tan^{-1}\frac{b-c}{1+bc} = \tan^{-1}b - \tan^{-1}c$ ....(ii)

By (i) + (ii)

$\tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc} = \tan^{-1}a - \tan^{-1}c$

20. **(2)**

Sol. $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = 1 \quad \Rightarrow 1 - 6x^2 = 5x$

$\Rightarrow 6x^2 + 5x - 1 = 0 \quad \Rightarrow x = -1, \frac{1}{6}$

But $x \neq -1$

$\therefore$ If $x = -1 \Rightarrow \tan^{-1}2x + \tan^{-1}3x > \frac{\pi}{2}$

21. **(1)**

Sol. $\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

also $\sin^{-1}x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow 0 \le \tan^{-1}x + \cot^{-1}x + \sin^{-1}x \le \pi$

22. **(4)**

Sol. $\cot^{-1}x < -\sqrt{3}$ $\quad \because \cot^{-1}x > 0$

$\forall\ x \in R \Rightarrow$ no solution

23. **(4)**

Sol. $\cos^{-1}x > \sin^{-1}x \quad \Rightarrow \frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x$

$\Rightarrow \sin^{-1}x < \frac{\pi}{4}$

$\Rightarrow -1 \le x < \sin\frac{\pi}{4} \quad \Rightarrow -1 \le x < \frac{1}{\sqrt{2}}$

24. **(2)**

Sol. $\sum_{r=1}^{n} \tan^{-1}\left(\frac{2^r - 2^{r-1}}{1+2^r \; . \; 2^{r-1}}\right)$

$= \sum_{r=1}^{n} \left\{\tan^{-1} 2^r - \tan^{-1} 2^{r-1}\right\} = \tan^{-1} 2^n - \frac{\pi}{4}$

25. **(1)**

Sol. $\sum_{n=1}^{\infty} \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

$= \lim_{k\to\infty} \sum_{n=1}^{k} \left\{\tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2\right\}$

$= \lim_{k\to\infty} \left\{ \tan^{-1}(k+1)^2 + \tan^{-1} k^2 - \tan^{-1} 1 - \tan^{-1} 0\right\}$

$= \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{4} - 0 = \frac{3\pi}{4}$

26. **(4)**

Sol. $\because \quad x^2 + y^2 + z^2 = r^2$

$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$

$= \tan^{-1}\left\{\frac{\frac{x^2y^2 + y^2z^2 + z^2x^2}{xyzr} - \frac{(xy)(yz)(zx)}{xyzr^3}}{1 - \left(\frac{x^2+y^2+z^2}{r^2}\right)}\right\}$

$= \tan^{-1}(\infty) = \pm \frac{\pi}{2}$

27. **(3)**

Sol. $f(x) = \cot^{-1}x$; $\quad f: R^+ \to \left(0, \frac{\pi}{2}\right)$

$g(x) = 2x - x^2 : R \to R$

$f(g(x)) = \cot^{-1}(2x - x^2)$, where $x \in (0, 1]$

Hence $f(g(x)) \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

28. **(4)**

Sol. $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$

$= \frac{\pi}{2} - \tan^{-1} x$

Domain $x \in [-1, 1]$

But given $x \geq 0$

$\Rightarrow x \in [0, 1]$

$\theta = \frac{\pi}{2} - \tan^{-1} x$

for $x \in [0, 1]$

$\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

29. **(3)**

Sol. $\sum_{i=1}^{2n} \sin^{-1} x_i = n\pi$

we know that $-\frac{\pi}{2} \leq \sin^{-1} x_i \leq \frac{\pi}{2}$

Equality holds good only when $\sin^{-1} x_i = \frac{\pi}{2} \; \forall$

i = 1, 2, 3......$2n$

$\Rightarrow \; x_i = 1 \; \forall \qquad i = 1, 2, 3,...., 2n$

$\Rightarrow \; \sum_{i=1}^{2n} x_i = 2n$

30. **(1)**

Sol. $([\cot^{-1}x] - 3)^2 \leq 0$

$\Rightarrow \; [\cot^{-1} x] = 3$

$\Rightarrow \; -\infty < x \leq \cot 3$

31. **(3)**

Sol. $\sin^{-1} x > \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$

$\Rightarrow \sin^{-1} > \frac{\pi}{4}$

$\Rightarrow x > \frac{1}{\sqrt{2}}$ also $|x| \leq 1$

$\Rightarrow \; x \in \left(\frac{1}{\sqrt{2}}, 1\right]$

32. **(2)**

Sol. $\cos[\tan^{-1}\{\sin(\cot^{-1}\sqrt{3})\}] = y$

$= \cos[\tan^{-1}(\sin\frac{\pi}{6})] = \cos\left[\tan^{-1} \frac{1}{2}\right]$

$= \cos\left[\cos^{-1} \frac{2}{\sqrt{5}}\right] = \frac{2}{\sqrt{5}}$

33. (2)

Sol. $\cos\left[\frac{1}{2}\cos^{-1}\cos\left(-\frac{14\pi}{5}\right)\right]$

$= \cos\left[\frac{1}{2}\cos^{-1}\cos\left(\frac{14\pi}{5}\right)\right]$ $\quad \because \cos\theta$

$= \cos(-\theta)$

$= \cos\left[\frac{1}{2}\left(\frac{14\ \pi}{5} - 2\pi\right)\right]$

since $\frac{14\pi}{5} \in (2\pi, 3\pi)$

$= \cos\left[\frac{2\pi}{5}\right]$

$= -\cos\left(\pi - \frac{2\pi}{5}\right) = -\cos\left(\frac{3\pi}{5}\right)$

$= -\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right) = \sin\left(\frac{\pi}{10}\right)$

34. (4)

Sol. Since $x \in \left(\frac{\pi}{2}, \pi\right)$

$\sin^{-1}[\cos(x + \pi - x) = \sin^{-1}(-1) = -\frac{\pi}{2}$

35. (2)

Sol. $\cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$,

$\frac{\pi}{2} < x < \pi$

Rationalize the term in the bracket

$= \cot^{-1}\left(\frac{2+2\sqrt{1-\sin^2 x}}{-2\sin x}\right) = \cot^{-1}\left(\frac{1-\cos x}{-\sin x}\right)$

$= \cot^{-1}\left(-\tan\frac{x}{2}\right) = \frac{\pi}{2} - \tan^{-1}\left(-\tan\frac{x}{2}\right)$

$= \frac{\pi}{2} + \tan^{-1}\tan\frac{x}{2}$

Since $\frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$= \frac{\pi}{2} + \frac{x}{2}$

36. (2)

Sol. $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x + \cos^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{2} \Rightarrow x = \frac{1}{\sqrt{5}}$

37. (2)

Sol. $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\frac{\sqrt{3}}{2}$

$\frac{\pi}{2} - 2\cos^{-1} x = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{6}$

$x = \frac{\sqrt{3}}{2}$

38. (1)

Sol. $(\tan^{-1} x)^2 + \left(\frac{\pi}{2} - \tan^{-1} x\right)^2 = \frac{5\pi^2}{8}$

$\Rightarrow 2(\tan^{-1}x)^2 - \pi\tan^{-1}x + \frac{\pi^2}{4} = \frac{5\pi^2}{8}$

$\Rightarrow 2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \frac{3\pi^2}{8} = 0$

$\Rightarrow \left(\tan^{-1} x - \frac{3\pi}{4}\right)\left(2\tan^{-1} x + \frac{\pi}{2}\right) = 0$

$\Rightarrow \tan^{-1}x = \frac{3\pi}{4}, -\frac{\pi}{4}$

$\Rightarrow x = -1$

39. (4)

Sol. $\tan\left[\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{2}{3}\right]$

$\tan\left[\tan^{-1}\frac{\left(\frac{3}{4}+\frac{2}{3}\right)}{\left(1-\frac{6}{12}\right)}\right]$

$= \tan\tan^{-1}\left(\frac{17}{6}\right) = \frac{17}{6}$

40. (4)

Sol. $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right)$

$= \tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x^2-1}{x^2-4}}\right]$

$= \tan^{-1}\left(\frac{4-2x^2}{3}\right) = \frac{\pi}{4}$

case- I

$\frac{4-2x^2}{3} = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$ ... (1)

If $\frac{(x-1)\ (x+1)}{(x-2)\ (x+2)} < 1$

$\Rightarrow\ x \in (-2, 2)$
...(2)

from (1) & (2) $x = \pm\frac{1}{\sqrt{2}}$

case-II If $\frac{(x-1)\ (x+1)}{(x-2)\ (x+2)} > 1$

$\Rightarrow\ x \in (-\infty, -2) \cup (2, \infty)$
....(3)

$\tan^{-1}\left(\frac{4-2x^2}{3}\right) + \pi = \frac{\pi}{4}$

$\frac{4-2x^2}{3} = 1$

$x = \pm\frac{1}{\sqrt{2}}$ .....(4)

from (3) & (4) $\Rightarrow x \in \phi$

case-III $\therefore \frac{(x-1)\ (x+1)}{(x-2)\ (x+2)} = 1$

no solution

$\Rightarrow\ x = \pm\frac{1}{\sqrt{2}}$ are the solutions

41. (3)

Sol. $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x,\quad x > 0$

$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}x$

$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \theta \quad \tan\theta = \frac{1-x}{1+x}$

$x = \frac{1}{\sqrt{3}}$

42. (3)

Sol. $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$

$\sin^{-1}1 - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$

$\Rightarrow \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{x}}$

$\Rightarrow \sqrt{x} = 2$

$\therefore x = 4$

43. (4)

Sol. $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = 4^o$

$x \neq 0$ taking tan on both side

$\frac{\sqrt{1+x^2}-1}{x} = \tan 4^o$

$\sqrt{1+x^2} = 1 + x\tan 4^o$

$1 + x^2 = 2x\tan 4^o + 1 + x^2\tan^2 4^o$

$x = 0, x = \frac{2\tan 4^o}{1-\tan^2 4}$, since $x \neq 0$

$x = \tan 8^o$

44. (2)

Sol. $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right), 0 \le x \le 1$

$0 \le x \le 1 \Rightarrow \frac{1-x}{1+x} = \frac{2}{1+x} - 1 \in [0, 1]$

$f_{min} = 0 \qquad f_{max} = \frac{\pi}{4}$

45. **(1)**

Sol. Let $\sin^{-1} x = \theta$

$\cos(2\theta) = \frac{1}{3}$

$\Rightarrow 1 - 2\sin^2\theta = \frac{1}{3}$

$\Rightarrow \sin^2\theta = \frac{1}{3} \Rightarrow x = \sin\theta = \pm\frac{1}{\sqrt{3}}$

46. **(3)**

Sol. $\cos^{-1} x = \tan^{-1} x$

$\Rightarrow x \in (0, 1]$

$\tan^{-1}\frac{\sqrt{1-x^2}}{x} = \tan^{-1} x$

taking tan on both side

$\sqrt{1-x^2} = x^2$

$\Rightarrow 1 - x^2 = x^4$

$\Rightarrow x^4 + x^2 - 1 = 0$

$x^2 = \frac{-1+\sqrt{5}}{2}$, since x^2 is +ve avoid negative result

$\sin(\cos^{-1} x) = \sin(\sin^{-1}\sqrt{1-x^2}) = \sqrt{1-x^2} = x^2$

$= \frac{\sqrt{5}-1}{2}$

47. **(3)**

Sol. $\tan(x + y) = 33$

$x + y = \tan^{-1} 33$

$\Rightarrow y = \tan^{-1} 33 - x = \tan^{-1} 33 - \tan^{-1}3 = \tan^{-1}\left(\frac{33-3}{1+99}\right) = \tan^{-1}\left(\frac{3}{10}\right)$

48. **(2)**

Sol. $\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3)$

$\tan^2(\tan^{-1}\sqrt{3}) + \cot^2(\cot^{-1}\sqrt{8})$

$3 + 8 = 11$

49. **(3)**

Sol. Using properties

$\therefore \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

$\Rightarrow \frac{a}{x} = \frac{x}{b}$

$\Rightarrow x = \sqrt{ab}$

statement-1 is true

$\tan^{-1}\left(\frac{m}{n}\right) + \tan^{-1}\left(\frac{1-\frac{m}{n}}{1+\frac{m}{n}}\right) = \tan^{-1}\frac{m}{n} + \tan^{-1} 1$

$- \tan^{-1}\frac{m}{n} = \frac{\pi}{4}$

50. **(1)**

Sol. $\sin^{-1}(ax) + \cos^{-1}y + \cos^{-1}(bxy) = \frac{\pi}{2}$

(1) $a = 1, \quad b = 0$

$\Rightarrow \sin^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(0) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x + \cos^{-1}y = 0$

$\Rightarrow \cos^{-1}y = -\sin^{-1}x$

$\Rightarrow \cos^{-1} y = \cos^{-1}\sqrt{1-x^2}$

$\Rightarrow x^2 + y^2 = 1$

(2) $\sin^{-1}(x) + \cos^{-1}y + \cos^{-1}(xy) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}(y) + \cos^{-1}(xy) = \cos^{-1}x.$

$\Rightarrow \cos^{-1}\left(xy^2 - \sqrt{(1-y^2)\ (1-x^2y^2)}\right)$

$= \cos^{-1}x.$

$\Rightarrow xy^2 - \sqrt{(1-y^2)(1-x^2y^2)} = x$

$\Rightarrow 1 - x^2 - y^2 + x^2y^2 = 0$

$\Rightarrow (1 - x^2)(1 - y^2) = 0$

(3) $\sin^{-1}(x) + \cos^{-1} y + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(2xy^2 - \sqrt{(1-y^2)(1-4x^2y^2)}\right)$

$= \cos^{-1}x.$

$\Rightarrow\ 2xy^2 - \sqrt{(1-y^2)(1-4x^2y^2)} = x$

$\Rightarrow\ 2xy^2 - x = \sqrt{(1-y^2)(1-4x^2y^2)}$

$\Rightarrow\ 4x^2y^4 + x^2 - 4x^2y^2 = 1 - y^2 - 4x^2y^2 + 4x^2y^4$

$\Rightarrow\ x^2 + y^2 = 1.$

(4) $\sin^{-1}(2x) + \cos^{-1}y + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow\ \cos^{-1}\left(2y^2x - \sqrt{(1-y^2)(1-4x^2y^2)}\right)$

$= \cos^{-1}(2x)$

$\Rightarrow\ 2y^2x - \sqrt{1-y^2-4x^2y^2+4x^2y^4} = 2x.$

$\Rightarrow\ 1 - 4x^2 - y^2 + 4x^2y^2 = 0$

$\Rightarrow\ (1-4x^2)(1-y^2) = 0.$

51. (2)

Sol. $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is defined if

$-1 \le x - 3 \le 1$

$\Rightarrow 2 \le x \le 4$ ...(1)

& $9 - x^2 > 0 \Rightarrow -3 < x < 3$ ...(2)

Hence from (1) & (2)

we get $2 \le x < 3$

$\therefore$ Domain = [2, 3)

52. (3)

Sol. Given that $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$

$\Rightarrow\ \cos^{-1}\left(\frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}\right) = \alpha$

$\Rightarrow\ \frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow 2\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = 2\cos\alpha - xy$

On squaring both sides, we get

$\frac{4(1-x^2)(4-y^2)}{4} = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow\ 4 - 4x^2 - y^2 + x^2y^2 = 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow\ 4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$

53. (4)

Sol. $f(x)$ is defined if $-1 \le \frac{x}{2} - 1 \le 1$ and $\cos x > 0$

or $0 \le \frac{x}{2} \le 2$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$ or

$0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore\ x \in \left[0, \frac{\pi}{2}\right)$.

54. (2)

Sol. Since $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow\ \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$

$\Rightarrow\ x = 3$

55. (1)

Sol. $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$x = \tan\theta$

$\frac{-\pi}{6} < \theta < \frac{\pi}{6}$

$\tan^{-1}y = \theta + \tan^{-1}\tan 2\theta \quad = \theta + 2\theta = 3\theta$

$y = \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$y = \frac{3x - x^3}{1 - 3x^2}$.

56. **(1)**

Sol. $y = \sqrt{\sin^{-1} 2x + \frac{\pi}{6}}$

For domain $\sin^{-1} 2x + \frac{\pi}{6} \geq 0$

$\Rightarrow -\frac{\pi}{6} \leq \sin^{-1} 2x \leq \frac{\pi}{2}$

$\Rightarrow -\frac{1}{2} \leq 2x \leq 1$

$\Rightarrow -\frac{1}{4} \leq x \leq \frac{1}{2}$

57. **(3)**

Sol. $\sqrt{1+x^2}$

$\left[\left\{x \cos \cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}^2 - 1\right]^{1/2}$

$= \sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{\frac{1}{2}}$

$= \sqrt{1+x^2} \, . \, x$

58. **(2)**

Sol. $\cot \sum_{n=1}^{23} \cot^{-1} \left(1+2+4+6+ \; . \; . \; . \; . \; . \; . \; +2n\right)$

$\cot \sum \cot^{-1}(1 + n(n + 1))$

$\cot \sum \tan^{-1} \frac{(n+1)-n}{1+n \; (n+1)}$

$\cot \sum_{n=1}^{23} \left(\tan^{-1}(n+1) - \tan^{-1} n\right)$

$\cot(\tan^{-1}24 - \tan^{-1}1)$

$\cot\left(\tan^{-1}\frac{24-1}{1+24}\right)$

$\cot\left(\cot^{-1}\frac{25}{23}\right) = \frac{25}{23}$

59. **(1)**

Sol. $2\sin^{-1} x + \cos^{-1} x = \frac{11\pi}{6}$

$\frac{\pi}{2} + \sin^{-1} x = \frac{11\pi}{6}$

$\sin^{-1} x = \frac{4\pi}{3}$

Never possible, so no solution.

60. **(1)**

Sol. $\sin^{-1}(\sin 5) > x^2 - 4x$

$\Rightarrow \sin^{-1}(\sin (2\pi - 5)) > x^2 - 4x$

$\Rightarrow 5 - 2\pi > x^2 - 4x$

$\Rightarrow x^2 - 4x - (5 - 2\pi) < 0$

$x = \frac{4 \pm \sqrt{16+4(5-2\pi)}}{2}$

$x = \frac{4 \pm 2\sqrt{4+5-2\pi}}{2} = 2 \pm \sqrt{9-2\pi}$

$\Rightarrow x \in \left(2-\sqrt{9-2\pi}, 2+\sqrt{9-2\pi}\right)$

$\Rightarrow |x-2| < \sqrt{9-2\pi}$

Integer Type Questions (61 to 71)

61. **(0)**

Sol. $\sin\left[\frac{\pi}{6} + \left(-\frac{\pi}{6}\right)\right] = \sin 0 = 0$

62. **(20)**

Sol. $\sum_{i=1}^{20} \sin^{-1} x_i = 10\pi$

$-\frac{\pi}{2} \leq \sin^{-1} x_i \leq \frac{\pi}{2}$

$\Rightarrow -10\pi \leq \sum_{i=1}^{20} \sin^{-1} x_i \leq 10\pi$

so $x_i = 1$

$\Rightarrow \sum_{i=1}^{20} x_i = 20$

63. **(3)**

Sol. $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \dfrac{3\pi}{2}$

$\Rightarrow \sin^{-1}\alpha = \sin^{-1}\beta = \sin^{-1}\gamma = \dfrac{\pi}{2}$

$\Rightarrow \alpha = \beta = \gamma = 1$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 3$

64. **(3)**

Sol. $\because \cot^{-1} x + \tan^{-1} x = \dfrac{\pi}{2}$

so $x = 3$

65. **(1)**

Sol. $\cos^{-1}\dfrac{3}{5} - \sin^{-1}\dfrac{4}{5} = \cos^{-1}x$

$\Rightarrow \cos^{-1}\dfrac{3}{5} - \cos^{-1}\dfrac{3}{5} = \cos^{-1}x$

$\Rightarrow 0 = \cos^{-1}x$

$\Rightarrow x = 1$

66. **(15)**

Sol. $\sec^2(\tan^{-1}2) = 1 + \tan^2(\tan^{-1}2) = 1 + (2)^2 = 5$ and

$\text{cosec}^2(\cot^{-1}3) = 1 + \cot^2(\cot^{-1}3) = 1 + (3)^2 = 10$

$\Rightarrow \sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3) = 5 + 10 = 15$

67. **(1)**

Sol. $\cot\left(\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{5} + \tan^{-1}\dfrac{1}{8}\right)$

$= \cot\left[\tan^{-1}\left(\dfrac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{10}}\right) + \tan^{-1}\dfrac{1}{8}\right]$

$\Rightarrow \cot\left[\tan^{-1}\dfrac{7}{9} + \tan^{-1}\dfrac{1}{8}\right]$

$= \cot\left[\tan^{-1}\dfrac{\left(\frac{7}{9}+\frac{1}{8}\right)}{\left(1-\frac{7}{72}\right)}\right] = \cot[\tan^{-1}1] = 1$

68. **(3)**

Sol. $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \theta$

$\Rightarrow \cot^{-1}\left(\dfrac{7.8-1}{7+8}\right) + \cot^{-1}18 = \theta$

$\Rightarrow \cot^{-1}\left(\dfrac{11}{3}\right) + \cot^{-1}18 = \theta$

$\Rightarrow \cot^{-1}\left(\dfrac{\frac{11}{3}\,.\,18-1}{\frac{11}{3}+18}\right) = \theta$

$\Rightarrow \cot\theta = 3$

69. **(5)**

Sol. $\cot^{-1}\dfrac{n}{\pi} > \dfrac{\pi}{6} \qquad \Rightarrow -\infty < \dfrac{n}{\pi} < \cot\dfrac{\pi}{6}$

$\Rightarrow n < \pi\sqrt{3} \qquad n \in N\ , \quad n_{\max} = 5$

70. **(3)**

Sol. $\sin^{-1}\left(\dfrac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \dfrac{\pi}{2}$

Taking sin on both side

$\dfrac{3\sin 2\theta}{5+4\cos 2\theta} = 1$

$\Rightarrow 3\sin2\theta = 5 + 4\cos2\theta$

$\Rightarrow \dfrac{6\tan\theta}{1+\tan^2\theta} = 5 + 4\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow \Rightarrow \tan^2\theta - 6\tan\theta + 9 = 0$

$\tan\theta = 3$

71. **(1)**

Sol. $\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$

$2\sin^{-1}x + \cos^{-1}(1-x) = 0 \qquad$ here $x \in [0,\ 1]$

for $x \in [0,\ 1] \quad 2\sin^{-1}x \in [0,\ \pi]$ and $\cos^{-1}(1-x) \in \left[0, \dfrac{\pi}{2}\right]$

There sum is equal to zero when both terms equal to zero it gives $x = 0$ is only solution.

MATRICES & DETERMINANTS

Single Option Correct Type Questions (01 to 57)

1. **(1)**

Sol. AB = O

$$\Rightarrow \begin{bmatrix} \cos^2\theta\cos^2\phi+\sin\theta\sin\phi\cos\theta\cos\theta & \cos^2\theta\cos\phi\sin\phi+\sin^2\phi\cos\theta\sin\theta \\ \cos^2\phi\cos\theta\sin\theta+\sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi+\sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi(\cos\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \cos(\theta-\phi) = 0 \Rightarrow \theta-\phi = (2n+1)\dfrac{\pi}{2}$

2. **(2)**

Sol. $A = 3I$

$\therefore AB = 3IB \qquad (\because IB = B)$

$AB = 3B$

3. **(4)**

Sol. $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

For $n = 2$ option (1), (2), (3) are not satisfied. Hence option (4) is correct.

4. **(1)**

Sol. $A^2 = A$

$\Rightarrow A^{-1}A^2 = A^{-1}A \Rightarrow A = I$

$\therefore (I+A)^{10} = (I+I)^{10} = (2I)^{10}$

$= 1024\,I = I + kI = (k+1)\,I$

$\therefore k + 1 = 1024$

$\Rightarrow k = 1023$

5. **(1)**

Sol. $(A+B)(A-B) = A^2 + BA - AB + B^2 \neq A^2 - B^2$

6. **(3)**

Sol. $A^T = A^2 - 2A \Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$

$\begin{bmatrix} 3a & c+2b \\ b+2c & 3d \end{bmatrix} = \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix}$

$\Rightarrow 3a = a^2 + bc,\ c + 2b = 2b,\ b + 2c = 2c$ and $3d = bc + d^2$

$\Rightarrow c = 0,\ b = 0,\ a = 0,3$ and $d = 0,3$

$\Rightarrow (a, d) = (0, 0), (0, 3), (3, 0), (3,3)$

but $a + d = 2$

so no such matrix is possible

7. **(1)**

Sol. $A = A'$ is symmetric

(1) $(AA')' = (A')'A' = AA'$ (so AA' is symmetric)

(3) $(A'A)' = A'(A')' = A'A$ (so $A'A$ is also symmetric)

8. **(1)**

Sol. $A^T = -A$

$(A^n)^T = (AAA \text{ ------- } A)^T = (A^T A^T A^T \text{ ------- } A^T)$

$= (A^T)^n$ for all $n \in N$

$(-A)^n = (-1)^n A^n$

$(A^n)^T = \begin{cases} A^n & \text{if } n \text{ is even} \\ -A^n & \text{if } n \text{ is odd} \end{cases}$

9. **(4)**

Sol. $|A|+|A^T|=2|A|\neq 0$

10. **(4)**

Sol. $AB = C \Rightarrow \det(A)\det(B) = \det(C) \Rightarrow \det(B) = -1$

11. **(1)**

Sol. For nth order determinant $\Delta = |C_{ij}| = D^{n-1}$

(1) For 3rd order determinant $\Delta = D^{3-1} = D^2$ ... (1)

(2) From (1) if $D = 0$ then $\Delta = 0$

(3) $D = 9 = 3^2$

$\Delta = (3^2)^2 = 3^4$ (Δ is not a perfect cube)

12. **(3)**

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $bc \neq 0$

Characteristic equation is $|A - xI| = 0$

$$\begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = 0$$

$(a-x)(d-x) - bc = 0$

$x^2 - x(a+d) + ad - bc = 0$

On comparing with the given equation

$x^2 + k = 0$

$a + d = 0,\ k = ad - bc = |A|$

13. **(1)**

Sol. If we interchange any two rows of a determinant in set B, its value becomes -1, hence it becomes a member of set C.

$\Rightarrow$ Number of elements in set B is equal to number of elements in set C.

$\Rightarrow$ Statement - 1 is true.

Also $B \cap C = \phi \subset A$

$\Rightarrow$ statement-2 is true.

But statement-2 is not a correct explanation of statement-1.

14. **(4)**

Sol. $A = \begin{vmatrix} 2 & 1+2i \\ 1-2i & 7 \end{vmatrix} \Rightarrow |A| = 14 - (1 - 4i^2) = 9$

statement 1 is true and obviously statement 2 is false.

15. **(4)**

Sol. $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{10}$

$= \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$

$\text{adj } A^{10} = \begin{bmatrix} 1 & 0 \\ -10 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 10 & 100 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 90 \\ 0 & 11 \end{bmatrix}$

$b_1 + b_2 + b_3 + b_4 = 22 + 90 = 112.$

16. **(3)**

Sol. Taking $C_3 \to C_3 - (C_1\alpha - C_2)$

we get

$$|A| = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha+1 \end{vmatrix}$$

$= (1 - 2\alpha)(ac - b^2)$

$\therefore$ non-invertible if $\alpha = \frac{1}{2}$ and if a, b, c are in G.P.

17. **(3)**

Sol. $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$A^3 = A^2 . A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ and

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\Rightarrow A^3 = A^{-1}$

18. **(2)**

Sol. $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$; $A^3 = 5A^2 - 6AI + I^2$

(by $|A - \lambda I| = 0$)

19. **(3)**

Sol. Since $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Now $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

and $BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

If $AB = BA \Rightarrow a = b$

Hence, $AB = BA$ is possible for infinitely many $B's$.

20. **(1)**

Sol. Given, $A^2 - B^2 = (A - B)(A + B)$

$\Rightarrow A^2 - B^2 = A^2 - B^2 + AB - BA$

$\Rightarrow AB = BA$

21. **(3)**

Sol. Since, $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 10\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$

$\Rightarrow |A^2| = \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{vmatrix}$

$= 25 \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 \\ 0 & \alpha^2 \end{vmatrix} = 625\alpha^2$

But $|A|^2 = 25$

$\therefore 625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$.

22. **(3)**

Sol. As $\det(A) = \pm 1$, A^{-1} exists and

$A^{-1} = \frac{1}{\det(A)}(adj\ A) = \pm(\text{adj } A)$

All entries in adj (A) are integers.

$\therefore A^{-1}$ has integer entries.

23. **(4)**

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\because A^2 = I)$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow b(a + d)$

$= 0$, $c(a + d) = 0$

and $a^2 + bc = 1$, $bc + d^2 = 1 \Rightarrow a = 1, d = -1, b = c = 0$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A \neq I, A \neq -I$

det $A = -1$ (Statement I is true)

Statement II tr $(A) = 1 - 1 = 0$,

Statement II is false.

24. **(3)**

Sol. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\Rightarrow$ det $(A) = a_1 (b_2c_3 - c_2b_3) - a_2 (b_1c_3 - c_1b_3) + a_3 (b_1c_2 - c_1b_2)$

$= a_1b_2c_3 - a_1c_2b_3 + a_2c_1b_3 - a_2 b_1c_3 + a_3b_1c_2 - a_3c_1b_2$

if any of the terms is non-zero, then det (A) will be non-zero and all the element of that term will be unity

Now there are 6 elements remaining out of which any one can be unity.

Hence number of non-singular matrices

$= \underbrace{{}^6C_1}_{\text{choosing any one triplet}} \times \underbrace{{}^6C_1}_{\text{choosing any one element}}$

Hence correct option is (3)

25. **(2)**

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} a^2 + bc & b(a+d) \\ c(a+d) & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $a + d = 0$ and $a^2 + bc = 1$

$\Rightarrow$ $\text{Tr}(A) = 0$

Statement-1 is true

Statement-2 $|A| = ad - bc = -a^2 - bc = -1$

Statement-1 is true statement-2 is false.

Hence correct option is (2)

26. **(1)**

Sol. $B = \begin{vmatrix} x & c & b \\ -c & x & a \\ b & -a & x \end{vmatrix}$

Transpose matrix formed by cofactors of B = $\begin{bmatrix} a^2 + x^2 & ab - cx & ac + bx \\ ab + cx & b^2 + x^2 & bc - ax \\ ac - bx & bc + ax & c^2 + x^2 \end{bmatrix}$

$\Rightarrow$ adj $B = A$

$\Rightarrow |adjB| = |A|$

$\Rightarrow |B|^2 = |A|$

27. **(2)**

$A' = A$, $B' = A$

$P = A(BA)$

$P' = (A(BA))' = (BA)' A' = (A'B') A' = (AB) A = A(BA)$

$\therefore$ $A(BA)$ is symmetric

similarly $(AB) A$ is symmetric

Statement(2) is correct but not correct explanation of statement (1).

28. **(4)**

Sol. $H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

If $H^k = \begin{bmatrix} \omega^k & 0 \\ 0 & \omega^k \end{bmatrix}$, then $H^{k+1} = \begin{bmatrix} \omega^{k+1} & 0 \\ 0 & \omega^{k+1} \end{bmatrix}$

So by mathematical induction

$H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$

29. **(3)**

Sol. adj(AB) = (adj B) (adj A)

so statement 2 is false

30. **(4)**

Sol. $A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$; $u_1 + u_2 = A^{-1} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}$ adj A ; $|A| = 1$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

Now $u_1 + u_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

31. **(4)**

Sol. $BB^T = B(A^{-1}A^T)^T$

$= B(A^T)^T (A^{-1})^T$

$= BA(A^{-1})^T$

$= A^{-1}A^TA(A^{-1})^T$

$= A^{-1}AA^T(A^{-1})^T$

$= IA^T(A^{-1})^T$

$= A^T(A^{-1})^T$

$= A^T(A^T)^{-1}$

$= I$

32. **(4)**

Sol. $AA^T = 9\text{I}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad a + 4 + 2b = 0, 2a + 2 - 2b = 0, a^2 + 4 + b^2 = 9$

$\Rightarrow \quad a = -2, b = -1.$

33. **(1)**

Sol. $A^2 - 5A + 7\text{I} = 0$

$\Rightarrow A^2 - 5A = -7\text{I}$

$\Rightarrow |A^2 - 5A| = |-7\text{I}|$

$\Rightarrow |A|\,|A-5\text{I}| = 7$

so $|A|$ can not be zero

$A^2 - 5A + 7\text{I} = 0 \qquad |A| \neq 0$

$\Rightarrow A - 5\text{I} = -7A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{7}(5\text{I} - A)$

Hence statement 1 is true

Now $A^3 - 2A^2 - 3A + \text{I} = A(A^2) - 2A^2 - 3A + \text{I}$

$= A(5A - 7\text{I}) - 2A^2 - 3A + \text{I}$

$= 3A^2 - 10A + \text{I}$

$= 5A - 20\text{I} \quad = 3((5A - 7\text{I}) - 10A + I$

$= 5(A - 4\text{I})$

Statement 2 also correct

34. **(4)**

Sol. $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\alpha^2 = 1, \ldots\ldots\ldots (1)$

$\alpha + 1 = 5 \ldots\ldots\ldots(2)$

$\alpha = \pm 1 \qquad \alpha = 4$

(1) and (2) not satisfied simultaneously so no real solution of α.

35. **(1)**

Sol. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PA\,P^T$ and

$X = P^TQ^{2005}\,P$

We observe that $Q = PA\,P^T$

$Q^2 = (P\,A\,P^T)\,(P\,A\,P^T) = PA\,(P^TP)$

$A\,P^T = PA\,(IA)\,P^T = P\,A^2\;P^T$

Proceeding in the same way, we get

$Q^{2005} = P\,A^{2005}\;P^T$

Also $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

And proceeding in the same way

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$$P^TP = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, $X = P^T Q^{2005} P = P^T (PA^{2005} P^T) P = (P^T P) A^{2005} (P^T P) = I A^{2005} I$

$$= A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

36. **(3)**

Sol. $|A^n| = |A|^n$; $|A^3| = 125$

$|A|^3 = 125 \Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 5$

$\alpha^2 - 4 = 5 \Rightarrow \alpha \equiv \pm 3$

37. **(4)**

Sol.

$P^T = 2P + I$

$\Rightarrow (P^T)^T = (2P + I)^T$

$\Rightarrow P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I$

$\Rightarrow 3P = -3I$

$\Rightarrow P = -I$

$\Rightarrow PX = -IX = -X$

38. **(2)**

Sol. $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$

$$\Rightarrow P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 16(1+2) & 8 & 1 \end{bmatrix},$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 16(1+2+3) & 12 & 1 \end{bmatrix}$$

$$P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 4\times 50 & 1 & 0 \\ 16(1+2+...+50) & 4\times 50 & 1 \end{bmatrix},$$

$$P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix}$$

$\because P^{50} - Q = \text{I}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix} - \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 200 - q_{21} = 0 \Rightarrow q_{21} = 200$

$20400 - q_{31} = 0 \Rightarrow q_{31} = 20400$ and

$200 - q_{32} = 0 \Rightarrow q_{32} = 200$

$\therefore \dfrac{q_{31} + q_{32}}{q_{21}} = \dfrac{20400 + 200}{200} = 103$

39. **(4)**

Sol. 1×1 matrix [0] i.e.1

2×2 matrix $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$ i.e $2 \times 1 = 2$

3×3 matrix i.e $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ $2 \times 2 \times 2 = 8$

Total = 1 + 2 + 8 = 11

40. **(2)**

Sol. Here $A^2 = A \Rightarrow a^2 + bc = a,\ b(a+d) = b$

$c(a+d) = c,\ bc + d^2 = d$

$\therefore abcd \neq 0 \qquad \therefore a + d = 1$

$\Rightarrow bc = ad$

$\Rightarrow bc - ad = 0$

$\Rightarrow |A| = 0$ Ans.

41. **(3)**

Sol. Here $AA' = A'A \Rightarrow a = b$

42. **(1)**

Sol. Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (a_1 b_2 c_3 + a_3 b_1 c_2 + a_2 b_3 c_1) - (a_1 b_3 c_2 + a_2 b_1 c_3 + a_3 b_2 c_1)$

$\therefore$ each element of Δ is either 0 or 2 , therefore the value of Δ cannot exceed 24

$\Delta = 24 \Rightarrow a_1 b_2 c_3 + a_3 b_1 c_2 + a_2 b_3 c_1 = 24 = 8 + 8 + 8$

$\Rightarrow a_1 = a_2 = a_3 = 2,\ b_1 = b_2 = b_3 = 2,\ c_1 = c_2 = c_3 = 2$

but in this case $\Delta = 0$

$$\therefore \Delta = \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{vmatrix} = 0\,(0-4) - 2\,(0-4) + 2\,(4-0)$$

$= 0 + 8 + 8 = 16$

43. **(3)**

Sol. Here $\Delta = \begin{vmatrix} abc & b^2c & c^2b \\ abc & c^2a & ca^2 \\ abc & a^2b & b^2a \end{vmatrix} = 0$

$\Rightarrow -a^2b^2c^2(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow (a+b+c)(a^2+b^2+c^2 - ab - bc - ca) = 0$

$\Rightarrow a + b + c = 0 \qquad \therefore a \neq b \neq c \neq 0$

44. **(3)**

Sol. Given can be written as

$$p + \frac{q}{\lambda} + \frac{r}{\lambda^2} + \frac{5}{\lambda^3} + \frac{t}{\lambda^4} = \begin{vmatrix} 1 + 3/\lambda & 1 - \frac{1}{\lambda} & 1 + \frac{3}{\lambda} \\ 1 + \frac{1}{\lambda^2} & \frac{2}{\lambda} - 1 & 1 - \frac{3}{\lambda} \\ 1 - \frac{3}{\lambda^2} & 1 + \frac{4}{\lambda} & 3 \end{vmatrix}$$

Taking limit $\lambda \to \infty$

$$p = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = -4$$

45. **(4)**

Sol. Let $A = \text{diag}(a_1\ a_2\ a_3\ ....a_n)$

$A^3 = \text{diag}\left(a_1^3, a_2^3, a_3^3, ...a_n^3\right)$

$\therefore \quad A^3 = A \qquad \therefore a_1^3 = a_1$

$\Rightarrow \quad a_1 = 0, \quad 1, -1$

$\therefore$ Total Number of diagonal matrix $= 3^n$

46. **(2)**

Sol. Here $f(x) = 2\sin^2 x + 2\cos^2 x = 2 \ \therefore f'(x) = 0$

$$\int_0^{\pi/2} (f(x) + f'(x))dx = \int_0^{\pi/2} 2dx = 2 \times \frac{\pi}{2} = \pi$$

47. **(3)**

Sol. $A + B = AB$

$\Rightarrow \quad I_n - A - B + AB = I_n \qquad \Rightarrow (I_n - A)(I_n - B) = I_n$

$\Rightarrow \quad (I_n - A)^{-1} = (I_n - B) \ \therefore (I_n - B)\ (I_n - A) = I_n$

$\Rightarrow \quad I_n - B - A + BA = I_n \qquad \Rightarrow A + B = BA$

$\therefore \quad AB = BA$

48. **(4)**

Sol. $\therefore B = -A^{-1}BA \qquad \therefore AB = -BA$

$\Rightarrow AB + BA = 0$

$\therefore \quad (A + B)^2 + A^2 + AB + BA + B^2 = A^2 + B^2$

49. **(3)**

Sol. As $PQ = kI$

$\Rightarrow Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(adjP)\ I$

$$\Rightarrow Q = \frac{k}{(20 + 12\alpha)} \begin{bmatrix} - & - & - \\ - & - & (-3\alpha - 4) \\ - & - & - \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\because q_{23} = \frac{-k}{8} \qquad \Rightarrow \qquad \frac{k}{(20 + 12\alpha)}(-3\alpha - 4) = \frac{-k}{8} \qquad \Rightarrow \qquad 2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3 \quad \Rightarrow \qquad \alpha = -1$

also $|Q| = \frac{k^3 |I|}{|P|} \quad \Rightarrow$

$\frac{k^2}{2} = \frac{k^3}{(20 + 12\alpha)}$

$(20 + 12\alpha) = 2k \qquad \Rightarrow 8 = 2k$

$\Rightarrow \quad k = 4$

(1) incorrect

(2) incorrect

(3) $|P(adjQ)| = |P|\ |adjQ| = P|\ |Q|^2 = 2^2(2^3)^2 = 2^9$ correct

(4) $|Q(adjP)| = |Q|\ |adjP| = |Q|\ |P|^2 = 2^3(2^3)^2 = 2^9$ incorrect

50. **(2)**

Sol. Make $C_1 \to C_1 + C_3$ we get

determinant $= f(\theta)$

$$= \begin{bmatrix} 1 & \tan\theta + \sec^2\theta & 3 \\ 0 & \cos\theta & \sin\theta \\ 0 & -4 & 3 \end{bmatrix}$$

$= 3\cos\theta + 4\sin\theta$

$\Rightarrow f'(\theta) = 0$

$\Rightarrow -3\sin\theta + 4\cos\theta = 0$

$\Rightarrow \tan\theta = \frac{4}{3}$

$\Rightarrow \theta = \tan^{-1}\frac{4}{3}$

$\Rightarrow f(\theta)$ is $\uparrow$ for $\left[0, \tan^{-1}\frac{4}{3}\right]$

and $\downarrow$ for $\left[\tan^{-1}\frac{4}{3}, \frac{\pi}{2}\right]$

max $f(\theta)$ is at $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow$ max $f(\theta) = 3\left(\frac{3}{5}\right) + 4\left(\frac{4}{5}\right) = 5$

minimum $f(\theta)$ is at $\theta = 0$

$\Rightarrow$ min $f(\theta) = 3$

51. (4)

Sol. $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

$\text{Adj}.A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$\Rightarrow |A| = -17 \quad \Rightarrow \quad A^{-1} = \frac{\text{AdjA}}{|A|}$

$A^{-1} = \frac{1}{17}\begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -1 & -7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3x+0+3z \\ 2x+y+0 \\ 4x+0+2z \end{bmatrix} = \begin{bmatrix} 8+2y \\ 1+z \\ 4+3y \end{bmatrix}$

$\Rightarrow 3x + 3z = 8 + 2y$

$\Rightarrow 2x + y = 1 + z$

$\Rightarrow 4x + 2z = 4 + 3y$

By Solving $x = 1$, $y = 2$ & $z = 3$

52. (1)

Sol. $AB = A$

Premultiplying by B

$BAB = BA$

$BB = B$

$B^2 = B$

$\Rightarrow B$ is idempotent

similarly on post mutliplying by A

$ABA = A^2$

$AB = A^2$

$A = A^2$

$\Rightarrow A$ is idempotent

53. (4)

Sol. For orthogonal matrix $AA' = I$

$$\Rightarrow \begin{vmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{vmatrix}\begin{vmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$\Rightarrow 4\beta^2 + \gamma^2 = 1,\ 2\beta^2 - \gamma^2 = 0,\ -2\beta^2 + \beta^2 = 0,$

$\alpha^2 - \beta^2 - \gamma^2 = 0,\ \alpha^2 + \beta^2 + \gamma^2 = 1$

$\therefore \quad \alpha = \pm\frac{1}{\sqrt{2}},\ \beta = \pm\frac{1}{\sqrt{6}},\ \gamma = \pm\frac{1}{\sqrt{3}}$

54. (3)

Sol. $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow A$

is nilpotent of order 3.

55. (1)

Sol. $|\text{adj } A| = |A|^{n-1} = |A|$, for $n = 2$

adj (A).adj(adj $(A)) = |\text{adj } (A)|\, I$

$\Rightarrow$ A.adj (A).adj(adj(A)) = |adj(A)|A

$\Rightarrow$ $|A|$.adj(adj (A)) = $|A|\ A$

$\Rightarrow$ adj(adj (A)) = A

56. **(4)**

Sol. **Statement-1:** Determinant of a skew symmetric matrix of odd order is zero

Statement-2 : det (A^T) = det(A)

det$(-A) = (-1)^n$ det (A) where A is a $n \times n$ order matrix

57. **(2)**

Sol. $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$A^2 - 3A - 10I = 0$

$A^2 = 3A + 10I$

$3A^2 + 12A = 3(3A + 10I) + 12A = 21A + 30I$

$21A + 30I = \begin{bmatrix} 42 & -63 \\ -84 & 21 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

adj $(21A + 30I) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Integer Type Questions (58 to 67)

58. **(8)**

Sol. $A(Adj\ A) = |A|\ I_3 = kI_3 \Rightarrow k = |A| = 8$

59. **(5)**

Sol. $|A|I = AA^T$

$\Rightarrow (10a + 3b)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$\Rightarrow 25a^2 + b^2 = 10a + 3b$ & $15a - 2b = 0$ & $10a + 3b = 13$

$\Rightarrow 10a + \dfrac{3.15a}{2} = 13$

$\Rightarrow 65a = 2 \times 13$

$\Rightarrow a = \dfrac{2}{5}$

$\Rightarrow 5a = 2$

$\Rightarrow 2b = 6$

$\Rightarrow b = 3$

60. **(1)**

Sol. $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$

$= \sin x\,(x^2 - x) - \cos x\,(x^3 - 2x^2) + \tan x\,(x^3 - 2x^3)$

$= (x^2 - x)\sin x - x^2(x - 2)\cos x - x^3 \tan x$

$\dfrac{f(x)}{x^2} = \left(1 - \dfrac{1}{x}\right)\sin x - (x - 2)\cos x - x\tan x$

$\therefore \lim\limits_{x \to 0} \dfrac{f(x)}{x^2} = 0 - 1 - 0 + 2 - 0 = 1.$

61. **(3)**

Sol. Here $\Delta = abc\left(1 + \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$

$= abc\left(1 + \dfrac{ab + bc + bc}{abc}\right) = abc + (ab+bc+ca)$

Now $a+b+c = -3$

$ab+bc+ca = 4$

$abc = -1$

$\therefore\ \Delta = -1 + 4 = 3$

62. **(2)**

Sol. $AB = B$

$\Rightarrow \begin{bmatrix} ap + bq \\ cp + dq \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$

$\Rightarrow ap + bq = p$

$cp + dq = q$

Eliminating p and q we get

$\Rightarrow ad - bc - (a+d) + 1 = 0$

$\Rightarrow ad - bc - 3 + 1 = 0$

$\Rightarrow ad - bc = 2 \quad \Rightarrow \quad |A| = 2$

63. **(8)**

Sol. Let A = diagonal (a, b, c), B is any other square matrix of order 3

$\therefore AB = BA$

$\therefore a = b = c$

given $a + b + c = 12$

$\therefore a = b = c = 4$

$\therefore |A| = abc = 4.\ 4.\ 4 = 64$

$\therefore |A|^{\frac{1}{2}} = 8$

64. (1)

Sol. Here $A^2 = 3A - 2I$ $\therefore$ $A^8 = 255A - 254I$

$\therefore \lambda = 255,$ $\mu = -254$

$\therefore$ $\lambda + \mu = 1$

65. (4)

Sol. $$\det(A) = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 2k+1 & -1 \end{vmatrix} \quad R_2 \to R_2 - R_3$$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ -2\sqrt{k} & 2k+1 & -1 \end{vmatrix} = (2k+1)^3$$

$\because$ B is a skew-symmetric matrix of odd order therefore $\det(B) = 0$

Now $\det(adj\ A) + \det(adj\ B) = 10^6$

$\Rightarrow$ $\{(2k+1)^3\}^2 + 0 = 10^6$

$\Rightarrow$ $2k + 1 = 10$, as $k > 0$

$\Rightarrow$ $k = 4.5$

$\Rightarrow$ $[k] = 4$

66. (2)

Sol. $$x.x^2 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 0 & 2 & 6x^3-1 \\ 0 & 6 & 2x^3-2 \end{vmatrix}$$

$\Rightarrow x^3(12x^3 + 2) = 10$

$6x^6 + x^3 - 5 = 0$

$\Rightarrow 6x^6 + 6x^3 - 5x^3 - 5 = 0$

$(6x^3 - 5)(x^3 + 1) = 0$

$\Rightarrow x^3 = -1,\ x^3 = \dfrac{5}{6}$

$x = -1,\ x = \left(\dfrac{5}{6}\right)^{1/3}$ so two solutions

67. (11)

Sol. $|P| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$

$|P| = |A|^2 = 16 \Rightarrow 2\alpha - 6 = 16 \Rightarrow \alpha = 11.$

APPLICATION OF DERIVATIVES

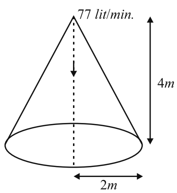
Single Option Correct Type Questions (01 to 63)

1. **(2)**

Sol. $V = \frac{1}{3}\pi r^2 h$ $\left(\because \frac{r}{h} = \frac{2}{4} = \frac{1}{2}\right)$

$V = \frac{1}{3}\pi \frac{h^3}{4} = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2 \frac{dh}{dt}$



$77 \times 10^3 = \frac{22}{7} \times \frac{1}{4} \times 70 \times 70 \times \frac{dh}{dt}$

($\because$ 1 litre = 10^3 *c.c.*)

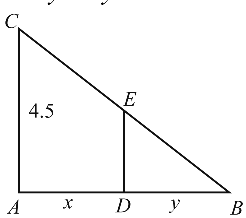
$\therefore \frac{dh}{dt} = 20$ *cm/min.*

2. **(3)**

Sol. Let AC be pole, DE be man and B be farther end of shadow as shown in figure

From triangles ABC and DBE

$\frac{4.5}{x+y} = \frac{1.5}{y}$ $3y = 1.5\,x$



$\frac{dy}{dt} = 2,$ $(x + y) = \frac{dx}{dt} + \frac{dy}{dt}$

3. **(3)**

Sol. $y = \frac{2X^2 - 1}{X^4}$ is even funciton.

Even function is nonmonotonic.

4. **(4)**

Sol. $f'(x) = 6(x^2 - 3x + 2) = 6(x - 2)(x - 1)$

for monotonically increasing, $f'(\underline{x}) > 0$

$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$

5. **(4)**

Sol. $f'(x) > 0$

$\Rightarrow -\sin x + x > 0 \Rightarrow x > \sin x \Rightarrow x \in (0, \infty)$

6. **(4)**

Sol. $f'(x) = 1 - \sin x \geq 0 \quad \forall \; x \in R$

$\Rightarrow$ $f(x)$ is *M.I.*

7. **(2)**

Sol. $f'(x) = 3x^2 + 2ax + b + 5\sin 2x \geq 0, \; \forall \; x \in R$

$\because$ $\sin 2x \geq -1$

$\Rightarrow$ $f'(x) \geq 3x^2 + 2ax + b - 5 \; \forall \; x \in R$

$\Rightarrow$ $3x^2 + 2ax + b - 5 \geq 0 \, \forall \; x \in R$

$\Rightarrow$ $4a^2 - 4 \cdot 3 \cdot (b - 5) < 0$

$\Rightarrow$ $a^2 - 3b + 15 < 0$

8. **(1)**

Sol. $f'(x) = 3(a + 2)x^2 - 6ax + 9a \leq 0 \; \forall \; x \in R$

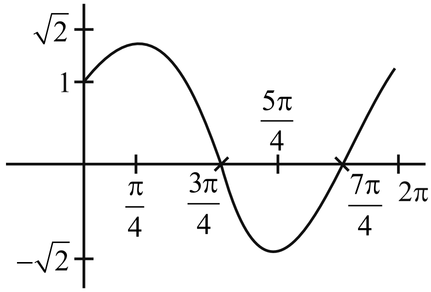
$\Rightarrow$ $a + 2 < 0$ $D \leq 0$

$\Rightarrow$ $a < -2$ $a \in (-\infty, -3] \cup [0, \infty)$

$\Rightarrow$ $a \in (-\infty, -3]$

9. **(2)**

Sol. $f(x) = \sin x + \cos x = \sqrt{2}\ \sin\left(\frac{\pi}{4} + x\right)$

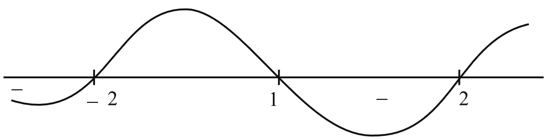


10. **(1)**

Sol. $f(x) = \log(x-2) - \frac{1}{x}$

$$f'(x) = \frac{1}{x-2} + \frac{1}{x^2} = \frac{x^2+x-2}{x^2(x-2)}$$

$$= \frac{x^2+2x-x-2}{x^2(x-2)} = \frac{(x+2)\ (x-1)}{x^2(x-2)}$$



but $\log(x-2)$ is defined when $x > 2$

$\Rightarrow$ $f(x)$ is *M.I.* for $x \in (2, \infty)$

11. **(3)**

Sol. The function $f(x) = x^3$ increases $\forall\ x$ and the function

$6x^2 + 15x + 5$ increases is

$g'(x) > 0 \qquad \Rightarrow 12x + 15 > 0$

$\Rightarrow \quad x > -5/4$

It is given that $f(x)$ increases less rapidly than $g(x)$, therefore function $\phi(x) = f(x) - g(x)$ is decreasing function, which implies that $\phi'(x) < 0$

$\Rightarrow 3x^2 - 12x - 15 < 0 \Rightarrow (x-5)\ (x+1) < 0$

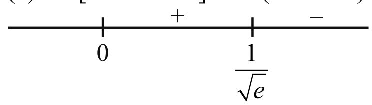
$\Rightarrow \quad -1 < x < 5$

Hence, x^3 increases less rapidly than $6x^2 + 15x + 5$ in the interval $(-1, 5)$

12. **(1)**

Sol. $f(x) = x^2\ ln\left(\frac{1}{x}\right) = -x^2\ ln\ x$

$f'(x) = -[2x\ ln\ x + x] = -x(2ln x + 1)$

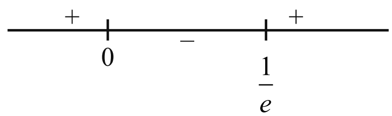


$\Rightarrow$ maximum at $x = \frac{1}{\sqrt{e}}$

13. **(4)**

Sol. $f'(x) = ln\ x + 1$

Local minima at $x = \frac{1}{e}$



No local maxima

14. **(2)**

Sol. $f(x) = a \sin x + \frac{1}{3} \sin 3x$

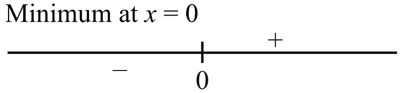
$f'(x) = a \cos x + \cos 3x$

at $x = \frac{\pi}{3}$, $f'(x) = 0 \Rightarrow \frac{a}{2} - 1 = 0 \Rightarrow a = 2$

15. **(3)**

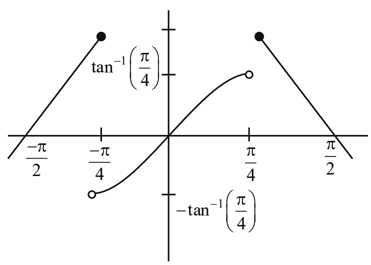
Sol. $f'(x) = (2^2 + 4^2x^2 + 6^2x^4 + ... + 100^2 x^{98})x$

Minimum at $x = 0$



16. **(3)**

Sol.



17. **(4)**

Sol. $f(1^-) \le f(1)$ and $f(1^+) \le f(1)$

$-2 + \log_2 (b^2 - 2) \le 5$

$0 < b^2 - 2 \le 128,\ 2 < b^2 \le 130$

18. **(4)**

Sol. Minimum value of $f(x)$ is $\frac{3}{2}$ at $x = \frac{1}{2}$

19. **(2)**

Sol. $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$

$\therefore\ f'(x) = 12x^3 - 24x^2 + 24x - 48 = 0$

$\Rightarrow\ x^3 - 2x^2 + 2x - 4 = 0$

$\Rightarrow\ (x^2 + 2)(x - 2) = 0$

$\Rightarrow\ x = 2 \in [0, 3]$

$\therefore\ f(0) = 25$

$f(2) = 48 - 64 + 48 - 96 + 25 = -39$

$f(3) = 243 - 216 + 108 - 144 + 25 = 16$

20. **(4)**

Sol. $f'(x) = \cos x - \sin 2x$

$f'(x) = 0$

$\Rightarrow\ \cos x = 0,\ \sin x = \frac{1}{2}$

$\Rightarrow\ x = \frac{\pi}{2},\ x = \frac{\pi}{6}$

$x = 0,$ $\quad f(0) = \frac{1}{2}$

$x = \frac{\pi}{6},$ $\quad f\left(\frac{\pi}{6}\right) = \frac{3}{4}$

$x = \frac{\pi}{2},$ $\quad f\left(\frac{\pi}{2}\right) = \frac{1}{2}$

Minimum $= \frac{1}{2}$, Maximum $= \frac{3}{4}$

21. **(1)**

Sol. $h = R(\sin\theta + 1)$

$v = \pi\ \frac{1}{3}\ (R\cos\theta)^2\ h = \frac{\pi R^3}{3} \cos^2\theta\ (1 + \sin\theta)$

$\frac{dh}{d\theta} = \frac{\pi R^3}{3}\ (\cos^3\theta - 2\sin\theta\cos\theta\ (1 + \sin\theta)]$

$= \frac{\pi R^3}{3} \cos\theta\ (\cos^2\theta - 2\sin\theta - 2\sin^2\theta)$

$= \frac{\pi R^3 \cos\theta}{3}\ (1 - 2\sin\theta - 3\sin^2\theta)$

$= (1 - 3\sin\theta)(1 + \sin\theta)\ \frac{1}{3}$

$\Rightarrow$ maximum when $\sin\theta = \frac{1}{3}$

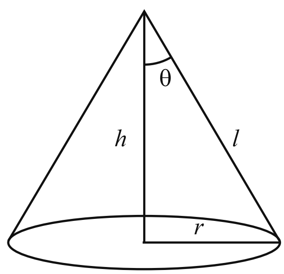
$\Rightarrow\ \frac{h}{2R} = \frac{2}{3}$

22. **(4)**

Sol. $h = l\cos\theta$

$r = l\sin\theta$

$V = \frac{1}{3}\ \pi r^2 h$



$V = \frac{1}{3}\ \pi l^3 \sin^2\theta\cos\theta$

$\frac{dV}{d\theta} = \frac{1}{3}\ \pi l^3\ (2\sin\theta\cos^2\theta - \sin^3\theta)$

$\frac{dV}{d\theta} = \frac{1}{3}\ \pi l^3 \sin\theta\ (2 - 3\sin^2\theta) = 0$ at

$\sin\theta = \sqrt{\frac{2}{3}} \Rightarrow \tan\theta = \sqrt{2}$

23. **(1)**

Sol. $R^2 + r^2 = h^2$

$R^2 = h^2 - r^2$

volume of cylinder ,

$V = \pi R^2 \ (2h) = \pi \ (2h) \ (\sqrt{r^2 - h^2}\ )^2$

$\frac{dV}{dh} = 2\pi\ (r^2 - h^2) + 2\pi h(-2h) = 0$

$\Rightarrow \quad r^2 = 3h^2$

$\Rightarrow \quad h = \frac{r}{\sqrt{3}}$

$\frac{d^2V}{dh^2} < 0 \quad \text{at } h = \frac{r}{\sqrt{3}}$

$\Rightarrow \quad V_{max} = 2\pi \ \frac{r}{\sqrt{3}}\left(r^2 - \frac{r^2}{3}\right) = \frac{4\pi r^3}{3\sqrt{3}}$

24. **(4)**

Sol. $f'(x) = -\pi \sin \pi x + 10 + 6x + 3x^2$

$f'(x) = (3x^2 + 6x + 6) + (4 - \pi \sin \pi x)$

positive positive

$f_{\min}(-2) = \cos 2\pi - 20 + 3 \times 4 - 8$

$= 13 - 28 = -15$

25. **(2)**

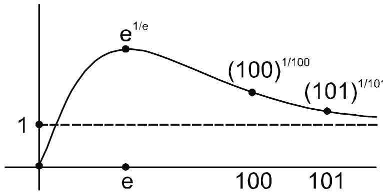
Sol. Assume $f(x) = x^{1/x}$ and let us examine monotonic nature of $f(x)$

$f'(x) = x^{1/x} . \left(\frac{1 - \ell nx}{x^2}\right)$

$f'(x) > 0 \Rightarrow x \in (0, e)$

and $f'(x) < 0 \Rightarrow x \in (e, \infty)$

Hence $f(x)$ is *M.D.* for $x \geq e$



and since 100 < 101

$\Rightarrow \quad f(100) > f(101)$

$\Rightarrow \quad (100)^{1/100} > (101)^{1/101}$

26. **(3)**

Sol. $f(x) = \begin{cases} \frac{1-x}{x^2}, & x < 1, \quad x \neq 0 \\ \frac{x-1}{x^2}, & x \geq 1 \end{cases}$

The given function is not differentiable at $x = 1$

$f'(x) = \begin{cases} \frac{1}{x^2} - \frac{2}{x^3}, & x < 1, \quad x \neq 0 \\ \frac{2}{x^3} - \frac{1}{x^3}, & x > 1 \end{cases}$

Now $f'(x) < 0$

$\Rightarrow \begin{cases} \frac{x-2}{x^3} < 0 & given \quad x < 1 \\ \frac{2-x}{x^3} < 0 & when \quad x > 1 \end{cases}$

$\Rightarrow x < 1$ or $x > 2 \quad \Rightarrow x \in (0, 1) \cup (2, \infty)$

27. **(4)**

Sol. $\frac{df(x)}{dx} = \left(\frac{1}{\sqrt{1-x^2}} + \frac{1}{1+x^2}\right)\frac{1}{\pi} + \frac{2}{2\sqrt{x}}$

Domain: $0 \leq x \leq 1$,

at $x = 0$ $f(x) = 0$,

at $x = 1$ $f(x) = (\sin^{-1} 1 + \tan^{-1} 1) / \pi + 2\sqrt{1}$

$= \frac{\frac{\pi}{2} + \frac{\pi}{4}}{\pi} + 2 = \frac{11}{4}$

$f(x) \in \left[0, \frac{11}{4}\right]$

28. **(4)**

Sol. If range of $f(x)$ is not R and c does not belong to range of $f(x)$ then it is not necessary to have one solution.

29. **(4)**

Sol. Given that f is a real valued function s.t

$f(x) f'(x) < 0 \ \forall \ x \in R$

Now, $\frac{d}{dx} |f(x)| = \frac{f(x)}{|f(x)|} f'(x)$

since $f(x) f'(x) < 0$

$\Rightarrow \frac{d}{dx} |f(x)| < 0$

$\Rightarrow |f(x)|$ is a decreasing function

30. **(3)**

Sol. $g(x)$ is monotonically increasing

$\Rightarrow g'(x) \geq 0$ & $f(x)$ is *M.D.* $\Rightarrow f'(x) \leq 0$ $\frac{d}{dx}$ (fog)

$(x) = \frac{d}{dx}\left((f(g(x)) \right) = f'((g\ \ (x))\ \ g'(x) \leq 0$

as $f'(x) \leq 0$ & $g'(x) \geq 0 \Rightarrow (fog)\ (x)$ is monotonically decreasing

Also $x + 1 > x - 1 \Rightarrow f(x+1) < f(x-1)$ as $f(x)$ is *M.D.*

$\Rightarrow \quad g(f(x+1) < g(f(x-1))$ as $g(x)$ is *M.I.*

31. **(3)**

Sol. $x > 1 \Rightarrow f(x) \geq f(1)$

$x > 1 \Rightarrow g(x) \leq g(1)$

$\Rightarrow \quad f(g(x)) \leq f(g(1))$

$\Rightarrow \quad h(x) \leq 1$ .... (i)

Range of $h(x)$ is subset of $[1, 10]$

$\Rightarrow \quad h(x) \geq 1$ .... (ii)

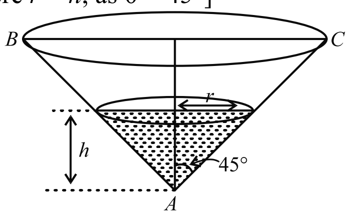
By (i), (ii) we have $h(x) = 1 \Rightarrow h(2) = 1$

32. **(1)**

Sol. We have $\frac{dV}{dt} = 2$

$\Rightarrow \quad \frac{d}{dt}\left(\frac{1}{3}\pi r^3\right) = 2$

[Here $r = h$, as $\theta = 45°$]



$\Rightarrow \quad \pi r^2 \frac{dr}{dt} = 2$

$\Rightarrow \quad \frac{dr}{dt} = \frac{2}{\pi r^2}$ ...... (i)

Now, perimeter $= 2\pi r = p$ (let)

$\Rightarrow \quad \frac{d}{dt}(2\pi r) = 2\pi\left(\frac{2}{\pi r^2}\right) = \frac{4}{r^2}$ ...... (ii)

(Using equation (i))

When $h = 2$ meters $\quad \Rightarrow r = 2$ meters

Hence $\frac{dp}{dt} = \frac{4}{4} = 1$ m/sec.

33. **(4)**

Sol. $f(x) = \frac{x^2 - 1}{x^2 + 1} = 1 - \frac{2}{x^2 + 1}$

$f'(x) = \frac{4x}{(x^2+1)^2}$

$-$ 0 $+$

$\Rightarrow f(x)$ is minimum at $x = 0$

$\Rightarrow f(x), x = 0$

$\Rightarrow$ min $(f(x)) = -1$

34. **(1)**

Sol. $f'(x) = \begin{cases} a \ ; \ x < 0\ , \\ 2x \ ; \ x > 0. \end{cases}$

$f'(x) > 0 \Rightarrow a > 0$

35. **(2)**

Sol. Here, $\frac{d^2y}{dx^2} = \frac{10}{9x^{1/3}}$

From the given points we find that (0, 0) is the point of the curve where $\frac{d^2y}{dx^2}$ does not exist but sign of $\frac{d^2y}{dx^2}$ changes about this point.

$\therefore$ (0, 0) is the required point.

36. **(1)**

Sol. $f'(x) = \begin{cases} \frac{x}{\sqrt{1-x^2}} & , \quad 0 < x < 1 \\ -1 & , \quad x > 1 \end{cases}$

Maximum of $f(x)$ exist at $x = 1$

37. **(2)**

Sol. $f'(x) = \frac{a}{x} + 2bx + 1$

$f'(-1) = 0$

$-a - 2b + 1 = 0$

$a + 2b = 1$

$f'(2) = 0$

$\frac{a}{2} + 4b + 1 = 0$

$\Rightarrow a + 8b + 2 = 0$

$-6b = 3 \Rightarrow b = \frac{-1}{2}$, $a = 2$

38. **(3)**

Sol. $f'(x) = 3x^2 - 3p2x + 3p^2 - 3$

$= 3((x-p)^2 - 1)$

$= 3(x-(p+1))\,(x-(p-1))$

$\Rightarrow p - 1 > -2$ and $p + 1 < 4$

$\Rightarrow p > -1$ and $p < 3$

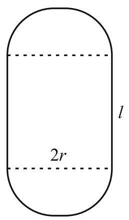
$\Rightarrow -1 < p < 3$

39. **(3)**

Sol. $2l + 2\pi r = 440$

$A = l\,2r = -2\pi r^2 + 440r$

$\frac{dA}{dr} = -4\pi r + 440 = 0$

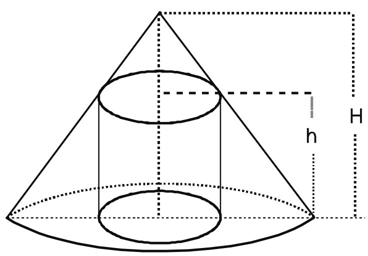


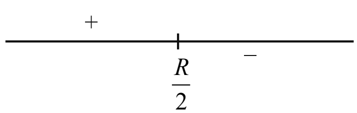
40. **(4)**

Sol. $\frac{H}{R} = \frac{H-h}{r}$

$S = 2\pi rh = 2\pi H\left(r - \frac{r^2}{R}\right)$

$\frac{dS}{dr} = 2\pi H\left(1 - \frac{2r}{R}\right)$

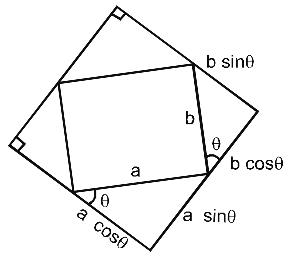




Maximum at $r = \frac{R}{2}$

41. **(2)**

Sol.



Area = $ab +$

$\left(\frac{1}{2}a^2 \sin\theta \ \cos\theta + \frac{1}{2}b^2 \sin\theta\cos\theta\right)$. 2

$= ab + \frac{(a^2+b^2)}{2} \ \sin 2\theta$

Maximum area is $ab + \frac{(a^2+b^2)}{2}$

42. **(4)**

Sol. $f'(x) = \frac{x^{1/x}}{x^2}\ (1 - lnx)$

$f'(x) \le 0$, when $x \ge e$

$\therefore$ $f(x)$ is decreasing function, when $x \ge e$

$\pi > e$

$\Rightarrow f(\pi) < f(e)$

$\pi^{1/\pi} < e^{1/e} \Rightarrow e^{\pi} > \pi^{e}$

$\therefore$ Statement-1 is True, Statement-2 is False

43. **(2)**

Sol. Area of ΔOPQ is minimum when (8,2) is midpoint of line. So, $P(16, 0)$, $Q(0,4)$

44. **(2)**

Sol. $f'(x) = 50x^{49} - 20x^{19}$

$x = 0$ is stationary point. Statement-2 is ture.

$f(0) = 0$

$f\left(\left(\frac{2}{5}\right)^{1/30}\right) = \left(\frac{2}{5}\right)^{5/3} - \left(\frac{2}{5}\right)^{2/3} < 0$

$f(1) = 0$

$\therefore$ Global maximum is 0. Statement-1 is true.

45. **(1)**

Sol. Let $g(x)$ be the inverse function of $f(x)$. Then $f(g(x)) = x$.

$\therefore f'(g(x)).\, g'(x) = 1 \quad i.e. \quad g'(x) = \dfrac{1}{f'(g(x))}$

$\therefore g''(x) = \dfrac{-1}{(f'(g(x))^2}.f''(g(x)).g'(x)$

In tatement-1 $f'(g(x)) > 0$ and $g'(x) > 0$

$\therefore g''(x) < 0$ and so the concavity of $f^{-1}(x)$ is downwards

$\therefore$ statement-1 is false

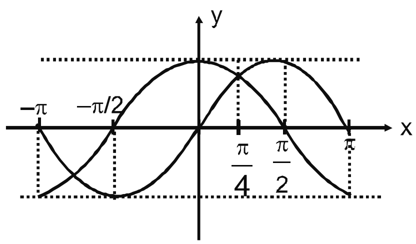
In statement-2 $f'(g(x)) > 0$ and $g'(x) < 0$

$\therefore g''(x) > 0$ and so the concavity of $f^{-1}(x)$ is upwards

$\therefore$ statement-2 is true

46. **(2)**

Sol.

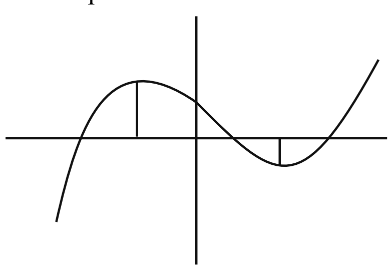


Figure

$f'(x) = \dfrac{1}{1+(\sin x + \cos x)^2} (\cos x - \sin x)$

47. **(1)**

Sol. Graph of $y = x^3 - px + q$ cuts x-axis at three distinct pints



$\dfrac{dy}{dx} = 0 \Rightarrow x = \pm\sqrt{\dfrac{p}{3}}$

Maxima at $x = -\sqrt{\dfrac{p}{3}}$, minima at $x = \sqrt{\dfrac{p}{3}}$

48. **(2)**

Sol. $P'(x) = 4x^3 + 3ax^2 + 2bx + c$ and $P'(0) = 0$

$\Rightarrow \quad c = 0.$

$P'(x) = x(4x^2 + 3ax + 2b)$

$D = 9a^2 - 32b < 0 \;\Rightarrow\; b > \dfrac{9a^2}{32} > 0$

($\because$ $P'(x) = 0$ has only one root $x = 0$)

$P(-1) < P(1)$

$\Rightarrow a > 0$

$P'(x)$ has only one change of sign. $\Rightarrow x = 0$ is a point of minima.

$P(-1) = 1 - a + b + d, \quad P(0) = d$

$P(1) = 1 + a + b + d$

$\Rightarrow P(-1) < P(1), P(0) < P(1), P(-1) > P(0)$

$\Rightarrow P(-1)$ is not minimum but $P(1)$ is maximum.

49. **(3)**

Sol. $\lim\limits_{x\to -1^+} f(x) = 1$

$f(-1) = k + 2$

$\lim\limits_{x\to(-1)^-} f(x) = k + 2$

$\because f$ has a local minimum at $x = -1$

$x = -1$

$\therefore f(-1^+) \geq f(-1) \leq f(-1^-)$

$1 \geq k + 2 \leq k + 2$

$\Rightarrow k \leq -1$

possible value of k is -1

50. **(4)**

Sol. Here; $f'(x) = 3x^2 + 6(a - 7)x + 3(a^2 - 9) = 0$

$\Rightarrow x = 7 - a \pm\sqrt{58-14a}$, $x_1 = 7 - a + \sqrt{58-14a}$, $x_2 = 7 - a - \sqrt{58-14a}$

$58 - 14a > 0 \Rightarrow 14a < 58 \Rightarrow a < \frac{29}{7}$

$f''(x) = 6x + 6(a - 7), f''(x_2) < 0 \Rightarrow$ at $x = a_2$,

$f(x)$ has a maxima

$\therefore x_2 > 0$

$\Rightarrow 7 - a - \sqrt{58-14a} > 0, \sqrt{58-14a} < 7 - a$

$\Rightarrow 58 - 14a < (7 - a)^2 \Rightarrow a^2 - 9 > 0$

$\Rightarrow a \in (-\infty,-3) \cup (3,\infty)$ & $a < \frac{29}{7}$

$\therefore a \in (-\infty, -3) \cup \left(3, \frac{29}{7}\right)$

51. (2)

Sol. $f(x) = \begin{cases} \frac{\tan x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

In right neighbourhood of '0'

$\tan x > x$

$\frac{\tan x}{x} > 1$

$\Rightarrow x = 0$ is point of minima

52. (3)

Sol. $V = \frac{4}{3}\pi r^3 \Rightarrow 4500\pi = \frac{4\pi r^3}{3}$

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right) \Rightarrow 45 \times 25 \times 3 = r^3$

$r = 15\ m$

after 49 min

$= (4500 - 49.72)\pi$

$= 972\ \pi\ m^3$

$972\ \pi = \frac{4}{3}\pi r^3$

$r^3 = 3 \times 243$

$= 3 \times 3^5$

$r = 9$

$72\ \pi = 4\pi \times 9 \times 9 \left(\frac{dr}{dt}\right)$

$\Rightarrow \frac{dr}{dt} = \left(\frac{2}{9}\right)$

53. (4)

Sol. Since coefficient of x^2 is (+ ve)

$\Rightarrow m(b) = -\frac{D}{4a}$

$\Rightarrow m(b) = -\frac{(4b^2 - 4(1+b^2))}{4(1+b^2)}$

$\Rightarrow m(b) = \frac{1}{1+b^2}$

$\Rightarrow b^2 \geq 0$

$\Rightarrow 1 + b^2 \geq 1$

$\Rightarrow 0 < \frac{1}{1+b^2} \leq 1$

$\Rightarrow m(b) \in (0, 1]$

54. (4)

Sol. $f(x) = 2x^3 + 3x + k$

$f'(x) = 6x^2 + 3 > 0 \quad \forall x \in R$

$\Rightarrow f(x)$ is strictly increasing function

$\Rightarrow f(x) = 0$ has only one real root, so two roots are not possible

55. (1)

Sol. $f(x) = \alpha \ell n|x| + \beta x^2 + x$

$f'(x) = \frac{\alpha}{x} + 2\beta x + 1$.

$= \frac{2\beta x^2 + x + \alpha}{x}$

Since $x = -1, 2$ are extreme points

$\Rightarrow f'(x) = 0$ at these points.

56. (2)

Sol. $4x + 2\pi r = 2$ ....(i)

$x^2 + \pi r^2$

$\Rightarrow$ So $f(r) = \left(\frac{1-\pi r}{2}\right)^2 + \pi r^2$

$\frac{df}{dr} = \pi^2 \frac{r}{2} - \frac{\pi}{2} + 2\pi r = 0 \Rightarrow r = \frac{1}{\pi + 4}$

using equation (i) $x = \frac{(1-\pi r)}{2}$

57. (2)

Sol. $f(x) = x^2 + \frac{1}{x^2}, g(x) = x - \frac{1}{x}$

$$h(x) = \frac{f(x)}{g(x)} = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{\left(x - \frac{1}{x}\right)}$$

$$x - \frac{1}{x} = t$$

$$h(t) = \frac{t^2 + 2}{t} = t + \frac{2}{t} \quad |t| \geq 2$$

$$AM \geq GM. \quad \frac{t + \frac{2}{t}}{2} \geq \sqrt{t.\frac{2}{t}}$$

$$t + \frac{2}{t} \geq 2\sqrt{2}$$

58. **(1)**

Sol. Let $f(x) = 3\sin x - 4\sin^3 x = \sin 3x$

The longest interval in which sin x is increasing, is of length $\frac{\pi}{3}$

59. **(3)**

Sol. $\lim\limits_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$. Put $x = 0$ and we get $\frac{0}{0}$ form. Also because 'f' is strictly increasing and differentiable. Apply L-Hospital rule, weget

$\lim\limits_{x \to 0} \frac{2xf'(x^2) - f'(x)}{f'(x)} = -1$. Since '$f$' is strictly increasing

60. **(1)**

Sol. $f'(x) = 3x^2 + 2bx + c$ whose discriminant is $4(b^2 - 3c)$ which is negative as $0 < b^2 < c$.

Thus $f(x)$ is always positive and $f(x)$ is strictly increasing

61. **(3)**

Sol. $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$

$g(u) = \tan^{-1}(e^u) - \cot^{-1}(e^u)$

$g(-u) = \cot^{-1}(e^u) - \tan^{-1}(e^u)$ odd function.

$g(-u) = -g(u)$

$$g'(u) = \frac{e^u}{1 + e^{2u}} + \frac{e^{-u}}{1 + e^{-2u}} > 0$$

Strictly increasing function

62. **(1)**

Sol. Here $f(g(\alpha^2 - 2\alpha)) > f(g(3\alpha - 4))$

$\Rightarrow \quad g(\alpha^2 - 2\alpha) > g(3\alpha - 4) \quad \therefore f$ is increasing

$\Rightarrow \quad \alpha^2 - 2\alpha < 3\alpha - 4$

$\therefore g$ is decreasing

$\Rightarrow \quad \alpha^2 - 5\alpha + 4 < 0$

$\Rightarrow \quad \alpha \in (1, 4)$

63. **(4)**

Sol. $g(x) = f(\tan^2 x - 2\tan x + 4)$

$= f(t^2 - 2t + 4),\ t \in (0, \infty)$

$g'(x) = f'(t^2 - 2t + 4)(2t - 2) > 0\ \forall\ t \in (0, \infty)$ as $f''(x) > 0\ \&\ f'(3) = 0$

$\Rightarrow t > 1$

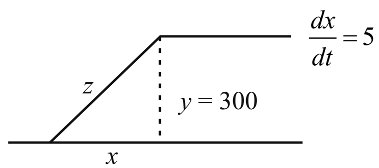
$\Rightarrow \tan x > 1 \Rightarrow x > \frac{\pi}{4}$

$\Rightarrow x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Integer Type Questions (64 to 73)

64. **(4)**

Sol.



From figure $z^2 = x^2 + y^2$

$$\frac{zdz}{dt} = \frac{xdx}{dt}$$

If $z = 500$ then $x = 400$

$$\Rightarrow 500\ \frac{dz}{dt} = 400(5)$$

$$\Rightarrow \frac{dz}{dt} = 4$$

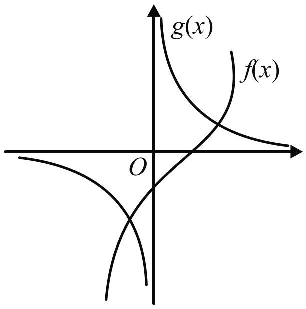
65. **(2)**

Sol. $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$

$\Rightarrow \left(e^{3x}-1\right)^2 - e^x\left(e^{3x}-1\right) = 12e^{2x}$

$\left(e^{3x}-1\right)\left(e^x - e^{-x} - e^{-2x}\right) = 12$

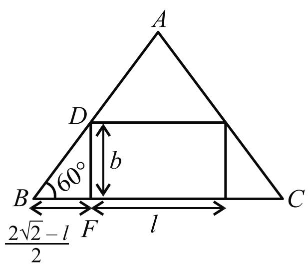
$\Rightarrow \underbrace{e^x - e^{-x} - e^{-2x}}_{\text{increasing (let } f(x))} = \dfrac{12}{\underbrace{e^{3x}-1}_{\text{decreasing (let } g(x))}}$



$\Rightarrow$ Number of real roots = 2.

66. (3)

Sol.



In ΔDBF, $\tan 60° = \dfrac{2b}{2\sqrt{2}-\ell}$

$\Rightarrow b = \dfrac{\sqrt{3}\left(2\sqrt{2}-\ell\right)}{2}$

A = Area of rectangle $= \ell \times b$

$A = \ell\dfrac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right)$

$\dfrac{dA}{d\ell} = \dfrac{\sqrt{3}}{2}\left(2\sqrt{2}-2\ell\right)$

Now, $\dfrac{dA}{d\ell} = 0$

$\Rightarrow \ell = \sqrt{2}$

$A = \ell \times b = \sqrt{2} \times \dfrac{\sqrt{3}}{2}\left(\sqrt{2}\right) = \sqrt{3}$

67. (2)

Sol. $f(x) = 3^{x+1} + 3^{-(x+1)}$

$f(x) = \dfrac{3^{x+1}+3^{-(x+1)}}{2} \geq \sqrt{3^{x+1}.\ 3^{-(x+1)}}$

$\Rightarrow \quad 3^{x+1} + 3^{-(x+1)} \geq 2$

68. (1)

Sol. Let $f(x) = \sin x \tan x - x^2$

$f'(x) = \cos x. \tan x + \sin x. \sec^2 x - 2x$

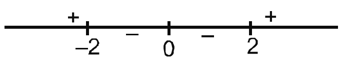
$\Rightarrow f'(x) = \sin x + \sin x \sec^2 x - 2x$

$\Rightarrow f''(x) = \cos x + \cos x \sec^2 x + 2\sec^2 x \sin x \tan x - 2$

$\Rightarrow f''(x) = (\cos x + \sec x - 2) + 2\sec^2 x \sin x \tan x$

69. (2)

Sol. $f'(x) = \dfrac{(x+2)\ (x-2)}{2x^2}$



$f'(x)$ changes sign as x crosses 2.

$f(x)$ has minima at $x = 2$.

70. (25)

Sol. $2r + l = 20 \Rightarrow 2r + r\theta = 20 \Rightarrow \theta = \dfrac{20-2r}{r}$

$A = \dfrac{\pi r^2\theta}{360} = \dfrac{r^2}{2} . \dfrac{20-2r}{r} = r(10-r)$

$A = 10r - r^2$

$\dfrac{dA}{dr} = 10 - 2r = 0 \Rightarrow r = 5$

$\therefore \theta = \dfrac{10}{5} = 2$

$\therefore$ Maximum area $= \dfrac{1}{2} \times 25 \times 2 = 25$ *sq. m.*

71. (2)

Sol. $f(x) = \begin{cases} (2+x)^2 & , \quad -3 < x \leq -1 \\ x^{2/3} & , \quad -1 < x < 2 \end{cases}$

one point of local maxima at $x = -1$

one point of local minima at $x = 0$

72. **(2)**

Sol. $x^2 = x\sin x + \cos x$

$f(x) = x^2$

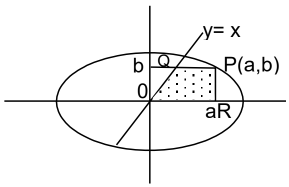
$g(x) = x\sin x + \cos x$

$g'(x) = \sin x + x\cos x - \sin x$

$g'(x) = x\cos x$

73. **(4)**

Sol.



$P(a, b) = p\,(\sqrt{3}\cos\theta, 2\sin\theta)$

Area trap $= \frac{1}{2}\ (PQ + OR)PR$

$= \frac{1}{2}\ ((\sqrt{3}\cos\theta - 2\sin\theta) + \sqrt{3}\cos\theta)\ 2\sin\theta$

$= 2\sqrt{3}\sin\theta\cos\theta - 2\sin^2\theta$

$A = \sqrt{3}\sin\theta\cos\theta - (1 - \cos 2\theta)$

$= \sqrt{3}\ \sin 2\theta + \cos 2\theta - 1$

$= 2\left(\frac{\sqrt{3}}{2}\sin 2\theta + \frac{1}{2}\cos 2\theta\right) - 1$

$= 2\cos\left(2\theta - \frac{\pi}{3}\right) - 1$

A will be maximum when $2\theta - \frac{\pi}{3} = 0$

$\Rightarrow \theta = \frac{\pi}{6}$

$\therefore\ P\left(\frac{3}{2}, 1\right)\ \therefore\ a = \frac{3}{2},\ b = 1$

$\therefore\ 2a + b = 4$

INDEFINITE INTEGRATION

Single Option Correct Type Questions (01 to 66)

1. **(2)**

Sol.

$$I = \int \frac{\ell n\left(\frac{x-1}{x+1}\right)}{x^2-1}\,dx \quad \text{put} \quad ln\left(\frac{x-1}{x+1}\right) = t$$

$$\Rightarrow \frac{2}{x^2-1}\,dx = dt$$

$$\Rightarrow I = \int t\frac{dt}{2} = \frac{t^2}{4} + C = \frac{1}{4}\log^2\left(\frac{x-1}{x+1}\right) + C$$

2. **(1)**

Sol. $I = \int \frac{\ell n\,(\tan x)}{\sin x\ \cos x}\,dx \quad \text{put } ln \tan x = t$

$$\Rightarrow \frac{1}{\sin x \cos x}dx = dt$$

$$I = \int t\ \ dt = \frac{t^2}{2} + C = \frac{1}{2}(\ell n \tan x)^2 + C$$

$$= \frac{1}{2}(\ell n \cot x)^2 + C = \frac{1}{2}(\ell n^2 \cot x) + C$$

$$= \frac{1}{2}\ell n^2(\sin x \sec x) + C$$

$$= \frac{1}{2}\ ln^2\ (\cos x\ \text{cosec}\ x) + C$$

3. **(2)**

Sol. $f(x) = \int \frac{2\sin x - \sin 2x}{x^3}\,dx$

$$f'(x) = \frac{2\sin x - \sin 2x}{x^3} = \frac{2\sin x}{x} . \frac{1-\cos x}{x^2}$$

$$= 2\left(\frac{\sin x}{x}\right) . \frac{2\sin^2\frac{x}{2}}{\frac{x^2}{4}\times 4}$$

$$= \frac{2\times 2}{4}\left(\frac{\sin x}{x}\right).\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2$$

$$\lim_{x\to 0} f'(x) = \lim_{x\to 0}\left(\frac{\sin x}{x}\right). \lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = 1$$

4. **(3)**

Sol. $I = \int x^3(x^6 + x^3 + 1)(2x^6 + 3x^3 + 6)^{1/3}\ dx$

$$= \int x^2(x^6 + x^3 + 1)(2x^9 + 3x^6 + 6x^3)^{1/3}\ dx$$

Let $2x^9 + 3x^6 + 6x^3 = t$

$18(x^8 + x^5 + x^2)\ dx = dt$

$I = \frac{1}{18}\int t^{1/3}dt = \frac{1}{18}.\frac{t^{4/3}}{4/3} + c = \frac{1}{24}\ (2x^9 + 3x^6 + 6x^3)^{4/3} + c$

5. **(3)**

Sol. $I = \int 2^{mx}.\ 3^{nx}\,dx$

$$= \int\left(2^m.\ 3^n\right)^x\ dx = \frac{2^{mx}.\ 3^{nx}}{\ell n\,(2^m.\ 3^n)} + c$$

6. **(1)**

Sol. $\int \frac{dx}{\sin x.\sin(x+\alpha)}$

$= \frac{1}{\sin\alpha}\int \frac{\sin(\alpha + x - x)}{\sin x\ \sin(x+\alpha)}dx$

$= \text{cosec}\ \alpha \int \frac{\sin(x+\alpha)\cos x - \cos(x+\alpha)\sin x}{\sin\ x\sin(x+\alpha)}$

$= \text{cosec}\ \alpha \left[\int \cot x\,dx - \int \cot(x+\alpha)\right] + C$

$= \text{cosec}\ \alpha\ [\log|\sin x| - \log|\sin(x+\alpha)|] + C$

$= \text{cosec}\ \alpha\ \log\left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$

7. **(2)**

Sol. $I = -\frac{1}{2}\cos 2x - \frac{\sin 2x}{2} + b =$

$-\frac{1}{\sqrt{2}}\sin\left(2x+\frac{\pi}{4}\right) + b = \frac{1}{\sqrt{2}}\sin\left(2x+\frac{5\pi}{4}\right) + b$

$a = -\frac{5\pi}{4},\ b \in R$

8. **(3)**

Sol. $\int (1+\tan x\ \tan(x+\alpha))\,dx$

$= \int \frac{\sin x\sin(x+\alpha)+\cos x\cos(x+\alpha)}{\cos x\cos(x+\alpha)}$

$= \int \frac{\cos(x+\alpha-x)}{\cos x\cos(x+\alpha)}dx$

$= \cot\alpha \int \frac{\sin(x+\alpha-x)}{\cos x\cos(x+\alpha)}dx$

$= \cot\alpha$

$\left[\int \frac{\sin(x+\alpha)\cos x}{\cos x\cos(x+\alpha)} - \frac{\cos(x+\alpha)\sin x}{\cos x\cos(x+\alpha)}dx\right]$

$= \cot\alpha\left[\int \tan(x+\alpha)\,dx - \int \tan x\,dx\right] = \cot\alpha$

$[\ell n|\sec(x+\alpha)| - \ell n|\sec x|]$

$= \cot\alpha\ ln\left|\frac{\cos x}{\cos(x+\alpha)}\right| + C = \cot\alpha\ ln$

$\left(\left|\frac{\sec(x+\alpha)}{\sec x}\right|\right) + C$

9. **(2)**

Sol. $\int 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}\,dx$

$= \int 2\sin x(\cos 2x + \cos x)dx$

$= \int 2\sin x\cos 2x\,dx + \int \sin 2x + c$

$= \int (\sin 3x - \sin x + \sin 2x) + c$

$= \cos x - \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

10. **(3)**

Sol. $\int \frac{x^2+\cos^2 x}{1+x^2}\text{cosec}^2 x\,dx$

$= \int \cos ec^2 x dx - \int \frac{1}{1+x^2}dx$

$= -\cot x - \tan^{-1}x + c = -\tan^{-1}x - \frac{\cos\text{ec x}}{\sec\text{ x}}$

$+ c$

11. **(3)**

Sol. $I = \int \sqrt{\frac{x-1}{x+1}} \times \frac{1}{x^2}\ dx$ Put $\frac{1}{x} = \cos 2\theta$

$\Rightarrow -\frac{dx}{x^2} = -2\sin 2\theta\ d\theta$

$I = \int \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\ 2\sin 2\theta\ d\theta$

$= \int 4\sin^2\theta\ d\theta = 2\int(1-\cos 2\theta)d\theta$

$= 2\theta - \sin 2\theta + C = \cos^{-1}\left(\frac{1}{x}\right) - \sqrt{1-\frac{1}{x^2}} + C$

12. **(1)**

Sol. $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}.\ dx = Ax + B\ ln|9e^{2x}-4| + C$

put $4e^x + 6se^{-x} = P(9e^x - 4e^{-x}) + Q(9e^x + 4e^{-x})$

$\Rightarrow 4 = 9P + 9Q$ and $6 = 4Q - 4P$

comparing, $P = -\frac{19}{36}, Q = \frac{35}{36}$

$I = -\frac{19}{36}\int dx + \frac{35}{36}\int \frac{9e^x + 4e^{-x}}{9e^x - 4e^{-x}}\, dx = -\frac{19}{36}$

$.x + \frac{35}{36}\, ln\left|\left(9e^x - 4e^{-x}\right)\right| + C$

$= -\frac{19}{36}x + \frac{35}{36}\, ln\left|\left(9e^{2x} - 4\right)\right| - \frac{35}{36}x + C =$

$\frac{35}{36}\, ln\left|\left(9e^{2x} - 4\right)\right| - \frac{54}{36}x + C$

$= \frac{35}{36}\, ln\left|\left(9e^{2x} - 4\right)\right| - \frac{3}{2}x + C$

So , $A = -\frac{3}{2}$, $B = \frac{35}{36}$, $C \in R$

13. **(2)**

Sol. $\int \frac{dx}{\sqrt{\sin^3 x(\sin x\cos\alpha - \cos x\sin\alpha)}}$

$= \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha - \sin\alpha\cot x}}\, dx$

$= \frac{2}{\sin\alpha}\sqrt{\cos\alpha - \sin\alpha\cot x} + C$

14. **(2)**

Sol. $\int \frac{\cos 4x + 1}{\cot x - \tan x}\, dx = A\cos 4x + B$

$= \int \frac{2\cos^2 2x\sin x\cos x}{\cos 2x}\, dx$

$= \int 2\cos 2x\sin x\cos x\, dx$

$= \frac{1}{2}\int \sin 4x\, dx = \frac{1}{2}\frac{(-\cos 4x)}{4} + B$

15. **(4)**

Sol. $\int \tan^4 x\, dx = \int \tan^2 x\ (\sec^2 x - 1)dx = \int \tan^2 x$

$d(\tan x) - \int (\sec^2 x - 1)\, d$

$x = \frac{1}{3}\tan^3 x - \tan x + x + c$

16. **(3)**

Sol. $I = \int \frac{dx}{2\sin^2 x + 2\sin x\cos x}$

$= -\frac{1}{2}\int \frac{-\operatorname{cosec}^2 x.dx}{(1 + \cot x)}$

Let $1 + \cot x = t \therefore -\operatorname{cosec}^2 x\, dx = dt$

$= -\frac{1}{2}\int \frac{(dt)}{t} = -\frac{1}{2}\, ln\, |t| + c$

$= -\frac{1}{2}\, ln\, |(1 + \cot x)| + c$

17. **(2)**

Sol. $\int \frac{\sin^2 x}{\cos^6 x}dx = \int \tan^2 x(1 + \tan^2 x)\ \sec^2 x\, dx$

$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c$

18. **(3)**

Sol. STATEMENT-1: Put $\sin x = t$

$\Rightarrow \cos x\, dx = dt$

Now $\int t^5\, dt = \frac{t^6}{6} + C = \frac{\sin^6 x}{6} + C$

STATEMENT-2: is false for $n = -1$

19. **(1)**

Sol. $I = \int ((\log_x e) - (\log_x e)^2)\, dx$

$= \int \left(\frac{1}{\ell nx} - \frac{1}{(\ell nx)^2}\right) dx$ put $ln\, x = t$

$\Rightarrow x = e^t \Rightarrow dx = e^t\, dt$

$\Rightarrow I = \int \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t\, dt = \frac{e^t}{t} + C = \frac{x}{\ell nx} + C = x$

$\log_x e + C$

20. **(1)**

Sol. $I = \int \frac{x^{n-1}}{x^n(x^n + 1)}\, dx$

Let $x^n = t$ $n.\ x^{n-1}\, dx = dt$

$\Rightarrow I = \frac{1}{n}\int \frac{1}{t(t+1)}dt = \frac{1}{n}\int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$= \frac{1}{n}[\ell n\,|t| - \ell n\,|t+1|] + C = \frac{1}{n}\ell n\left|\frac{x^n}{x^n + 1}\right| + C$

21. **(2)**

Sol. Put $x - \alpha = t \Rightarrow dx = dt$

$\Rightarrow \int \frac{\sin x}{\sin(x + \alpha)}\, dx = \int \frac{\sin(\alpha + t)}{\sin t}\, dt = \sin\alpha$

$\int \cot t\ dt + \cos\alpha \int dt$

= sin α. *ln* |sin *t*| + *t* cos α + *C*

= sin α. *ln* sin (*x* – α)| + (*x* – α) cos α + *C*

⇒ *A* = cos α & *B* = sin α.

22. **(4)**

Sol. $\int \frac{dx}{\cos x - \sin x} = \frac{1}{\sqrt{2}} \int \frac{dx}{\cos\left(x + \frac{\pi}{4}\right)}$

$= \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx$

$= \frac{1}{\sqrt{2}}\ ln \left| \tan\left(\frac{x}{2} + \frac{3\pi}{8}\right) \right| + C$

23. **(1)**

Sol. $\int \frac{dx}{\cos x + \sqrt{3}\sin x} = \frac{1}{2} \int \frac{dx}{\cos\left(x - \frac{\pi}{3}\right)}$

$= \frac{1}{2} \int \sec\left(x - \frac{\pi}{3}\right) dx = \frac{1}{2}\ ln \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

24. **(4)**

Sol. $\int \frac{5\tan x}{\tan x - 2} dx$

$= \int \frac{5\sin x}{\sin x - 2\cos x} dx$

$= \int \frac{(\sin x - 2\cos x) + 2(\cos x + 2\sin x)}{(\sin x - 2\cos x)} dx$

$= \int dx + 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x} dx = x + 2\ ln\ |(\sin x - 2\cos x)| + k \Rightarrow a = 2$

25. **(3)**

Sol. $\int f(x)dx = \psi(x)$

$I = \int x^5 f(x^3) dx$

put $x^3 = t \Rightarrow x^2 dx = \frac{dt}{3}$

$= \frac{1}{3} \int t f(t) dt$

$= \frac{1}{3}\left[t\psi(t) - \int \psi(t) dt\right]$

$= \frac{1}{3}\left[x^3\psi(x^3) - 3\int x^2\psi(x^3) dx\right] + c =$

$\frac{1}{3} x^3\psi(x^3) - \int x^2\psi(x^3) dx + c$

26. **(4)**

Sol. $\int \left(1 + x - \frac{1}{x}\right) e^{\left(x + \frac{1}{x}\right)} dx$

$\int e^{x+\frac{1}{x}} dx + \int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$

Applying integration by parts in (I)

We get,

$\int e^{x+\frac{1}{x}} dx = x.\ e^{x+\frac{1}{x}} - \int x\left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx$

$= x\ e^{x+\frac{1}{x}} - \int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$

Thus $\int e^{x+\frac{1}{x}} dx + \int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx = x\ e^{x+\frac{1}{x}} + C.$

27. **(2)**

Sol. $I_n = \int \tan^n x dx = \int \tan^{n-2}(\sec^2 x - 1) dx$

$\int (\tan x)^{n-2} \sec^2 x dx -$

$\int (\tan x)^{n-2} dx = \frac{(\tan x)^{n-1}}{n-1} - I_{n-2}$

$I_n + I_{n-2} = \frac{(\tan x)^{n-1}}{n-1}$

put $n = 6$

$I_4 + I_6 = \frac{1}{5}\tan^5 x = a\tan^5 x + bx^5 + c$

$\Rightarrow a = \frac{1}{5}\ b = 0\ c = 0$

$\therefore (a, b) = \left(\frac{1}{5}, 0\right)$

28. **(3)**

Sol. $\because f(x) = \int e^x (x-1)(x-2)\,dx$

$\Rightarrow f'(x) = e^x(x-1)(x-2) \le 0 \;\Rightarrow\; 1 \le x \le 2$

$\Rightarrow x \in [1, 2]$

29. **(4)**

Sol. $I = \int \dfrac{\left(\dfrac{1}{x^3} - \dfrac{1}{x^5}\right)}{\sqrt{2 - \dfrac{2}{x^2} + \dfrac{1}{x^4}}}\,dx$ put $2 - \dfrac{2}{x^2} + \dfrac{1}{x^4}$

$= z \Rightarrow \left(\dfrac{4}{x^3} - \dfrac{4}{x^5}\right) dx = dz$

$\Rightarrow I = \dfrac{1}{4}\int \dfrac{dz}{\sqrt{z}}$

$= \dfrac{1}{2}\sqrt{z} + c$

$\Rightarrow \dfrac{1}{2}\sqrt{2 - \dfrac{2}{x^2} + \dfrac{1}{x^4}} + C$

$= \dfrac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

30. **(1)**

Sol. $f(f(x)) = \dfrac{f(x)}{(1 + (f(x))^n)^{1/n}}$

$= \dfrac{x/(1+x^n)^{1/n}}{\left(1 + \dfrac{x^n}{1+x^n}\right)^{1/n}} = \dfrac{x}{(1+2x^n)^{1/n}}$

Also, $f(f(f(x))) = \dfrac{x/(1+2x^n)^{1/n}}{\left(1 + \dfrac{x^n}{1+2x^n}\right)^{1/n}}$

$= \dfrac{x}{\left(1+3x^n\right)^{1/n}} \Rightarrow g(x) = \dfrac{x}{(1+nx^n)^{1/n}}$

Hence $I = \int x^{n-2} \dfrac{x}{(1+nx^n)^{1/n}}\,dx$

$= \int \dfrac{x^{n-1}}{(1+nx^n)^{1/n}}\,dx$

Let $1 + n\,x^n = t \Rightarrow x^{n-1}\,dx = \dfrac{1}{n^2}\,dt$

$\Rightarrow I = \dfrac{1}{n^2}\int \dfrac{dt}{t^{1/n}} = \dfrac{1}{n^2}\dfrac{t^{\frac{-1}{n}+1}}{1 - \frac{1}{n}}$

$= \dfrac{1}{n(n-1)}(1 + nx^n)^{1-1/n} + K$

31. **(4)**

Sol. Statement - 1 $F(x)$

$= \int (\sin^2 x)\,dx = \int \dfrac{1 - \cos 2x}{2}\,dx$

$= \dfrac{x}{2} - \dfrac{\sin 2x}{4} + c$

Which is aperiodic function. Hence statement is false.

Statement – 2 $\sin^2 x$ is periodic with the period π, hence statement is true.

32. **(3)**

Sol. $J - I = \int \dfrac{(e^{3x} - e^x)\,dx}{e^{4x} + e^{2x} + 1}$

Let $e^x = t \Rightarrow e^x\,dx = dt$

$\Rightarrow J - I = \int \dfrac{t^2 - 1}{t^4 + t^2 + 1}\,dt$

$= \int \dfrac{1 - \frac{1}{t^2}}{t^2 + \frac{1}{t^2} + 1}\,dt$

put $t + \dfrac{1}{t} = u \Rightarrow \left(1 - \dfrac{1}{t^2}\right) dt = du$

$\Rightarrow J - I = \int \dfrac{du}{u^2 - 1} = \dfrac{1}{2}\, ln\left|\dfrac{u-1}{u+1}\right| + C = \dfrac{1}{2}$

$ln\left|\dfrac{t^2 - t + 1}{t^2 + t + 1}\right| + C = \dfrac{1}{2}\, ln\left|\dfrac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1}\right| + C$

33. **(3)**

Sol. Put sec x + tan $x = t$

(sec x tan x + sec^2x) $dx = dt$

sec x. $t\,dx = dt$

$\sec x - \tan x = \frac{1}{t}$

$\sec x = \frac{t+\frac{1}{t}}{2}$

$\int \frac{\sec x.dt}{t^{9/2}.t} = \int \frac{1}{2}\frac{\left(t+\frac{1}{t}\right)}{t\ .\ t^{9/2}}dt$

$= \frac{1}{2}\int\left(\frac{1}{t^{9/2}}+\frac{1}{t^{13/2}}\right)\ dt$

$= -\frac{1}{2}\left[\frac{2}{7t^{7/2}}+\frac{2}{11t^{11/2}}\right]+k$

$= -\frac{1}{t^{11/2}}\left[\frac{t^2}{7}+\frac{1}{11}\right]+k$

34. **(1)**

Sol. $I = \int \overset{II}{e^x}.\overset{I}{\ell n(4x+1)}\,dx + \int e^x.\frac{16}{(4x+1)^2}dx$

$= e^x\,\ell n(4x+1) -$

$\int \frac{4}{4x+1}.e^x dx + 16\int \frac{e^x}{(4x+1)^2}dx$

$= e^x\,\ell n(4x+1) -$

$\frac{4e^x}{4x+1} - 16\int\frac{1}{(4x+1)^2}.e^x dx + 16\int\frac{e^x}{(4x+1)^2}dx$

$= e^x\left(\ell n(4x+1) - \frac{4}{4x+1}\right)+C$

35. **(3)**

Sol.

$I = \int\left(e^{\left(x^2-\frac{1}{x}\right)} + x(2x+\frac{1}{x^2}).e^{\left(x^2-\frac{1}{x}\right)}\right)dx$

$\therefore \int (f(x)+xf'(x))\,dx = x\,f(x) + C$

so, $I = xe^{\left(x^2-\frac{1}{x}\right)} + C$

36. **(1)**

Sol. Let, $\sin^2 x = t$

$2\sin x\cos x\,dx = dt$

$\Rightarrow I = \frac{1}{2}\int e^t\left(1+1-t\right)dt$

$= \frac{1}{2}.\ 2\int e^t\,dt - \frac{1}{2}\int t.e^t\,dt$

$= e^t - \frac{1}{2}(te^t - e^t) + C = \frac{3}{2}e^t - \frac{t}{2}e^t$

$= \frac{1}{2}e^{\sin^2 x}(3-\sin^2 x)+C$

37. **(2)**

Sol. $I = \int\frac{1}{(\sqrt{x})^2+(\sqrt{x})^7}dx$

$= \int\frac{1}{(\sqrt{x})^7\left(1+\frac{1}{(\sqrt{x})^5}\right)}dx$

$\Rightarrow -\frac{5}{2}\frac{1}{(\sqrt{x})^7}dx = dt$

$I = -\frac{2}{5}\int\frac{1}{1+t}dt$

$= -\frac{2}{5}\ell n\left(1+\frac{1}{x^{5/2}}\right)+C$

$= \frac{2}{5}\ell n\left(\frac{x^{5/2}}{x^{5/2}+1}\right)+C$

so $\alpha = \frac{2}{5}$ and $\beta = \frac{5}{2}$

38. **(4)**

Sol. Let $\tan^{-1}x = t$

$\frac{1}{1+x^2}dx = dt$

$\Rightarrow I = \int x^3.\frac{1}{1+x^2}dx = \int\frac{x(x^2+1-1)}{1+x^2}dx$

$= \int x dx - \int\frac{x}{1+x^2}dx = \frac{x^2}{2} - \frac{1}{2}\ell n(1+x^2)+C$

39. **(3)**

Sol. $I = -e^{-x}\ell n(e^x+1) - \int \frac{1}{e^x+1}.e^x(-e^{-x})\,dx$

$= -e^{-x}\ \ell n(e^x+1) + \int \frac{e^{-x}}{1+e^{-x}}\,dx$

put $1 + e^{-x} = t$

$= -e^{-x}\,dx = dt$

$\Rightarrow I = -e^{-x}\ell n(e^x+1) \quad -\ell n(1+e^x) \quad + \ell ne^x + C$

$= x - (e^{-x}+1)\ \ell n(e^x+1) + C$

40. **(3)**

Sol. put $\ell n(g(x)) - \ell n(f(x)) = t$

$\Rightarrow \left(\frac{g'(x)}{g(x)} - \frac{f'(x)}{f(x)}\right)dx = dt$

$\Rightarrow I = \int t\,dt = \frac{t^2}{2} + C$

$= \frac{1}{2}\left(\ell n(g(x)) - \ell n(f(x))\right)^2 + C$

41. **(3)**

Sol. $f(x) = \int \frac{(1+x^2)-(1-\sin^2 x)}{1+x^2}.\sec^2 x\,dx$

$= \int \sec^2 x\,dx - \int \frac{1}{1+x^2}\,dx$

$f(x) = \tan x - \tan^{-1}x + C$

$\because f(0) = 0 \Rightarrow c = 0$

so $f(1) = \tan 1 - \frac{\pi}{4}$

42. **(4)**

Sol. put $\cot^{-1}x = t$

$-\frac{1}{1+x^2}\,dx = dt$

$\Rightarrow I = \int e^{\tan^{-1}x}(1+x+x^2).\left(-\frac{1}{1+x^2}\,dx\right)$

$I = -\int e^{\tan^{-1}(x)}\left(1+\frac{x}{1+x^2}\right)dx$

$= -\int\left(e^{\tan^{-1}x} + x.\frac{e^{\tan^{-1}x}}{1+x^2}\right)dx = -xe^{\tan^{-1}x} + C$

43. **(3)**

Sol. $I_n = x(\ell nx)^n - \int n\frac{(\ell nx)^{n-1}}{x}\,x\,dx$

$= x(\ell nx)^n - n\int(\ell nx)^{n-1}\,dx$

$\Rightarrow I_n + nI_{n-1} = x(\ell nx)^n$

put $n = 5$

$= n = 5$

44. **(3)**

Sol. $I = \int \frac{\cos x}{\sin^2 x(1+\sin^5 x)^{4/5}}\,dx$

$= \int \frac{\cos x}{\sin^6 x(1+\sin^{-5} x)^{4/5}}\,dx$

put $1 + \sin^{-5}x = t^5 \Rightarrow -5\sin^{-6}x\cos x\,dx = 5t^4\,dt$

$\Rightarrow I = -\int \frac{1}{(t^5)^{4/5}}.t^4\,dt = -(1+\sin^{-5}x)^{1/5} + C$

$= -\frac{1}{\sin x}\ (\sin^5 x + 1)^{1/5} + C$

45. **(4)**

Sol. $f'(x) = \frac{\sqrt{x^2+1}+x}{(x^2+1)-x^2}$

integrating both sides *w.r.t. x*

$\int f'(x)\,dx = \int(\sqrt{x^2+1}+x)\,dx$

$\Rightarrow f(x)$

$= \frac{x^2}{2} + \frac{x}{2}\sqrt{x^2+1} + \frac{1}{2}\ell n(x+\sqrt{x^2+1}) + C$

$f(0) = -\frac{\sqrt{2}+1}{2}$

Now $f(1) = \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2}\ell n\ (1+\sqrt{2})\ - \frac{\sqrt{2}+1}{2}$

$= \frac{1}{2}\ \ell n(\sqrt{2}+1)$

$= \ell n\ (\sqrt{\sqrt{2}+1})$

46. **(3)**

Sol. $\therefore \vec{a}.\vec{b} = 0$

$\Rightarrow f(x).\,f'(x) - (f'(x))^2 = 0$

$\Rightarrow \ell n(f'(x)) = \ell n(f(x)) + \ell nC$

Integrating both sides *w.r.t.*

$\ell n(f'(x)) = \ell n(f(x)) + \ell nC$

$f'(x) = Cf(x)$

$f(0) = 1, f'(0) = 2$

$\Rightarrow C = 2 \Rightarrow \dfrac{f'(x)}{f(x)} = 2$

$\Rightarrow \int \dfrac{f'(x)}{f(x)}dx = \int 2dx$

$\ell n(f(x)) = 2x + C_1$

put $x = 0 \Rightarrow C_1 = 0 \Rightarrow f(x) = e^{2x}$

47. **(2)**

Sol. $I = \int\left(\sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}} + \right.$

$\left.\sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}dx\right)$

$= \int\left(\left|\sin\frac{x}{2} + \cos\frac{x}{2}\right| + \left|\sin\frac{x}{2} - \cos\frac{x}{2}\right|\right)\ dx$

$\therefore\ \frac{x}{2} \in \left(\frac{\pi}{8}, \frac{\pi}{4}\right)$

$\Rightarrow I = \int 2\cos\frac{x}{2}dx = 4\sin\frac{x}{2} + C$

48. **(1)**

Sol. $I =$

$\int \overset{\text{I}}{(\cos x)^{-2017}}.\overset{\text{II}}{\operatorname{cosec}^2 x}\,dx - 2017\int(\cos x)^{-2017}\ dx$

$= (-\cot x).(\cos x)^{-2017} -$

$\int(-2017).(\cos x)^{-2018}.(-\sin x)(-\cot x)\,dx$

$-\,2017\int(\cos x)^{-2017}dx$

$I = \dfrac{-\cot x}{(\cos x)^{2017}} +$

$2017\int(\cos x)^{-2017}dx - \int(2017)(\cos x)^{-2017}dx$

$I = -\dfrac{\operatorname{cosec} x}{(\cos x)^{2016}} + C$

49. **(1)**

Sol. $f(x) + xf'(x) = 6f(x).\,f'(x)$

$\Rightarrow f(x) = (6f(x) - x)\,f'(x)$

Now $I = \int \dfrac{2x(x - 6f(x)) + f(x)}{(6f(x) - x)(x^2 - f(x))^2}\ dx =$

$-\int \dfrac{2x(6f(x) - x) - (6f(x) - x).f'(x)}{(6f(x) - x)(x^2 - f(x))^2}\ dx$

$= -\int \dfrac{2x - f'(x)}{(x^2 - f(x))^2}\ dx$

put $x^2 - f(x) = t$

$(2x - f'(x)\ dx = dt$

so $I = \dfrac{1}{x^2 - f(x)} + C$

50. **(2)**

Sol. $I_{4,\,3} = \int \cos^4 x.\sin 3x\,dx$

$= -\dfrac{1}{3}\ \cos 3x.\cos^4 x -$

$\int 4\cos^3 x(-\sin x)\dfrac{(-\cos 3x)}{3}\,dx$

$= -\dfrac{1}{3}\cos 3x.\cos^4 x - \dfrac{4}{3}\int \cos^3 x\sin x\cos 3x\ dx$

$= -\dfrac{\cos 3x.\cos^4 x}{3} - \dfrac{4}{3}$

$\int \cos^3 x.\sin x(4\cos^3 x - 3\cos x)\,dx$

$= -\dfrac{\cos 3x.\cos^4 x}{3} - \dfrac{4}{3}$

$\int \cos^4 x.\sin x(4(1 - \sin^2 x) - 3)\,dx$

$= -\dfrac{\cos 3x.\cos^4 x}{3} - \dfrac{4}{3}\int \cos^4 x.\sin x\ (1 - 4\sin^2 x)\ dx$

$$= -\frac{\cos 3x.\cos^4 x}{3} - \frac{4}{3}\int \cos^4 x.(\sin 3x - 2\sin x)\ dx$$

$$= -\frac{1}{3}\cos 3x.\cos^4 x - \frac{4}{3}\ I_{4,3} + \frac{4}{3}\int \cos^3 x.\sin 2x)\ dx$$

so $\frac{7}{3}I_{4,3} - \frac{4}{3}\ I_{3,2} = -\frac{\cos 3x.\cos^4 x}{3} + C$

51. (2)

Sol. $I = \int \sin(2016x + x).(\sin x)^{2015}\ dx$

$= \int ((\sin x)^{2015}.\cos x).\sin(2016x)\ \ dx +$

$\int (\sin x)^{2016}.\cos(2016x)\ \ dx$

$= \sin(2016x)\ \frac{(\sin x)^{2016}}{2016}\ .-$

$\int (2016).\cos(2016x).\frac{(\sin x)^{2016}}{2016}dx +$

$\int (\sin x)^{2016}.(\cos(2016x))\,dx$

$I = \frac{\sin(2016x)(\sin x)^{2016}}{2016} + C$

52. (2)

Sol. $I = \int \frac{1+x^2+2x}{x(x^2+1)}dx = \int\left(\frac{1}{x} + \frac{2}{x^2+1}\right)dx$

$I = \ell n\,|\,x\,| + 2\tan^{-1} x + C$

53. (2)

Sol. $I = \int (6x^5 + 5x^4 + 4x^3)(x^4 + x^5 + x^6)^6\ dx$

Let $x^6 + x^5 + x^4 = t$

$(6x^5 + 5x^4 + 4x^2)\ dx = dt$

$I = \int t^6 dt\ = \frac{(x^4+x^5+x^6)^7}{7} + C$

$= \frac{x^{28}(1+x+x^2)^7}{7} + C$

54. (4)

Sol. $I = \frac{1}{\sqrt{2}}\int \frac{(\cos x - \sin x)}{1 + (\sin x + \cos x)^2}\ dx$

put $\sin x + \cos x = t$

$(\cos x - \sin x)\ dx = dt$

$I = \frac{1}{\sqrt{2}}\int \frac{1}{1+t^2}\ dt$

$\frac{1}{\sqrt{2}}\ \tan^{-1}(\sin x + \cos x) + C$

so $A.\ f(x) = \frac{1}{\sqrt{2}}\ (\sin x + \cos x) + C$

Range $\in [-1, 1]$

55. (1)

Sol. $I = \int \frac{e^x(x-2)}{x^3\left(1 + \frac{e^x}{x^2}\right)}dx$

put $1 + \frac{e^x}{x^2} = t$

$\frac{x^2e^x - e^x 2x}{x^4}\ dx = dt \Rightarrow\ \frac{e^x}{x^3}\ (x-2)dx = dt$

so $I = \int \frac{1}{t}dt =\ \ell n\left(1 + \frac{e^x}{x^2}\right) + C$

56. (1)

Sol. $I = \int \frac{\ell n\,|\,x\,|}{x\sqrt{1+\ell n\,|\,x\,|}}\ dx$ Put $1 + ln\,|x| = t^2$, then

$\frac{1}{x}\ \ dx = 2t\ dt$

$\Rightarrow I = \int \frac{t^2-1}{t}\ .2t\ dt\ \ = 2\left[\frac{t^3}{3} - t\right]\ + C = \frac{2}{3}$

$\sqrt{1+\ell n\,|\,x\,|}\ \ (ln\,|x| - 2) + C$

57. (4)

Sol. $I = \int \sqrt{\frac{1-\cos x}{\cos\alpha - \cos x}}\ dx\ \ 0 < \alpha < x < \pi$

$$= \int \frac{\sqrt{2}\sin\frac{x}{2}dx}{\sqrt{2\cos^2\frac{\alpha}{2} - 1 - 2\cos^2\frac{x}{2} + 1}}$$

$$= \int \frac{\sin\frac{x}{2}\,dx}{\sqrt{\cos^2\frac{\alpha}{2} - \cos^2\frac{x}{2}}} \quad \text{put}$$

$$\cos\frac{x}{2} = t \Rightarrow -\frac{1}{2}\sin\frac{x}{2}\,dx = dt$$

$$\Rightarrow I = \int \frac{-2\,dt}{\sqrt{\cos^2\frac{\alpha}{2} - t^2}}$$

$$= -2\sin^{-1}\left(\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}}\right) + C$$

58. (2)

Sol. $I = \int \frac{\cos^3 x}{\sin^2 x + \sin x} dx = \int \frac{\cos x.(1-\sin^2 x)}{\sin x(1+\sin x)} dx$

Put $\sin x = t$, then $\cos x\, dx = dt$

$\Rightarrow I = \int \frac{(1-t)(1+t)\,dt}{t(1+t)} = ln\,|\,t\,| - t + C = ln\,|\sin x| - \sin x + C$

59. (1)

Sol. $I = 2\int \sin x\,.\,\operatorname{cosec} 4x$

$dx = 2\int \frac{\sin x\,dx}{4\sin x \cos x \cos 2x}$

$= \frac{1}{2}\int \frac{dt}{(1-t^2)(1-2t^2)}$

Put $\sin x = t$, then $\cos x\, dx = dt$

$\Rightarrow I = \frac{1}{2}\int \frac{dt}{(1-t^2)(1-2t^2)}$

$= \frac{1}{2}\left[-\int \frac{1}{1-t^2}dt + \int \frac{2}{1-2t^2}dt\right]$

[By partial fraction]

$= \frac{1}{2}\left[-\frac{1}{2}\ell n\left|\frac{1+t}{1-t}\right| + \frac{2}{2\sqrt{2}}\ell n\left|\left(\frac{1+\sqrt{2}t}{1-\sqrt{2}\,t}\right)\right|\right]$

$= \frac{1}{2\sqrt{2}}\ell n\left|\left(\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right)\right|$

$-\frac{1}{4}\ell n\left|\left(\frac{1+\sin x}{1-\sin x}\right)\right| + C$

60. (3)

Sol. $\int \{\sec^2 x + \tan^2 x + 2\tan x \sec x\}^{1/2}\,dx$

$= \int (\sec x + \tan x)\,dx$

$= ln\,|(\sec x + \tan x)| + ln\,|\sec x| + C = ln\,|\sec x(\sec x + \tan x)| + C$

61. (2)

Sol. $\int \frac{\sin^8 x - \cos^8 x}{\sin^4 x + \cos^4 x}dx = \int (\sin^4 x - \cos^4 x)\,dx =$

$\int (\sin^2 x - \cos^2 x)\;dx = -\frac{1}{2}\sin 2x + C$

62. (2)

Sol. $\int \frac{x^3 - 1}{x^3 + x}dx = \int \left(1 - \frac{1}{1+x^2} - \frac{1}{x} + \frac{x}{x^2+1}\right)dx = x$

$-\tan^{-1}x - ln\,|x| + \frac{1}{2}\,ln\,(x^2 + 1) + C$

63. (1)

Sol. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

put $\sqrt{x} = \cos 2t$

$\Rightarrow \quad dx = -4\sin 2t \cos 2t\,dt$

$= \int \sqrt{\frac{1-\cos 2t}{1+\cos 2t}}\,(-4\sin 2t\cos 2t)\,dt = -4$

$\int \frac{\sin t}{\cos t}.2\sin t\cos t\cos 2t\,.dt$

$= -4\int (1-\cos 2t)\cos 2t\,dt$

$= -4\int \cos 2t\;dt + 4\int \cos^2 2t\;dt$

$= -\frac{4}{2}\sin 2t + 2\int (\cos 4t + 1)\;dt = -2\sin 2t + 2$

$\times \frac{\sin 4t}{4} + 2t + C$

$= -2\sqrt{1-x} + \sqrt{x}\sqrt{1-x} + \cos^{-1}\sqrt{x} + C$

64. **(1)**

Sol. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}dx$

$\int \frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}dx$

Let $\frac{x-1}{x+2}=t \Rightarrow \frac{(x+2)-(x-1)}{(x+2)^2}\ dx = dt$

$\Rightarrow \frac{3}{(x+2)^2}dx = dt = \frac{1}{3}\int t^{-3/4}dt = \frac{1}{3}\frac{t^{1/4}}{(1/4)} +$

$c = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$

65. **(4)**

Sol. $\int \frac{3x^4-1}{(x^4+x+1)^2}\ dx = \int \frac{x(4x^3+1)-(x^4+x+1)}{(x^4+x+1)^2}$

$dx = \int \frac{d}{dx}\left(\frac{-x}{(x^4+x+1)}\right)\ dx = -\frac{x}{x^4+x+1}$

66. **(2)**

Sol. Let $I=\int \frac{1+x^4}{\left(1-x^4\right)^{3/2}}dx$

$= \int \frac{1+x^4}{x^3\left(\frac{1}{x^2}-x^2\right)^{3/2}}dx$

$=\int \frac{x+\frac{1}{x^3}}{\left(\frac{1}{x^2}-x^2\right)^{3/2}}dx$

Put $\frac{1}{x^2} - x^2 = t \Rightarrow \left(\frac{-2}{x^3}-2x\right)\ dx = dt$

$\Rightarrow I=-\frac{1}{2}\int \frac{dt}{t^{3/2}} = t^{-1/2} + C = \frac{1}{\sqrt{\frac{1}{x^2}-x^2}} + C$

Integer Type Questions (67 to 74)

67. **(1)**

Sol. $\because\ g'(x) = f(x)$ here

so $\int f(x)\ dx = g(x) + \phi(x)$

$\Rightarrow \int g'(x)\,dx\ = g(x) + \phi(x)$

$\Rightarrow g(x) + c = g(x) + \phi(x)$

$\Rightarrow \phi(x) = c \qquad \phi(0) = 1$

so $\phi(x) = 1$

68. **(0)**

Sol. Let $I = \int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$

Dividing the numerator and denominator by $\cos^4 x$

$I = \int \frac{2\tan x.\sec^2 x}{\tan^4 x+1}dx$

Let $\tan x = u$

$(\sec^2 x)dx = du$

$= \int \frac{2u}{1+u^4}du$

Let $u^2 = z$

$2u\ du = dz$

$I = \int \frac{dz}{1+z^2}$

$\therefore\ I = \tan^{-1} z + c$

$\therefore\ I = \tan^{-1} u^2 + c$

$\therefore\ I = \tan^{-1} (\tan^2 x) + c$

69. **(2)**

Sol. Using Partial Fractions

70. **(16)**

Sol. $I = \int \frac{dx}{\cos^3 x\sqrt{4\tan x\cos^2 x}}$

$I = \frac{1}{2}\int \frac{dx}{\cos^4 x\sqrt{\tan x}} = \frac{1}{2}\int \frac{\sec^4 x dx}{\sqrt{\tan x}}$

Put $\tan x = t$. Then $\sec^2 x\ dx = dt$

Therefore,

$I = \frac{1}{2}\int \frac{(1+t^2)dt}{t^{1/2}} == \frac{1}{2}\int (t^{-1/2}+t^{3/2})dt$

$= \frac{1}{2}\left(2t^{1/2} + \frac{2}{5}t^{5/2}\right) + k$

$= (\tan x)^{1/2} + \frac{1}{5}(\tan x)^{5/2} + k$

Comparing this with the given equation, we get
$A = 1/2$, $B = 5/2$ and $C = 1/5$
Therefore,
$A + B + C = 1/2 + 5/2 + 1/5 = 3 + 1/5 = 16/5$

71. **(1)**

Sol. $I = \int \frac{dx}{5 + 4\cos x}$

$= \int \frac{dx}{5 + 4\frac{\left(1 - \tan^2 \frac{x}{2}\right)}{\left(1 + \tan^2 \frac{x}{2}\right)}}$

$= \int \frac{\sec^2 \frac{x}{2} dx}{9 + \tan^2 \frac{x}{2}}$

Put $\tan \frac{x}{2} = t$

$\Rightarrow \sec^2 \frac{x}{2} dx = 2dt$

$\Rightarrow I = \int \frac{2dt}{9 + t^2}$

$= \frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right) + C$

$= \frac{2}{3}\tan^{-1}\left(\frac{\tan \frac{x}{2}}{3}\right) + C$

Ans. is 1

72. **(15)**

Sol. $I = \int \frac{1}{x^{13}(1 + x^{-4})^{1/2}} dx$

$1 + x^{-4} = t^2$

$-4x^{-5} dx = 2tdt$

$\Rightarrow I = -\frac{1}{4}\int \frac{2t}{x^8 t} dt \quad \Rightarrow \quad I = -\frac{1}{2}\int (t^2 - 1)^2\, dt$

$\Rightarrow I = -\frac{1}{2}\int (t^4 - 2t^2 + 1)\, dt$

$\Rightarrow I = \frac{t^5}{-10} - \frac{t^3}{-3} - \frac{t}{2} + k$

so $c - b - a = 2 - (-3) - (-10) = 15$

73. **(60)**

Sol. $g(x) = \int \frac{3x + 2}{\sqrt{x - 9}} dx$

Let $\sqrt{x - 9} = t \Rightarrow x - 9 = t^2$

$\Rightarrow dx = 2tdt$

$\Rightarrow g(x) = \int \frac{3(t^2 + 9) + 2}{t} .2tdt$

$g(x) = 2\int (29 + 3t^2)\, dt$

$= 2[29t + t^3] + C$

$g(x) = 2\left[29\sqrt{x - 9} + (x - 9)^{3/2}\right] + C$

$\therefore g(13) = 132$

$\Rightarrow 132 = 2(58 + 8) + C$

$\Rightarrow C = 0$

so, $g(10) = 60$

74. **(1)**

Sol. $I = \int \frac{\sqrt{x}}{\sqrt{x}(1 + \sqrt{x})^{2017}} dx$

$1 + \sqrt{x} = t$

$\frac{1}{\sqrt{x}} dx = 2dt$

$\Rightarrow I = 2\int \frac{t - 1}{t^{2017}} dt = 2\int (t^{-2016} - t^{-2017})\, dt$

$= 2\left[\frac{1}{(-2015)(1 + \sqrt{x})^{2015}} - \frac{1}{(-2016)(1 + \sqrt{x})^{2016}}\right] + C$

$= 2\left[\frac{1}{2016(1 + \sqrt{x})^{2016}} - \frac{1}{2015(1 + \sqrt{x})^{2015}}\right] + C$

so, $\alpha - \beta = 1$

DEFINITE INTEGRATION AND APPLICATION OF INTEGRALS

Single Option Correct Type Questions (01 to 60)

1. **(1)**

Sol. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$

Put $\sqrt{x} = t \Rightarrow \frac{dx}{2\sqrt{x}} = dt$

$2\int_0^{\sqrt{2}} 3^t\,dt = 2\left[\frac{3^t}{\log 3}\right]_0^{\sqrt{2}} = \frac{2}{\ell n3}(3^{\sqrt{2}} - 1)$

2. **(2)**

Sol. $\int_0^{\pi/4} x\tan x\sec^2 x\,dx$

$= \left(x\frac{\tan^2 x}{2}\right)_0^{\pi/4} - \int_0^{\pi/4}\frac{\tan^2 x}{2}\,dx$

$= \frac{\pi}{4} - \frac{1}{2}$

3. **(1)**

Sol. $\int_{\ell n\pi - \ell n2}^{\ell n\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}\,dx$

$= \frac{3}{2}\int_{\pi/3}^{2\pi/3}\frac{dt}{1-\cos 2t} = \frac{3}{2}\int_{\pi/3}^{2\pi/3}\frac{dt}{1-(1-2\sin^2 t)}$

$= \frac{3}{2.2}\int_{\pi/3}^{2\pi/3}\text{cosec}^2 t\;\,dt = \frac{3}{4}[-\cot\;\,t]_{\pi/3}^{2\pi/3}$

$= -\frac{3}{4}\left[-\sqrt{3}\;-\frac{1}{\sqrt{3}}\right] = \frac{3}{4}\left(\frac{4}{\sqrt{3}}\right) = \sqrt{3}$

4. **(1)**

Sol. $\int_1^2 e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)\,dx$

$\therefore\; \left.\frac{e^x}{x}\right|_1^2 = \mathbf{e}\left(\frac{e}{2} - 1\right)$

5. **(4)**

Sol. Let $I = \int_0^\infty e^{-ax^2}\,dx$

Put $\sqrt{a}\;\;x = t \;\Rightarrow\; dx = \frac{dt}{\sqrt{a}}$

then $I = \frac{1}{\sqrt{a}}\int_0^\infty e^{-t^2}\,dt = \frac{1}{\sqrt{a}}\int_0^\infty e^{-x^2}\,dx$

$= \frac{1}{\sqrt{a}}\;\frac{\sqrt{\pi}}{2} = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

6. **(1)**

Sol. $I_1 = \int_e^{e^2}\frac{dx}{\ell nx}$

Let $ln\,x = t$

$\Rightarrow\; x = e^t \Rightarrow dx = e^t dt$

$I_1 = \int_1^2 \frac{e^t}{t}\,dt = I_2$

7. **(1)**

Sol. $\frac{\pi}{6} < x < \frac{\pi}{3} \Rightarrow \frac{1}{2} < \sin x < \frac{\sqrt{3}}{2}$

$\Rightarrow 1 < 2$

$\sin x < \sqrt{3}$

$\therefore [2 \sin x] = 1$

8. **(2)**

Sol. $\int_0^{1.5} [x^2]\ dx = \int_0^1 0.dx + \int_1^{\sqrt{2}} 1.\ dx + \int_{\sqrt{2}}^{1.5} 2\ dx = 0$

$+ [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{1.5}$

$= 0 + \sqrt{2} - 1 + 2(1.5 - \sqrt{2}) = \sqrt{2} - 1 + 3 - 2\sqrt{2}$

$= 2 - \sqrt{2}$

9. **(3)**

Sol. $I = \int_{\ell n\lambda}^{-\ell n\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right)[g\ (x) + g\ (-x)]} dx = 0$

Since $\frac{f\left(\frac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right)[g\ (x) + \int(-x)]}$ is an odd function

10. **(4)**

Sol. $I = \int_{2-\log 3}^{3+\log 3} \frac{\log(4+x)}{\log(4+x) + \log(9-x)} dx$ ......(1)

using $\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\ dx$

$I = \int_{2-\log 3}^{3+\log 3} \frac{\log(9-x)}{\log(9-x) + \log(4+x)} dx$ ......(2)

From equation (1) and (2)

$2I = \int_{2-\log 3}^{3+\log 3} dx = 1 + 2\log 3$

$\therefore I = \frac{1}{2} + \log 3$

11. **(4)**

Sol. $\int_{-2}^{10} \text{sgn}\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right)$

$= 6\int_0^2 \text{sgn}\left\{\frac{x}{2}\right\}\ dx$

$\therefore$ period of $\{x\}$ is 1

$\{x\} = 6{\cdot}2 = 12$

12. **(4)**

Sol. $\because 0 \le \left|\frac{\sin x}{2}\right| \le \frac{1}{2} \quad \therefore \left[\left|\frac{\sin x}{2}\right|\right] = 0$

$I = \int_0^{2n\pi} |\sin x|\, dx = 2n \int_0^{\pi} (\sin x)\ dx = 4n \int_0^{\pi/2} \cos x\ dx = 4n$

13. **(2)**

Sol. $I = \frac{\int_x^{x+h} \ell n^2 t\ dx}{h}$

Using L hospital we get

$I = \lim_{h \to 0} \ell n^2(x+h) = \ell n^2 x$

14. **(4)**

Sol. $f(x) = 1 + x + \int_1^x (\ell n^2 t + 2\ell nt)\ dt$

Differentiate both sides *w.r.t. x*

by using Leibinitz theorem

$f'(x) = 1 + ln^2 x + 2ln\, x = 0$

$(1 + ln\, x)^2 = 0 \qquad \therefore ln\, x = -1$

$\therefore x = \frac{1}{e}$

15. **(4)**

Sol. $\lim_{x\to 0} \dfrac{\sqrt{\cos x^3}.3x^2}{1-\sqrt{\cos x}}$

$= 12$

16. **(2)**

Sol. We know that $\tan x > x \ \forall\ x \in (0,1)$

$\Rightarrow \dfrac{\tan x}{\sqrt{x}} > \sqrt{x}$

$\Rightarrow \int_0^1 \dfrac{\tan x}{\sqrt{x}} dx > \int_0^1 \sqrt{x} dx$

$\Rightarrow I > \left[\dfrac{x^{3/2}}{3/2}\right]_0^1 \qquad \Rightarrow I > \dfrac{2}{3}$

17. **(3)**

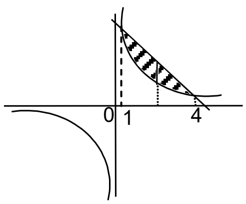
Sol. $A = \int_1^4 \left(5 - x - \dfrac{4}{x}\right) dx$

$= \left[5x - \dfrac{x^2}{2} - 4\ell nx\right]_1^4$

$= 20 - 8 - 4ln\ 4 - 5 + \dfrac{1}{2} + 0$

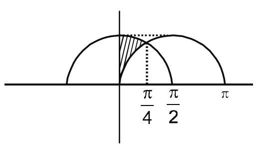
$= 7 + \dfrac{1}{2} - 4ln\ 4$

$= \dfrac{15}{2} - 4\ell n4$



18. **(3)**

Sol.



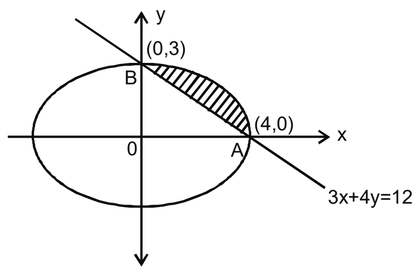
From figure

Area $= \int_0^{\pi/4} (\cos x - \sin x) dx = \sqrt{2} - 1$

19. **(2)**

Sol. Area $= \int_0^4 \left(3\sqrt{1 - \dfrac{x^2}{16}} - \dfrac{1}{4}(12 - 3x)\right) dx$

$= \dfrac{3}{4}\left[\dfrac{x}{4}\sqrt{16 - x^2} + \dfrac{16}{2}\sin^{-1}\left(\dfrac{x}{4}\right)\right]_0^4 - \dfrac{1}{4}\left[12x - \dfrac{3x^2}{2}\right]_0^4$



$= \dfrac{3}{4}\left[(0 + 4\pi) - (0 + 0)\right] - \dfrac{1}{4}[48 - 24]$

$= 3\pi - 6 = 3(\pi - 2)$

20. **(3)**

Sol. $I =$

$\int_0^1 \dfrac{dx}{(x + \cos\alpha)^2 + \sin^2\alpha} = \dfrac{1}{\sin\alpha}\left[\tan^{-1}\left(\dfrac{x + \cos\alpha}{\sin\alpha}\right)\right]_0^1 = \dfrac{1}{\sin\alpha}\left(\alpha - \dfrac{\alpha}{2}\right)$

$= \dfrac{\alpha}{2\sin\alpha}$

21. **(2)**

Sol. Let $\tan^{-1} x = t \Rightarrow \dfrac{dx}{1 + x^2} = dt$

$\therefore \quad I = \int_0^{\pi/4} \dfrac{t \ \tan \ t \cdot dt}{\sqrt{1 + \tan^2 t}} = \int_0^{\pi/4} t \cdot \sin t \ \ dt$

$= -\dfrac{\pi}{4\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \dfrac{4 - \pi}{4\sqrt{2}}$

22. **(3)**

Sol. $I = \int_{1/e}^{\tan x} \dfrac{t}{1 + t^2} \ dt + \int_{1/e}^{\cot x} \dfrac{1}{t(1 + t^2)} \ dt$

put $t = \frac{1}{x}$

$$I = \int_{1/e}^{\tan x} \frac{t}{1+t^2}\, dt + \int_{e}^{\tan x} \frac{1}{\frac{1}{x}\left(1+\frac{1}{x^2}\right)} \cdot \left(-\frac{1}{x^2}\right)$$

dx

$$I = \int_{1/e}^{\tan x} \frac{t}{1+t^2} + \int_{\tan x}^{e} \frac{x}{1+x^2}\, dx$$

$$= \int_{1/e}^{e} \frac{t}{1+t^2}\, dt = \left(\frac{1}{2}\ell n\ (1+t^2))\right)_{1/e}^{e} = 1$$

23. **(3)**

Sol. $$\int_0^2 f(x)\, dx = \int_0^{1/2} 1.dx + \int_{1/2}^{2/3} 1.dx + \int_{2/3}^{3/4} 1.dx + ... + \int_{\frac{n-1}{n}}^{\frac{n}{n+1}} 1.dx +$$

$$.........+ \int_1^2 1.\ dx$$

$$= \left(\frac{1}{2}\right) + \left(\frac{2}{3} - \frac{1}{2}\right) + \left(\frac{3}{4} - \frac{2}{3}\right) + ...$$

$$+ \left(\frac{n}{n+1} - \frac{n-1}{n}\right) + ...+1 = \frac{n}{n+1} + ...+1$$

as $n \to \infty$

taking limit $n \to \infty$

we get $\int_0^2 f(x)\ dx = 1+1 = 2$

24. **(2)**

Sol. $$\sum_{r=1}^{100} \left(\int_0^1 f(r-1+x)\ dx\right)$$

$$= \int_0^1 f(x)\ dx + \int_0^1 f(1+x)\ dx + \int_0^1 f(2+x)$$

$$dx + ...+ \int_0^1 f(99+x)\ dx$$

$$= \int_0^1 f(x)\ dx + \int_1^2 f(x)\ dx + ...+ \int_{99}^{100} f(x)\ dx$$

{using shifting property}

$$= \int_0^{100} f(x)\ dx = a$$

25. **(2)**

Sol. $I = \int_0^{\infty} \left[2\ e^{-x}\right] dx$

$\therefore$ $2e^{-x}$ decreases for $x \in [0, \infty)$

$\Rightarrow$ $0 < 2e^{-x} \le 2\ \forall\ x \in [0, \infty)$

for $x > ln2$, $[2e^{-x}] = 0$

$$\Rightarrow I = \int_0^{\ell n2} [2e^{-x}]\ dx + \int_{\ell n2}^{\infty} [2e^{-x}]\ dx$$

$$= \int_0^{\ell n2} 1\ .\ dx + \int_{\ell n2}^{\infty} 0.dx = ln2$$

26. **(2)**

Sol. $$\int_{-\frac{\pi}{2}}^{\pi/2} \frac{|x|\, dx}{8\cos^2 2x+1} = 2\int_0^{\pi/2} \frac{x\ dx}{8\cos^2 2x+1} = 2\text{I}$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) dx}{8\cos^2(\pi-2x)+1}$$

$$I = \frac{\pi}{2}\int_0^{\pi/2} \frac{dx}{8\cos^2 2x+1} - \text{I} = \pi \int_0^{\pi/4} \frac{dx}{8\cos^2 2x+1}$$

$$\therefore I = \frac{\pi}{2}\int_0^{\pi/4} \frac{dx}{8\cos^2 2x+1}$$

$$= \frac{\pi}{4}\int_0^{\pi/4} \frac{2\sec^2 2x\ dx}{9+\tan^2 2x}$$

$$= \frac{\pi}{4} \cdot \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan 2x\right)_0^{\pi/4} = \frac{\pi^2}{24}$$

$\therefore$ given integral $= \frac{\pi^2}{12}$

27. **(1)**

Sol. $x = \tan\theta$

$$\int_0^{\pi/2} 2\ell n\ \sec\theta\ d\theta$$

$$= -2\int_0^{\pi/2} \ell n\ \cos\theta\ d\theta = -2\left(-\frac{\pi}{2}\ell n2\right)$$

$= \pi ln2$

28. **(1)**

Sol. $I_1 = \int_0^{\pi} x \; f(\sin^3 x + \cos^2 x)dx$ ....(1)

$= \int_0^{\pi} (\pi - x) \; f(\sin^3(\pi - x) + \cos^2(\pi - x)dx$

.........(2)

(1) + (2)

$2I_1 = \pi \int_0^{\pi} f(\sin^3 x + \cos^2 x)dx$

$2I_1 = 2\pi \int_0^{\pi/2} f(\sin^3 x + \cos^2 x)dx$

$\therefore I_1 = \pi \int_0^{\pi/2} f(\sin^3 x + \cos^2 x)dx$

29. **(2)**

Sol. $\int_{-1}^{-1+6(1/2)} \{2x\}\, dx = 6 \int_0^{1/2} \{2x\}\, dx = 6 \int_0^{1/2} 2x dx$

$= 12 \left[\frac{x^2}{2}\right]^{1/2} = \frac{12}{8} = \frac{3}{2}$

30. **(2)**

Sol. $f'(x) = \frac{1}{\ell n x^3}. \; 3x^2 - \frac{1}{\ell n(x^2)}.2x$

$= \frac{x^2}{\ell nx} - \frac{x}{\ell nx} = \frac{x^2 - x}{\ell nx}$

for increasing $f'(x) > 0$

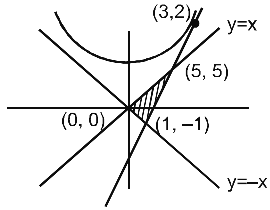
$\Rightarrow \quad \frac{x^2 - x}{\ell nx} > 0$

31. **(1)**

Sol. $\left.\frac{dy}{dx}\right|_{(3,2)} = \frac{3}{2}$. Tangent $y = \frac{3x}{2} - \frac{5}{2}$

$y = x, \qquad y = \frac{3x}{2} - \frac{5}{2}$

$\Rightarrow \quad (5, 5)$



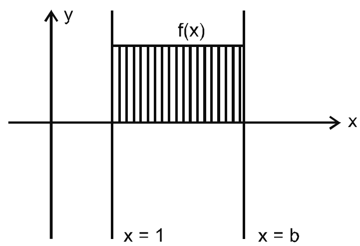
Figure

$y = -x, \qquad y = \frac{3x}{2} - \frac{5}{2} \Rightarrow \quad (1, -1)$

closed figure formed is right angled triangle. Its area is $\frac{1}{2}(\sqrt{2}) \; (5\sqrt{2}) \; = 5$

32. **(3)**

Sol. Required area $(b - 1) \sin (3b + 4) = \int_0^{b} f(x) \; dx$



diff. *w.r.t. b*

$3(b - 1) \cos(3b + 4) + \sin(3b + 4) = f(2)$

$\Rightarrow \; f(x) = 3(x - 1) \cos (3x + 4) + \sin (3x + 4)$

33. **(1)**

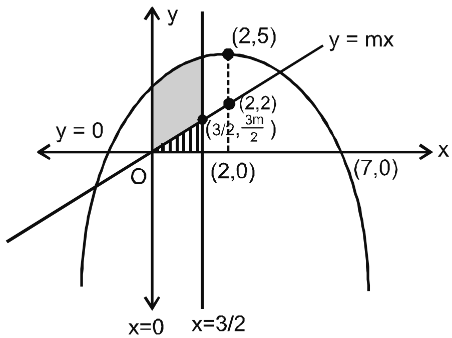
Sol. $y = 1 + 4x - x^2 \Rightarrow (x - 2)^2 = -(y - 5)$

vertex (2, 5)

$A = \int_0^{3/2} (1 + 4x - x^2) \; dx = \left(x + \frac{4x^2}{2} - \frac{x^3}{3}\right)_0^{3/2} = \frac{39}{8}$

...........(1)

$\Delta OAB = \frac{1}{2} \times \frac{3}{2} \times \frac{3}{2} \times m \; = \frac{9}{8} m$ ....(2)

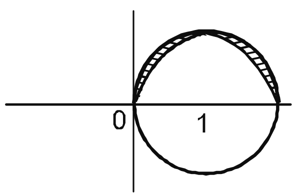


From (1) & (2) $\frac{9}{8}m = \frac{1}{2}\left(\frac{39}{8}\right)$

$\Rightarrow m = \frac{39}{2\times 9} = \frac{13}{6}$

34. (1)

Sol.



$(x-1)^2 + y^2 = 1$

Area of circle is πr^2

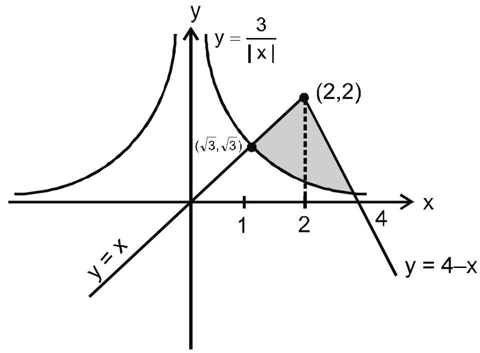
Required area = $\frac{\pi}{2} - \int_0^2 \sin\frac{\pi x}{2}dx$

$= \frac{\pi}{2} - \frac{4}{\pi}$

35. (2)

Sol. $A = \int_{\sqrt{3}}^{2}\left(x - \frac{3}{x}\right) dx + \int_2^3\left(4 - x - \frac{3}{x}\right) dx$

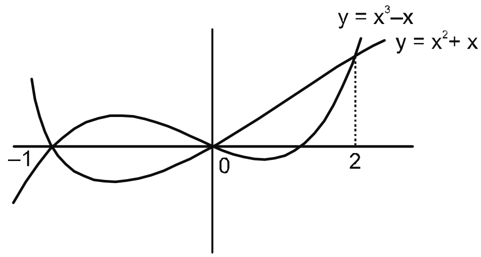
$= \left(\frac{x^2}{2} - 3\ell nx\right)_{\sqrt{3}}^{2} + \left(4x - \frac{x^2}{2} - 3\ell nx\right)_2^3$



$= \frac{4 - 3\ell n3}{2}$

36. (2)

Sol.



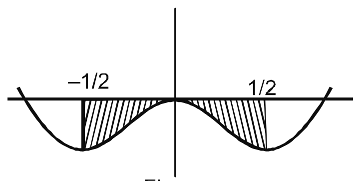
Area =

$\int_{-1}^{0} (x^3 - x) - (x^2 + x)\ dx + \int_0^2 (x^2 + x - (x^3 - x))\ dx$

$= \frac{37}{12}$

37. (4)

Sol. $\frac{dy}{dx} = 8x^3 - 2x,\ \frac{dy}{dx} = 0 \quad \Rightarrow (4x^2 - 1)\,x = 0$



Figure

$\Rightarrow x = -\frac{1}{2}, 0, \frac{1}{2}$

Required area = $-2\int_0^{1/2}(2x^4-x^2)\ dx = \frac{7}{120}$

38. **(2)**

Sol. $\int_0^t \{x\}\ dx = \int_0^{[t]}\{x\}\ dx + \int_{[t]}^t \{x\}\ dx = [t]$

$\int_0^1 x\ dx + \int_0^{\{t\}} x\ dx = \frac{[t]}{2}+\frac{\{t\}^2}{2}$

$\therefore$ statement-2 is true.

$\int_0^{5.5}\{x\}\ dx = \frac{5}{2}+\frac{(.5)^2}{2} = \frac{21}{8}$

$\therefore$ statement-1 is true and is explained by statement-2.

39. **(2)**

Sol. Let $I = \int_{-3\pi/2}^{-\pi/2}[(x+\pi)^3+\cos^2(x+3\pi)]\ dx$ .....(i)

and

$I = \int_{-3\pi/2}^{-\pi/2}\left[\left(-\frac{\pi}{2}-\frac{3\pi}{2}-x+\pi\right)^3+\cos^2\left(-\frac{\pi}{2}-\frac{3\pi}{2}-x+3\pi\right)\right]dx$

$\Rightarrow I = \int_{-3\pi/2}^{-\pi/2}[-(x+\pi)^3+\cos^2(\pi-x)]\ dx$ ...(ii)

(i) + (ii)

$2I = \int_{-3\pi/2}^{-\pi/2} 2\cos^2 x\ dx$

$= \int_{-3\pi/2}^{-\pi/2}(1+\cos 2x)\ dx$

$\Rightarrow I = \frac{\pi}{2}$

40. **(1)**

Sol. (I) $\to$ (S); (II) $\to$ (S); (III) $\to$ (P); (IV) $\to$ (R)

(1) $\int_{-1}^1 \frac{dx}{1+x^2} = \left(\tan^{-1}x\right)_{-1}^1 = \frac{\pi}{4}-\left(-\frac{\pi}{4}\right) = \frac{\pi}{2} \to$ (S)

(2) $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \left(\sin^{-1}x\right)_0^1 = \frac{\pi}{2}$

$\to$ (S)

(3) $\int_2^3 \frac{dx}{1-x^2} = \frac{1}{2}\left(\ell n\left|\frac{1+x}{1-x}\right|\right)_2^3 = \frac{1}{2}\ \ell\text{n}\left(\frac{2}{3}\right) \to$ (P)

(4) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}} = \left(\sec^{-1}x\right)_1^2 = \sec^{-1}2 -$

$\sec^{-1}(1) = \frac{\pi}{3} \to$ (R)

41. **(1)**

Sol. Since, $f(x) = \int_1^x \frac{\log t}{1+t}\ dt$

and $F(e) = f(e) + f\left(\frac{1}{e}\right)$

$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t}dt + \int_1^{1/e}\frac{\log t}{1+t}dt$

By putting $t = 1/x \Rightarrow dt = -1/x^2\ dx$

$= \int_1^e \frac{\log t}{1+t}dt + \int_1^e \frac{\log t}{(1+t)t}dt$

$= \int_1^e \frac{\log t}{t}dt = \left[\frac{(\log t)^2}{2}\right]_1^e$

$= \frac{1}{2}$

42. **(2)**

Sol. Since, $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \frac{x}{\sqrt{x}}dx$,

because $x \in (0, 1)$, $x > \sin x$

$I < \int_0^1 \sqrt{x}\ dx = \frac{2}{3}\left[x^{3/2}\right]_0^1$

$\Rightarrow I < \dfrac{2}{3}$

and $J = \int_0^1 \dfrac{\cos x}{\sqrt{x}}dx < \int_0^1 x^{-\frac{1}{2}}dx = 2$

$J < 2$

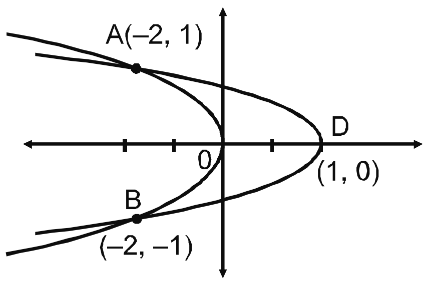
43. (4)

Sol. $x + 2y^2 = 0 \Rightarrow y^2 = -\dfrac{1}{3}(x-1)$

[Left handed parabola with vertex at (1, 0)]

Solving the two equations we get the points of intersection as (–2, 1), (–2, –1)

The required area is *AOBDA*, given by



$= \left|\int_{-1}^{1}(1-y^2)dy\right| = \left|\left[y - \dfrac{y^3}{3}\right]_{-1}^{1}\right|$

$= 2 \times \dfrac{2}{3} = \dfrac{4}{3}$ Sq. units.

44. (3)

Sol. Let $I = \int_0^{\pi}[\cot x]dx$ ...(i)

$\Rightarrow I = \int_0^{\pi}[\cot(\pi - x)]dx = \int_0^{\pi}[-\cot x)]dx$ ...(ii)

(i) + (ii)

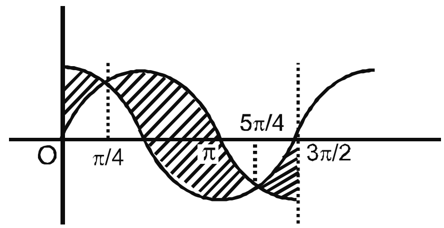
$2I = \int_0^{\pi}[\cot x]dx + \int_0^{\pi}[-\cot x)]dx$

$= \int_0^{\pi}(-1)dx = -\pi$

$I = -\dfrac{\pi}{2}$

45. (4)

Sol. Required area $= \int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx + \int_{5\pi/4}^{3\pi/2}(\cos x - \sin x)dx$

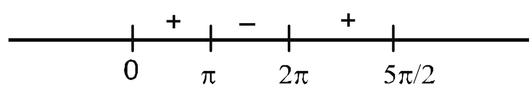


$= 2\left[\sin x + \cos x\right]_0^{\pi/4} + \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4} = 4\sqrt{2} - 2$

46. (4)

Sol. $f(x) = \int_0^x \sqrt{t}\sin t\,dt$

$f'(x) = \sqrt{x}\sin x$



local maximum at π

and local minimum at 2π

47. (3)

Sol. $\int_0^1 x\,[x^2]\,dx + \int_1^{\sqrt{2}} x\,[x^2]\,dx + \int_{\sqrt{2}}^{1.5} x\,[x^2]\,dx$

$\int_0^1 x.\,0\,dx + \int_1^{\sqrt{2}} x\,dx + \int_{\sqrt{2}}^{1.5} 2x\,dx$

$$0 + \left[\frac{x^2}{2}\right]_1^{\sqrt{2}} + \left[x^2\right]_{\sqrt{2}}^{1.5}$$

$$\frac{1}{2}(2-1) + (2.25-2)$$

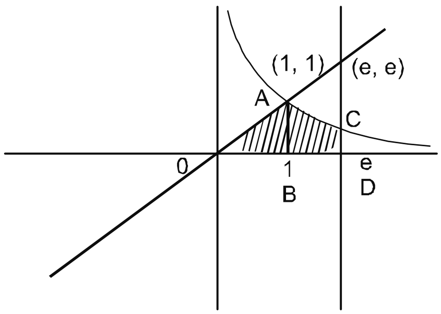
$$\frac{1}{2} + .25$$

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

48. **(3)**

Sol. Required area

$= OAB + ACDB$



$$= \frac{1}{2}\times 1\times 1 + \int_1^e \frac{1}{x}dx$$

$$= \frac{1}{2} + (\ell nx)_1^e$$

$$= \frac{3}{2} \text{ Sq. unit}$$

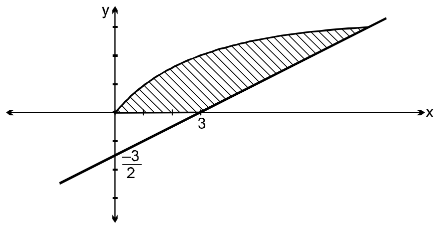
49. **(1)**

Sol. $y = \sqrt{x}$ ....(1)

and $2y - x + 3 = 0$ ....(2)

On solving both $y = -1, 3$

Required area = $\int_0^3 \left\{(2y+3) - y^2\right\} dy$



$$= y^2 + 3y - \frac{y^3}{3}\Big|_0^3$$

$= 9 + 9 - 9$

$= 9.$

50. **(2)**

Sol. $I = \int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}$

$$dx = \int_0^{\pi} |1 - 2\sin x/2|\ dx$$

$$= \int_0^{\pi/3} |1-2\sin x/2| + \int_{\pi/3}^{\pi} |1-2\sin x/2|\, dx$$

$$= \int_0^{\pi/3} (1-2\sin x/2)dx + \int_{\pi/3}^{\pi} (2\sin x/2 - 1)dx$$

$$= \left(x + 2\frac{\cos\frac{x}{2}}{\frac{1}{2}}\right)_0^{\pi/3} + \left(-2\frac{\cos\frac{x}{2}}{\frac{1}{2}} - x\right)_{\pi/3}^{\pi}$$

$$= \left(\frac{\pi}{3} + 4\frac{\sqrt{3}}{2}\right) - (4) + (0 - \pi)$$

$$-\left(\pi - 4\times\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right)$$

$$= \frac{\pi}{3} + 2\sqrt{3} - 4 - \pi + 2\sqrt{3} + \pi/3$$

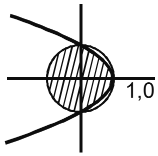
$$= -4 - \pi/3 + 4\sqrt{3}$$

51. **(3)**

Sol. Intersection of $x^2 + y^2 = 1$ & $y^2 = 1 - x$ is $x = 0, 1$

The required portion is shaded as shown.

Area of region is area of semi-circle plus area bounded by parabola & y-axis.



Area of semi-circle is . $\frac{\pi}{2}$

Area bounded by parabola = $\frac{2}{3}$ of corresponding rectangle = $\frac{2}{3} \times 1 \times 2 = \frac{4}{3}$

Hence total area = $\frac{\pi}{2} + \frac{4}{3}$.

Method - 1

Required area = area of semi circle + area bounded by parabola

$$= \frac{\pi}{2} + \int_0^1 (1-y^2)\ dy = \frac{\pi}{2} + 2\left(y - \frac{y^3}{3}\right)_0^1$$

$$= \frac{\pi}{2} + 2\left(1 - \frac{1}{3}\right) \Rightarrow \frac{\pi}{2} + \frac{4}{3}$$

52. (3)

Sol. $I = \int_2^4 \frac{\log x^2}{\log x^2 + \log\left(x^2 - 12x + 36\right)} dx$

$$I = \frac{2}{2}\int_2^4 \frac{\log x}{\log x + \log(6-x)} dx \quad \text{...(i)}$$

$$I = \int_2^4 \frac{\log(6-x)}{\log(6-x) + \log x} dx \qquad \left\{\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right\}$$

...(ii)

Equation (i) & (ii) gives

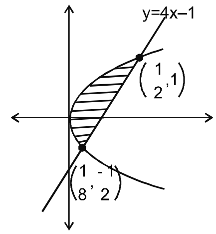
$$2I = \int_2^4 \frac{\log x + \log(6-x)}{\log x + \log(6-x)} dx = \int_2^4 dx = 2$$

Hence $I = 1$

53. (4)

Sol. $\int_{-1/2}^{2} \left(\frac{y+1}{4} - \frac{y^2}{2}\right) dy$

$$\Rightarrow \frac{1}{4}\left\{\frac{y^2}{2} + y\right\}_{-1/2}^{1} - \frac{1}{6}\{y^3\}_{-1/2}^{1}$$



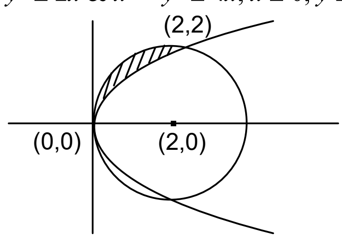
$$\Rightarrow \frac{1}{4}\left\{\left(\frac{1}{2}+1\right) - \left(\frac{1}{8} - \frac{1}{2}\right)\right\} - \frac{1}{6}\left\{1 + \frac{1}{8}\right\}$$

$$\Rightarrow \frac{1}{4}\left\{\frac{3}{2} + \frac{3}{8}\right\} - \frac{1}{6}\left\{\frac{9}{8}\right\}$$

$$\Rightarrow \frac{15}{32} - \frac{6}{32} = \frac{9}{32}\ \text{Sq-unit}$$

54. (1)

Sol. $y^2 \geq 2x$ & $x^2 + y^2 \leq 4x;\ x \geq 0,\ y \geq 0$



$x^2 + 2x = 4x$

$x^2 - 2x = 0$

$x = 0, 2$

$$\int_0^2 \left(\sqrt{4x - x^2} - \sqrt{2x}\right) dx = \pi - \frac{8}{3}$$

$$\int_0^2 \left(\sqrt{4-(x-2)^2} - \sqrt{2}\sqrt{x}\right) dx$$

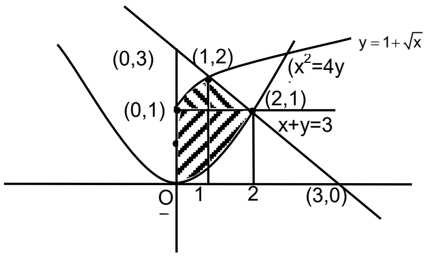
$$\left(\left(\frac{(x-2)}{2}\sqrt{4x-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-2}{2}\right)\right) - \frac{\sqrt{2}(x^{3/2})}{3}(2)\right)_0^2$$

$$\left(\frac{-2\sqrt{2}}{3}(2^{3/2})-\left(2\sin^{-1}(-1)\right)\right)$$

$$\frac{-2\sqrt{2}}{3}(2\sqrt{2})-2\left(\frac{-\pi}{2}\right) = \pi-\frac{8}{3} \text{ Sq-unit}$$

55. (4)

Sol.



$y = 1+\sqrt{x}$

$(y-1)^2 \le x$

Required area = $\int_0^1\left(1+\sqrt{x}\right)dx+\int_1^2(3-x)dx-\int_0^2\frac{x^2}{4}dx$

$$=\left(x+\frac{2x^{3/2}}{3}\right)\Bigg|_0^1+\left(3x-\frac{x^2}{7}\right)\Bigg|_1^2-\left(\frac{x^3}{12}\right)_0^2$$

$$= 1+\frac{2}{3}+\left\{(6-2)-\frac{5}{2}\right\}-\frac{8}{12}$$

$$= 1+\frac{2}{3}+\left(4-\frac{5}{2}\right)-\frac{2}{3} = 1+\frac{3}{2} = \frac{5}{2}$$

56. (1)

Sol. $L = \lim\limits_{x\to\frac{\pi}{4}}\dfrac{\int_2^{\sec^2 x} f(t)\ dt}{x^2-\frac{\pi^2}{16}}\left(\frac{0}{0}\ form\right)$

By *L*. Hospital rule

$$L = \lim_{x\to\frac{\pi}{4}}\frac{(f(\sec^2 x))2\sec^2 x\tan x-0}{2x}$$

$$L = \frac{8f(2)}{\pi}$$

57. (1)

Sol. $\int_0^1\frac{x^4(1-x)^4}{1+x^2}dx = \int_0^1\frac{x^4[(1+x^2)-2x]^2}{1+x^2}\ dx$

$$= \int_0^1\frac{x^4[(1+x^2)^2-4x(1+x^2)+4x^2]}{1+x^2}\ dx$$

$$= \int_0^1 x^4\left[(1+x^2)-4x+\frac{4x^2}{1+x^2}\right]\ dx$$

$$= \int_0^1\left[x^6+x^4-4x^5+\frac{4x^6}{1+x^2}\right]\ dx$$

Now on polynomial division of x^6 by $1+x^2$, we obtain

$$= \int_0^1\left[x^6+x^4-4x^5+4\left[(x^4-x^2+1)-\frac{1}{1+x^2}\right]\right]\ dx$$

$$= \int_0^1\left[\left(x^6-4x^5+5x^4-4x^2+4\right)-\frac{4}{1+x^2}\right]\ dx$$

$$= \left[\frac{x^7}{7}-\frac{4x^6}{6}+\frac{5\ .\ x^5}{5}-\frac{4x^3}{3}+4x\right]_0^1-4\left[\tan^{-x} x\right]_0^1$$

$$= \left(\frac{1}{7}-\frac{4}{6}+1-\frac{4}{3}+4\right)-4\left(\frac{\pi}{4}\right)$$

$$= \left[\frac{1}{7}-\frac{12}{6}+5\right]-\pi$$

$$= \left(\frac{1}{7}+3\right)-\pi = \frac{22}{7}-\pi$$

58. (2)

Sol. $f(x) = e^x\left(2+\int_0^x\sqrt{t^4+1}\ \ dt\right)$

Let $g(x) = f^{-1}(x) \Rightarrow g(f(x)) = x$

$\Rightarrow g'(f(x))\,f'(x) = 1$

$\Rightarrow g'(2) = \dfrac{1}{f'(0)}$ $(\because f(0) = 2)$

Now $f'(x) = e^x\left(2+\int_0^x \sqrt{t^4+1}\,dt\right) + e^x\sqrt{x^4+1}$ (Applying Leibinitz Rule)

$\Rightarrow f'(0) = 2 + 1 = 3$

$\Rightarrow g'(2) = \frac{1}{3}$

$\Rightarrow (f^{-1})'(2) = \frac{1}{3}$

59. **(2)**

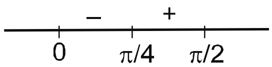
Sol. Given $y = \sin x + \cos x$

$x \in [0, \pi/2]$

$\frac{dy}{dx} = \cos x - \sin x$

$y = |\cos x - \sin x|$

$$= \begin{cases} \cos x - \sin x & x \in [0, \pi/4] \\ \sin x - \cos x & x \in [\pi/4, \pi/2] \end{cases}$$



required area y =

$$\int_0^{\pi/4} \left|(\sin x + \cos x) - (\cos x - \sin x)\right| dx + \int_{\pi/4}^{\pi/2} |2\cos x|\, dx$$

$$= \int_0^{\pi/4} |2\sin x|\, dx + \int_{\pi/4}^{\pi/2} |2\cos x|\, dx$$

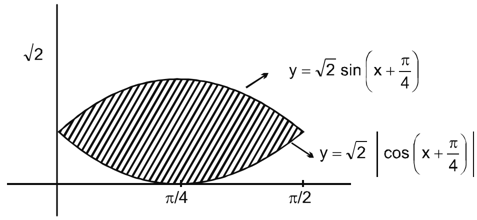
$\therefore \quad = 2\,(-\cos x)_0^{\pi/4} + 2(\sin x)_{\pi/4}^{\pi/2} = 2$

$$\left[-\frac{1}{\sqrt{2}}+1+1-\frac{1}{\sqrt{2}}\right]$$

$$= 2\left(2-\frac{2}{\sqrt{2}}\right)$$

$$= 2\,(2-\sqrt{2})$$

$$= 4 - 2\sqrt{2}$$



$$= 2\sqrt{2}\,(\sqrt{2}-1)$$

60. **(1)**

Sol. $I = \int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17}\, dx$

Put $ln\,(\tan x/2) = t \Rightarrow \tan\frac{x}{2} = e^t$

$\Rightarrow \quad \sin x = \frac{2e^t}{1+e^{2t}}$

$\operatorname{cosec} x = \frac{e^t + e^{-t}}{2}$

$$I = 2\int_{\ell n(\sqrt{2}-1)}^{0} (e^t + e^{-t})^{16}.dt$$

$$= 2\int_{-\ell n(\sqrt{2}+1)}^{0} (e^t + e^{-t})^{16}.dt$$

since$(e^t + e^{-t})^{16}$ is an even function

$$\int_{-a}^{0} = \int_0^a$$

Hence $I = \int_0^{\ell n(\sqrt{2}+1)} 2(e^t + e^{-t})^{16}\, dt$

Integer Type Questions (61 to 75)

61. **(9)**

Sol. $\alpha = \int_0^1 e^{9x+3\tan^{-1}x}.\left(\frac{12+9x^2}{1+x^2}\right) dx$

$\Rightarrow \alpha = \left(e^{9x+3\tan^{-1}x}\right)_0^1$

$\Rightarrow \alpha = e^{9+\frac{3\pi}{4}} - 1$

$\Rightarrow ln\,(1+\alpha) = 9 + \dfrac{3\pi}{4}$

62. **(0)**

Sol. $I = \displaystyle\int_{-1}^{2} \frac{x[x^2]}{2+[x+1]}dx = \int_{-1}^{2} \frac{x[x^2]}{3+[x+1]}dx$

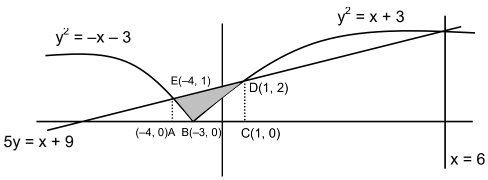
$= \displaystyle\int_{-1}^{0} \frac{0}{3-1}dx + \int_{0}^{1} \frac{0}{3+0}dx + \int_{1}^{\sqrt{2}} \frac{x.1}{3+1}dx$

$= \dfrac{1}{4}\left[\dfrac{x^2}{2}\right]_1^{\sqrt{2}} = \dfrac{2-1}{8} = \dfrac{1}{4}$

$\therefore \quad 4I - 1 = 0$

63. **(3)**

Sol.



Area *ABE* (under parabola) = $\displaystyle\int_{-4}^{-3} \sqrt{-x-3}\ \text{dx} = \frac{2}{3}$

Area *BCD* (under parabola) = $\displaystyle\int_{-3}^{1} \sqrt{x+3}\ \text{dx} = \frac{16}{3}$

Area of trapezium *ACDE* = $\dfrac{1}{2}(1+2)5 = \dfrac{15}{2}$

Required area = $\dfrac{15}{2} - \dfrac{16}{3} - \dfrac{2}{3} = \dfrac{3}{2}$

64. **(2)**

Sol. $\displaystyle\int_{0}^{\pi/2} \sqrt{1+\sin 2x}\ \text{dx} = \int_{0}^{\pi/2} (\sin x + \cos x)dx$

$= \left[-\cos x + \sin x\right]_0^{\pi/2} = 2$

65. **(0)**

Sol. $-\tan 1 < x < 0 \Rightarrow \tan(-1) < x < 0$

$\Rightarrow \quad -1 < \tan^{-1} x < 0$

$\Rightarrow \quad 0 < -\tan^{-1} x < 1$

$\therefore \quad [-\tan^{-1} x] = 0$

66. **(7)**

Sol. $\displaystyle\int_{-1}^{3} |x-2| + [x]\ dx$

$= \displaystyle\int_{-1}^{0} (2-x-1)\ dx + \int_{0}^{1} (2-x)\ dx + \int_{1}^{2} (2-x)+1\ dx + \int_{2}^{3} (x-2+2)\ dx$

$= \left[x - \dfrac{x^2}{2}\right]_{-1}^{0} + \left[2x - \dfrac{x^2}{2}\right]_{0}^{1} + \left[3x - \dfrac{x^2}{2}\right]_{1}^{2} + \left[\dfrac{x^2}{2}\right]_{2}^{3}$

$= -\left(-1-\dfrac{1}{2}\right) + \left(2-\dfrac{1}{2}\right) + \left(6-2\left(3-\dfrac{1}{2}\right)\right) + \dfrac{9}{2} - 2$

$= \dfrac{3}{2} + \dfrac{3}{2} + 4 - \dfrac{5}{2} + \dfrac{5}{2} = 7$

67. **(1)**

Sol. $I = \displaystyle\int_{-1}^{1} \frac{\cot^{-1} x}{\pi}\ dx \Rightarrow \int_{-1}^{1} \frac{\cot^{-1}(-x)}{\pi}\ dx$

$\Rightarrow \displaystyle\int_{-1}^{1} \frac{\pi - \cot^{-1} x}{\pi}\ dx$

$\Rightarrow I = \displaystyle\int_{-1}^{1} 1\ dx - \int_{-1}^{1} \frac{\cot^{-1} x}{\pi}\ dx$

$I = x|_{-1}^{1} - I$

$2I = 1 - (-1) = 2 \Rightarrow I = 1$

68. **(110)**

Sol. $I = \displaystyle\int_{0}^{11} 11^{\{x\}} dx = \int_{0}^{11\times 1} 11^{\{x\}} dx = 11\int_{0}^{1} 11^{\{x\}} dx$

$\{\because \{x\}\ is\ periodic\ with\ period\ 1\}$

$= 11\displaystyle\int_{0}^{1} 11^x dx = 11\left[\frac{11^x}{\ell n 11}\right]_0^1 = 11\left[\frac{11}{\ell n 11} - \frac{1}{\ell n 11}\right]$

$= \dfrac{110}{\ell n 11} = \dfrac{k}{\ell n 11}$

$\Rightarrow k = 110$

69. (40)

Sol. $I = \int_0^{20\frac{\pi}{2}} \left(|\sin x| + |\cos x|\right) dx$

$= 20 \int_0^{\frac{\pi}{2}} \left(\sin x + \cos x\right) dx$

$\left[\because \text{ period of } |\sin x| + |\cos x| = \frac{\pi}{2}\right] = 40$

70. (1)

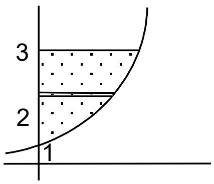
Sol. $A = \int_1^3 x dy$

$A = \int_1^3 \ell ny\, dy$

$= \left[y \ln y - y\right]_1^3$

$= 3\, ln\, 3 - 3 - 0 + 1$

$= 3\, ln\, 3 - 2$



71. (3)

Sol. $I = \int_4^{10} \frac{[(x-14)^2]}{[x^2]+[(x-14)^2]} \; dx$

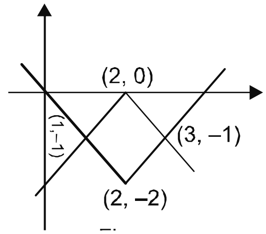
$\Rightarrow 2I = \int_4^{10} dx = 6 \quad \Rightarrow I = 3$

72. (0)

Sol. Using properties $\int_{-1}^{1} \frac{e^x+1}{e^x-1}$ dx = 0

73. (2)

Sol. $|y + 1| = 1 - |x - 2|$



$y = -1 \pm (1 - |x - 2|)$

$y = -|x - 2| \; (y \geq -1)$

$y = -2 - |x - 2| \qquad (y \leq -1)$

Area =

$\frac{1}{2}\left(\begin{vmatrix} 2 & 3 \\ -2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 2 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ -1 & -2 \end{vmatrix}\right)$

(formula from coordinate geometry)

$= \frac{1}{2}$ (4) = 2 Sq. unit

74. (1)

Sol. $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\, dt}{x \sin x}$

$= \lim_{x\to 0} \frac{\tan t|_0^{x^2}}{x \sin x} = \lim_{x\to 0} \frac{\tan x^2}{x^2} \cdot \frac{x}{\sin x} = 1$

75. (1)

Sol. $\int_2^3 f'(x) f''(x)\, dx + \int_1^3 f''(x)\, dx$

$= \frac{\left(f'(x)\right)^2}{2}\Bigg|_2^3 + f'(x)\Big|_1^3$

$= \frac{1}{2}\left[\left(f'(3)\right)^2 - \left(f'(2)\right)^2\right] + f'(3) - f'(1)$

$= \frac{1}{2}\left[\left(\tan\frac{\pi}{4}\right)^2 - \left(\tan\frac{\pi}{3}\right)^2\right] + \tan\frac{\pi}{4} - \tan\frac{\pi}{6}$

$= \frac{1}{2}[1-3] + 1 - \frac{1}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$

DIFFERENTIAL EQUATIONS

Single Option Correct Type Questions (01 to 64)

1. **(1)**

Sol: $\frac{dy}{dx} + 2\frac{y}{x} = 0$

$x^2y = C$ put $x = 1, y = 1$ and we get $C = 1$

put $x = 2 \Rightarrow y = \frac{1}{4}$

2. **(1)**

Sol: $\frac{dy}{1+y^2} + \frac{dx}{\sqrt{1-x^2}} = 0$

$\Rightarrow \tan^{-1} y + \sin^{-1} x = c$

3. **(3)**

Sol: $\frac{ydx - xdy}{y^2} = dx + \frac{dy}{y^2} \Rightarrow d\left(\frac{x}{y}\right) = dx + \frac{dy}{y^2}$

$\Rightarrow \frac{x}{y} = x - \frac{1}{y} + k$

$\Rightarrow x = xy - 1 + ky \Rightarrow (x + 1)(1 - y) = cy$

4. **(4)**

Sol: $(x^2 + y^2)\, dy = xy\, dx$

$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ put $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{v}{1+v^2} \Rightarrow \int \frac{1+v^2}{v^3} dv = -\int \frac{dx}{x}$

$\Rightarrow \ln v - \frac{1}{2v^2} = -\ln x + c$

$\Rightarrow$ put $x = 1$ and $y = 1$

$\therefore c = -\frac{1}{2}$

$\therefore \ln\frac{y}{x} - \frac{1}{2}\frac{x^2}{y^2} = -\ln x - \frac{1}{2}$ put $y = e$

$\therefore x = \sqrt{3}\, e$

5. **(1)**

Sol: $\frac{dx}{dy} = x + y + 1$

$\Rightarrow \frac{dx}{dy} - x - y - 1 = 0$ I.F. $= e^{-\int dy} = e^{-y}$

$(\text{I.F.}) = e^{-\int dy} = e^{-y}$

$\Rightarrow e^{-y}\frac{dx}{dy} - xe^{-y} - ye^{-y} - e^{-y} = 0$

$\Rightarrow \int d\,(xe^{-y}) = \int (e^{-y} + ye^{-y})\, dy$

$\Rightarrow xe^{-y} = -e^{-y} - ye^{-y} + \int e^{-y}\, dy$

$\Rightarrow xe^{-y} = -e^{-y} - ye^{-y} - e^{-y} + C$

$\Rightarrow x = -1 - y - 1 + Ce^{y}$

$\Rightarrow x + y + 2 = Ce^{y}$

6. **(3)**

Sol: $\frac{dy}{dt} - \frac{t}{t+1}y = \frac{1}{t+1}$

$I.F = e^{-\int \frac{t+1-1}{t+1} dt} = e^{-t + \ell n(t+1)} = (t+1)\, e^{-t}$

solution is $(t + 1)e^{-t}\, y = -e^{-t} + c$

put $t = 0$ and $y = -1 \Rightarrow c = 0$

$\therefore 2e^{-1}\, y = -e^{-1}$ put $t = 1$ $y = -\frac{1}{2}$

7. **(2)**

Sol: (1) $\frac{dy_1}{dx} + f(x)\, y_1 = 0 \Rightarrow f(x) = \frac{-1}{y_1}\frac{dy_1}{dx}$

(2) $\frac{dy}{dx} - \frac{1}{y_1}\frac{dy_1}{dx} \cdot y = r(x)$

$e^{-\int \frac{1}{y_1}\frac{dy_1}{dx}dx} = e^{-\int \frac{dy_1}{y_1}} = \frac{1}{y_1}$

$\Rightarrow \frac{y}{y_1} = \int \frac{r(x)\,dx}{y_1} + c$

$y = y_1 \int \frac{r(x)\ dx}{y_1} + cy_1$

8. **(3)**

Sol: $\frac{dy_1}{dx} + fy_1 = r$

$\frac{dy_2}{dx} + fy_2 = r$

$\frac{d}{dx}(y_1 + y_2) + f(y_1 + y_2) = 2r$

here $\frac{dy}{dx} + f(x)y = 2r$

9. **(3)**

Sol: Given DE can be written as $\frac{dy}{dx} - \left(1 + \frac{f'(x)}{f(x)}\right)$

$y = f(x)$

Which is $L.D.E.$

$I.F. = \frac{e^{-x}}{f(x)}$

General solution $y\frac{e^{-x}}{f(x)} = \int f(x)\frac{e^{-x}}{f(x)}dx +$

$c = -e^{-x} + c$

$\Rightarrow y = -f(x) + ce^{x} f(x)$

10. **(1)**

Sol: $(2x - 10y^3)\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{10\ \ y^3 - 2x}$

$\Rightarrow \frac{dx}{dy} = \frac{10\ \ y^3 - 2x}{y}$

$\frac{dx}{dy} = 10\,y^2 - 2\ \frac{x}{y} \Rightarrow \frac{dx}{dy} + \frac{2}{y}\ x = 10\,y^2$

$\Rightarrow xy^2 = 10\ \frac{y^5}{5} + c$

$\Rightarrow xy^2 = 2y^5 + c$

11. **(2)**

Sol: $\sin y\ \frac{dy}{dx} = \cos y\,(1 - x\cos y)$

put $\cos y = t - \sin y\ \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow -\frac{dt}{dx} = t(1 - tx) \Rightarrow \frac{dt}{dx} = t^2x - t$

$\Rightarrow \frac{dt}{dx} + t = t^2x$

$\frac{1}{t^2}\frac{dt}{dx} + \frac{1}{t} = x\ \ \frac{1}{t} = v$

$-\frac{dv}{dx} + v = x \Rightarrow \frac{dv}{dx} - v = -x$

I.F. $= e^{\int -dx} = e^{-x}$

Now solution $v.e^{-x} = \int -xe^{-x}dx + C$

$ve^{-x} = xe^{-x} - \int e^{-x}dx + c$

$ve^{-x} = xe^{-x} + e^{-x} + C$

$\Rightarrow 1/t = x + 1 + Ce^{x}$

12. **(4)**

Sol: $\sec^2 y\ \frac{dy}{dx} + \tan y = 1$ put $\tan y = t$, $\sec^2$

$y\ \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} = 1 - t \Rightarrow ln\,(1 - t) = -c\,x$

$\Rightarrow 1 - t = e^{-cx} \Rightarrow t = 1 - e^{-cx}$

$\Rightarrow \tan y = 1 + ce^{-x}$

13. **(2)**

Sol: $xdy = ydx + y^2\,dy \Rightarrow \frac{x\ \ dy - y\ \ dx}{y^2} = dy$

$\Rightarrow -d\left(\frac{x}{y}\right) = dy$

$-\dfrac{x}{y} = y + c$ put $x = 1$ $y = 1$ $\Rightarrow c = -2$

$-\dfrac{x}{y} = y - 2$ put $y = -3$

$\therefore \quad \dfrac{x}{3} = -5 \Rightarrow x = -15$

14. **(2)**

Sol: $2x^3dx + 2y^3dy - (xy^2\,dx + x^2y\,dy) = 0$

$d\left(\dfrac{x^4}{2}\right) + d\left(\dfrac{y^4}{2}\right) - \dfrac{1}{2}\,d(x^2y^2) = 0$

$\Rightarrow \quad d(x^4 + y^4 - x^2y^2) = 0$

$\Rightarrow \quad x^4 + y^4 - x^2y^2 = c$

15. **(1)**

Sol: $\dfrac{xdy - ydx}{x^2 + y^2} + dx = 0 \Rightarrow \dfrac{\dfrac{xdy - ydx}{x^2}}{1+\left(\dfrac{y}{x}\right)^2} + dx = 0$

$\Rightarrow \quad \dfrac{d\left(\dfrac{y}{x}\right)}{1+\left(\dfrac{y}{x}\right)^2} + dx = 0$

$\Rightarrow \quad d\left(\tan^{-1}\left(\dfrac{y}{x}\right)\right) + dx = 0$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{y}{x}\right) + x = c$

16. **(1)**

Sol: $e^y dx + xe^y dy - 2ydy = 0$

17. **(4)**

Sol: $\tan^{-1}\dfrac{y}{x} = \dfrac{mx^2}{2} + C$

$\dfrac{1}{1+\dfrac{y^2}{x^2}} \cdot \dfrac{\left(x\dfrac{dy}{dx} - y\right)}{x^2} = mx$

$\Rightarrow \quad \dfrac{x^2}{x^2 + y^2} \cdot \dfrac{\left(x\dfrac{dy}{dx} - y\right)}{x^2} = mx$

$\Rightarrow \quad \dfrac{x\dfrac{dy}{dx} - y}{x^2 + y^2} = mx$

statement-1 is false

statement-2 is linear form of differential equation

$I.F. = e^{\int \frac{1}{x}dx} = x$

$\therefore\ y.\,x = \int x\sin x + c$

$xy = -x\cos x + \int \cos x + c$

$xy = -x\cos x + \sin x + c$

$x(y + \cos x) = \sin x + c$

$\therefore$ statement-2 is true.

18. **(4)**

Sol: Here, slope of tangent

$\dfrac{dy}{dx} = \dfrac{(x+1)^2 + y - 3}{(x+1)}$

$\Rightarrow \quad \dfrac{dy}{dx} = (x + 1) + \dfrac{(y-3)}{(x+1)}$, put $x + 1 = X$ and

$y - 3 = Y \left(here\ \dfrac{dy}{dx} = \dfrac{dY}{dX}\right)$

$\therefore \quad \dfrac{dY}{dX} = X + \dfrac{Y}{X} \Rightarrow \dfrac{dY}{dX} - \dfrac{1}{X}\,Y = X$

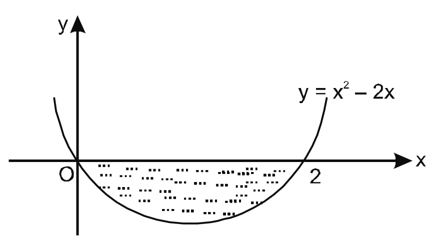
where integrating factor

$= e^{\int -\frac{1}{x}dX} = e^{-\ell n X} = \dfrac{1}{X}$

$\therefore$ Solution is,

$Y.\dfrac{1}{X} = \int X \cdot \dfrac{1}{X}dX + c \Rightarrow \dfrac{Y}{X} = X + c$

$y - 3 = (x + 1)^2 + c(x + 1)$, which passes through (2, 0)



$-3 = 9 + 3c$

$\Rightarrow \quad c = -4$

$\therefore$ Required curve

$y = (x+1)^2 - 4(x+1) + 3$

$\Rightarrow \quad y = x^2 - 2x$

Drawing curve

Thus, required area

$$= \left| \int_0^2 (x^2 - 2x)\; dx \right| = \left| \left(\frac{x^3}{3} - x^2 \right)_0^2 \right|$$

$= \frac{4}{3}$ sq. units

19. **(2)**

Sol: Since $\frac{d^2y}{dx^2} = e^{-2x}$

$\Rightarrow \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$

$\Rightarrow y = \frac{e^{-2x}}{4} + cx + d$

20. **(2)**

Sol: $(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$

$\Rightarrow (1+y^2)\frac{dx}{dy} + x = e^{\tan^{-1}y}$

$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$

$I.F. = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

Therefore , required solution is

$xe^{\tan^{-1}y} = \int e^{\tan^{-1}y}.\frac{e^{\tan^{-1}y}}{1+y^2}dy + k_1$

$\Rightarrow \quad xe^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{1+y^2}.\; dy + k_1$

$\Rightarrow \quad xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + k_1$

$\Rightarrow \quad 2x\; e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

21. **(2)**

Sol: $y\,dx + (x + x^2y)dy = 0$

$\Rightarrow y\,dx + xdy = -x^2y\,dy$

$\Rightarrow \frac{y\;dx + x\;dy}{x^2y^2} = -\frac{1}{y}\,dy$

$\Rightarrow d\left(-\frac{1}{xy}\right) = -\frac{1}{y}\,dy$

On integrating, we get

$-\frac{1}{xy} = -\log y + c$

$\Rightarrow \quad -\frac{1}{xy} + \log y = c$

22. **(2)**

Sol: $x\frac{dy}{dx} = y(\ell n\, y - \ell n\, x + 1)$

$\therefore \frac{dy}{dx} = \left(\frac{y}{x}\right)\left(\ell n\left(\frac{y}{x}\right) + 1\right)$

Now, put $\frac{y}{x} = t \qquad \frac{y}{x} = t$

$\Rightarrow y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\therefore \quad t + x\frac{dt}{dx} = t\;\ell n\, t + t$

$\Rightarrow \quad \frac{dt}{t\,\sin t} = \frac{dx}{x} \qquad \Rightarrow ln\;(ln\;t) = ln\;x + ln\;c$

$\Rightarrow \quad ln\;t = cx$

$\Rightarrow \quad ln\left(\frac{y}{x}\right) = cx$

23. **(4)**

Sol: Given equation can be rewritten as

$\frac{dy}{dx} - \frac{1}{x}.y = 1$

Now, $I.F. = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

$\therefore$ Required solution is $y\left(\frac{1}{x}\right) = \int \frac{1}{x}dx = \log x + c$

Since, $(y) = 1 \;\Rightarrow 1 = \log 1 + c \;\Rightarrow c = 1$

$\therefore \quad y = x\log x + x$

24. **(4)**

Sol: $\cos x\, dy - y \sin x\, dx = -y^2\, dx$

$\cos x\, dy + y d(\cos x) = -y^2\, dx$

$$\frac{d(y\cos x)}{y^2\cos^2 x} = -\frac{dx}{\cos^2 x}$$

$$-\frac{1}{y\cos x} = -\tan x + c$$

$-\sec x = y(-\tan x + c)$

$\sec x = y(\tan x + k)$

25. **(2)**

Sol: $\frac{dv(t)}{dt} = k(T-t)$

$$\int dv(t) = \int(-kT)dt + \int ktdt$$

$$V(t) = -kTt + k\frac{t^2}{2} + c$$

at $t = 0$ $C = I$

$$V(T) = -kTt + \frac{kt^2}{2} + I$$

Now at $t = T$

$$V(T) = -kT^2 + k\frac{T^2}{2} + I$$

$$V(T) = I - \frac{1}{2}kT^2$$

26. **(3)**

Sol: $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

$$I.F. = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$$

$$x.\, e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$$

$$\frac{-1}{y} = t$$

$$\Rightarrow \frac{1}{y^2}dy = dt$$

$$\Rightarrow I = -\int te^t dt = e^t - te^t = e^{-\frac{1}{y}} + \frac{1}{y}e^{-\frac{1}{y}} + c$$

$$\Rightarrow xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y}e^{-\frac{1}{y}} + c$$

$$\Rightarrow x = 1 + \frac{1}{y} + c.e^{1/y}$$

since $y(1) = 1$

$$\therefore c = -\frac{1}{e}$$

$$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$$

27. **(1)**

Sol: $2\frac{dp(t)}{900 - p(t)} = -dt$

$-2\ln(900 - p(t)) = -t + c$

when $t = 0, p(0) = 850$

$-2\ln(50) = c$ $\quad \therefore 2\ln\left(\frac{50}{900-p(t)}\right) = -t$

$900 - p(t) = 50\, e^{t/2}$

$p(t) = 900 - 50\, e^{t/2}$

let $p(t_1) = 0$

$0 = 900 - 50\, e^{\frac{t_1}{2}}$ $\quad \therefore t_1 = 2\ln 18$

28. **(3)**

Sol: $dP = (100 - 12\sqrt{x})dx$

By integrating

$$\int dP = \int\left(100 - 12\sqrt{x}\right)dx$$

$P = 100x - 8x^{3/2} + C$

When $x = 0$ then $P = 2000$

$\Rightarrow C = 2000$

Now when $x = 25$ then P is

$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000$

$= 2500 - 8 \times 125 + 2000$

$= 4500 - 1000$

$\Rightarrow P = 3500$

29. **(3)**

Sol: $p'(t) = \frac{1}{2}p(t) - 200$

$$p'(t) - \frac{1}{2}p(t) = -200$$

$I.F = e^{-\frac{1}{2}t}$

Hence solutions is

$p(t)\ e^{-\frac{1}{2}t} = \int -200e^{-t/2}dt = 400\,e^{-\frac{1}{2}t} + C$

or $p(t) = 400 + Ce^{t/2}$

Since $p(0) = 100$

$\Rightarrow\ 100 = 400 + C \Rightarrow C = -300$

Thus $p(t) = 400 - 300\,e^{t/2}$.

30. (3)

Sol: $y(1 + xy)\,dx = xdy$

$ydx - xdy + xy^2dx = 0$

$y^2d\left(\frac{x}{y}\right) + xy^2dx = 0$

$\frac{x}{y} + \frac{x^2}{2} = C$ ....(i)

(1, –1) satisfies

$-1 + \frac{1}{2} = C \Rightarrow C = -\frac{1}{2}$

Put in (i) $x = -\frac{1}{2}$

(i) $x = -\frac{1}{2}$

$\frac{-\frac{1}{2}}{y} + \frac{\frac{1}{4}}{2} = -\frac{1}{2} \Rightarrow \frac{-1}{2y} = \frac{-1}{2} - \frac{1}{8}$

$\frac{1}{2y} = \frac{5}{8}$

$y = \frac{4}{5}$

31. (1)

Sol: $\frac{dy}{dx} + \cot x\, y = 4x \operatorname{cosec} x$

$I.F. = e^{\int \cot x dx} = \sin x$

$y\,(\sin x) = \int 4x \operatorname{cosec} x . \sin x dx + C$

$y \sin x = 2x^2 + C$

$\because y\left(\frac{\pi}{2}\right) = 0$

$C = \frac{-\pi^2}{2}$

$y \sin x = 2x^2 - \frac{-\pi^2}{2}$

so $y\left(\frac{\pi}{6}\right) = 2\left(\frac{2\pi^2}{36} - \frac{\pi^2}{2}\right) = 2\pi^2\left(\frac{1}{18} - \frac{1}{2}\right)$

$= -\frac{8\pi^2}{9}$

32. (1)

Sol: $\lim\limits_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1 \ \left(\frac{0}{0}\ form\right.$

using L. Hospital $\lim\limits_{t\to x} \frac{2t\,f(x) - x^2 f'(t)}{1} = 1$

(differentiating with respect to t)

So $2x\,f(x) - x^2 f'(x) = 1$

$y = f(x)$

So $2xy - x^2\frac{dy}{dx} = 1$

$x^2\frac{dy}{dx} - 2xy = -1$

$\frac{dy}{dx} - \frac{2y}{x} = \frac{-1}{x^2}$

This is linear differential equation.

Integrating factor = $e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

So $\frac{y}{x^2} = \int \frac{-1}{x^4}dx$

$\frac{y}{x^2} = \frac{1}{3x^3} + c \qquad \therefore\ f(1) = 1 \Rightarrow\ c = \frac{2}{3}$

$y = \frac{1}{3x} + \frac{2}{3}x^2$

33. (3)

Sol: $\int \frac{y}{\sqrt{1-y^2}}\,dy = x + c$

$\Rightarrow\ -\sqrt{1-y^2} = x + c$

$\Rightarrow\ 1 - y^2 = (x + c)^2$

$\Rightarrow\ (x + c)^2 + y^2 = 1$

34. (2)

Sol: $I.F. = e^{\int \frac{x}{x^2-1}dx} = e^{\frac{1}{2}\int \frac{2x}{x^2-1}dx} = e^{\frac{1}{2}\ell n|x^2-1|}$

$= e^{\frac{1}{2}\ell n(1-x^2)} = \sqrt{1-x^2}$

$\therefore \quad y\sqrt{1-x^2} = \int \frac{x^4+2x}{\sqrt{1-x^2}} \times \sqrt{1^2-x^2}\,dx + c$

$y\sqrt{1^2-x^2} = \frac{x^5}{5} + x^2 + c$

$x = 0, y = 0 \Rightarrow c = 0$

$y = \frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}}$

$\therefore \quad I = \int_0^{\frac{\sqrt{3}}{2}} \left(\frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}} + \frac{\frac{-x^5}{5}+x^2}{\sqrt{1-x^2}} \right) dx$

$= 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}} dx$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$= 2\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta\cos\theta}{\cos\theta} d\theta$

$= \int_0^{\frac{\pi}{3}} (1-\cos 2\theta)\,d\theta = \left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\frac{\pi}{3}}$

$= \frac{\pi}{3} - \frac{1}{2} \times \sin\frac{2\pi}{3} = \frac{\pi}{3} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

35. (3)

Sol: order-4, degree-15

36. (4)

Sol: The degree of differential equation is not defined.

37. (2)

Sol: $\frac{dy}{dx} = \cot x \cot y$

$\int \tan y\,dy = \int \cot x\,dx$

$ln \sec y = ln \sin x + ln\, c$

$\sec y = c \sin x$

38. (4)

Sol: $\frac{dy}{dx} = \frac{y^2-y-2}{x^2+2x-3}$

$\frac{dy}{(y-2)(y+1)} = \frac{dy}{(x+3)(x-1)}$

$\Rightarrow \int \frac{dy}{(y-2)(y+1)} = \int \frac{dy}{(x+3)(x-1)}$

$\Rightarrow \frac{1}{3}\int\left(\frac{1}{y-2} - \frac{1}{y+1}\right)dy = \frac{1}{4}\int\left(\frac{1}{x-1} - \frac{1}{x+3}\right)dx$

$\Rightarrow \frac{1}{3}\ell n\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\ell n\left|\frac{x-1}{x+3}\right| + c$

39. (2)

Sol: $\frac{dy}{dx} = \ell n(x+1)$

$\int dy = \int \ell n(x+1)dx$

$y = x\, ln\,(x+1) - \int \frac{x}{x+1}dx$

$= x\, ln\,(x+1) - \int (1 - \frac{1}{x+1})dx$

$= x\, ln\,(x+1) - x + ln\,(x+1) + c$

Now at $x = 0, y = 3$

$3 = 0 + c \Rightarrow c = 3$

$\therefore y = (x+1)\, ln\,|x+1| - x + 3$

40. (2)

Sol: $\frac{dy}{dx} = e^{x+y} \Rightarrow \frac{dy}{dx} = e^x.e^y$

$\Rightarrow \int e^{-y}dy = \int e^x dx$

$\Rightarrow -e^{-y} = e^x + c$

$\Rightarrow e^x + e^{-y} = c$

41. (3)

Sol: $\frac{dy}{dx} = (2x+y)^2$

Let $2x + y = t$

$2 + \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} - 2 = t^2$

$\Rightarrow \frac{dt}{t^2+2} = dx$

$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} = x + c$

$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{2x+y}{\sqrt{2}} \right) = x + c$

42. (1)

Sol: $\frac{dx}{dy} = x + y - 3 \Rightarrow \frac{dx}{dy} - x = y - 3$

$I.F = e^{-\int dy} = e^{-y}$

solution of differential equation is

$xe^{-y} = \int (y-3)e^{-y} dy + c$

$\Rightarrow x = ce^{y} - y + 2$

43. (1)

Sol: $\frac{dy}{dx} = \frac{y/x}{1 - 2\sqrt{y/x}}$

Let $y = vx \Rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v$

$\therefore x\frac{dv}{dx} = \frac{v}{1-2\sqrt{v}} - v = \frac{2v^{3/2}}{1-2\sqrt{v}}$

$\Rightarrow \frac{1-2\sqrt{v}}{2v^{3/2}} dv = \frac{dx}{x}$

on integration

$-v^{-1/2} - \ell n v = \ell n x + c$

or $-\sqrt{\frac{x}{y}} - \ell n y + \ell n x = c + \ell n x$

or $\ell n y + \sqrt{\frac{x}{y}} = c$

44. (4)

Sol: $\sin\left(\frac{y}{x}\right)\frac{dy}{dx} = \frac{y}{x}\sin\frac{y}{x} - 2$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\sin v\left(v + x\frac{dv}{dx}\right) = v \sin v - 2$

$x \sin v \frac{dv}{dx} = -2$

$\sin v\, dv = -2 \frac{dx}{x}$

$-\cos v = -2\, ln\, x + c \Rightarrow \cos v = 2\, ln\, x + c$

$\Rightarrow \cos \frac{y}{x} = ln\, x^2 + c$

45. (1)

Sol: $\frac{dx}{dy} + \frac{x^2 - xy + y^2}{y^2} = 0$

$\Rightarrow \frac{dx}{dy} + \left(\frac{x}{y}\right)^2 - \left(\frac{x}{y}\right) + 1 = 0$

Put $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

Now $v + y\frac{dv}{dy} + v^2 - v + 1 = 0$

$\Rightarrow \frac{dv}{v^2+1} + \frac{dy}{y} = 0$

$\Rightarrow \int \frac{dv}{v^2+1} + \int \frac{dy}{y} = 0$

$\Rightarrow \tan^{-1}(v) + ln\, y + c = 0$

$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right) + ln\, y + c = 0$

46. (3)

Sol: $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$

$e^{-y}\frac{dy}{dx} + \frac{1}{x}e^{-y} = \frac{1}{x^2}$

Put $e^{-y} = t \Rightarrow -e^{-y}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow -\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2} \Rightarrow \frac{dt}{dx} - \frac{t}{x} = -\frac{1}{x^2}$

$I.F. = e^{\int -\frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$\Rightarrow \frac{t}{x} = \int -\frac{1}{x^3} \Rightarrow \frac{t}{x} = \frac{1}{2x^2} + c$

$\Rightarrow \frac{e^{-y}}{x} = \frac{1}{2x^2} + c \Rightarrow 2xe^{-y} = cx^2 + 1$

47. (2)

Sol: $\frac{dy}{dx} = \frac{y}{2y\ell ny + y - x} \Rightarrow \frac{dx}{dy} + \frac{x}{y} = (2\ell ny + 1)$

$I.F.\ e^{\int \frac{1}{y}dy} = e^{\ell ny} = y$

solution is $yx = \int (2\ell ny + 1)ydy + c$

$xy = y^2 \ln y + c$

48. (2)

Sol: $\frac{dx}{dy} = -\left(\frac{1 + x(1+y^2)}{y + y^3}\right)$

$= -\left[\frac{1}{y(y^2+1)} + \frac{x(1+y^2)}{y(1+y^2)}\right]$

$\frac{dx}{dy} + \frac{x}{y} = \frac{-1}{y(1+y^2)}$

$I.F. = e^{\int \frac{1}{y}dy} = y$

The solution is

$xy = -\int \frac{1}{y(1+y^2)} .y\, dy + c$

$xy = -\tan^{-1}y + c$

49. (2)

Sol: $\frac{1}{x^3}\frac{dx}{dy} + \frac{1}{yx^2} = 1$

Let $\frac{1}{x^2} = t \Rightarrow -\frac{2}{x^3}\frac{dx}{dy} = \frac{dt}{dy}$

$-\frac{1}{2}\frac{dt}{dy} + \frac{t}{y} = 1$ or $\frac{dt}{dy} - \frac{2}{y} t = -2$

$I.F. = e^{-\int \frac{2}{y}dy} = e^{-2\ell ny} = \frac{1}{y^2}$

Solution is, $t\left(\frac{1}{y^2}\right) = -2\int \frac{1}{y^2}dy + c$

$\frac{t}{y^2} = \frac{2}{y} + c$

$2x^2 y + c\, x^2y^2 = 1$

50. (1)

Sol: $\frac{dy}{dx} = \frac{2xy}{x^2 - 1 - 2y} \Rightarrow x^2dy - (1 + 2y)\, dy = 2xydx$

or $2xydx - x^2dy = -(1 + 2y)\, dy$

or $\frac{yd(x^2) - x^2dy}{y^2} = -\left(\frac{1}{y^2} + \frac{2}{y}\right)dy$

$d\left(\frac{x^2}{y}\right) = -\left(\frac{1}{y^2} + \frac{2}{y}\right)dy$

Integrating, $\frac{x^2}{y} = \frac{1}{y} - 2\ell n\, y + c$

51. (2)

Sol: $xy^4dx + ydx = xdy$

$xdx + \frac{ydx - xdy}{y^4} = 0$

or $x^3dx + \left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right) = 0$

integration, $\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 = c$

$\Rightarrow 3x^4y^3 + 4x^3 = cy^3$

52. (3)

Sol: $\frac{xdx + ydy}{(x^2+y^2)^2} = \left(\frac{ydx - xdy}{y^2}\right) \times \frac{y^2}{x^2}$

or $\int \frac{d(x^2+y^2)}{(x^2+y^2)^2} = -2\int d\left(\frac{y}{x}\right)$

$-\frac{1}{(x^2+y^2)} = -2\frac{y}{x} + c$ or $\frac{2y}{x} = \frac{1}{x^2+y^2} + c$

53. (1)

Sol: $\frac{dv}{dt} = -k(4\pi r^2)$

Put $v = \frac{4}{3}\pi r^3$ or $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

Hence $\frac{dr}{dt} = -k$

54. (3)

Sol: Order = Highest order derivative present in D.E.

$= 2 = m$ (given)

Degree = Highest power of highest order derivative which is $\frac{d^2y}{dx^2}$

$= 1 = n$

$\Rightarrow \quad m > n$

55. (3)

Sol: $\frac{dy}{dx} = \frac{xy+y}{xy+x} = \frac{y(x+1)}{x(y+1)}$

$\Rightarrow \quad \left(\frac{x+1}{x}\right)dx = \left(\frac{y+1}{y}\right)dy$

$\Rightarrow \quad \left(1+\frac{1}{x}\right)dx = \left(1+\frac{1}{y}\right)dy$

Integrating both side, we get

$\Rightarrow \quad x + \log x = y + \log y + \log c$

$\Rightarrow \quad x - y + \log x = \log y\, c$

$\Rightarrow \quad x - y = \log\frac{cy}{x}$

$\Rightarrow \quad e^{x-y} = \frac{cy}{x}$

$cy = xe^{x-y}$

$\Rightarrow \quad y = \frac{1}{c}x.e^{x-y}$

$= c_1 xe^{x-y}$

56. (4)

Sol: $x^2 {y_1}^2 + xy\, y_1 - 6y^2 = 0$

It is quadratic equation in y_1

$y_1 = \frac{-xy \pm \sqrt{x^2y^2 + 24y^2x^2}}{2x^2} = \frac{-xy \pm 5xy}{2x^2}$

$y_1 = -\frac{3y}{x} \quad y_1 = \frac{2y}{x}$

$\frac{dy}{dx} = \frac{-3y}{x} \qquad \frac{dy}{dx} = \frac{2y}{x}$

$-\frac{dy}{y} = 3\frac{dx}{x} \qquad \frac{dy}{dx} = \frac{2y}{x}$

$-\ln y = 3\ln x + \ln c \qquad \ln y = 2\ln x + \ln c$

$x^3y = C \qquad y = cx^2$

$\ln y = c + 2\ln x$

$\frac{1}{2}\ln y = \ln c_1 + \ln x$

57. (1)

Sol: $\frac{dm}{dx} = m$

$\ln m = x \quad \Rightarrow m = c_1e^x$

$\frac{dy}{dx} = c_1e^x$

$y = c_1e^x + c_2$

58. (2)

Sol: Let $t = \frac{d^2y}{dx^2}$

$\frac{dt}{dx} = 8t$

$\ln t = 8x + c$ here at $x = 0, c = 0$

$t = e^{8x} \Rightarrow \frac{d^2y}{dx^2} = e^{8x}$

now put $m = \frac{dy}{dx}$

$\frac{dm}{dx} = e^{8x}$

$m = \frac{e^{8x}}{8} + c_1$

at $x = 0$, $c_1 = \frac{-1}{8} \quad x = 0 \ c_1 = \frac{-1}{8}$

$\frac{dy}{dx} = \frac{e^{8x}}{8} - \frac{1}{8}$

$y = \frac{e^{8x}}{64} - \frac{x}{8} + c_2$

at $x = 0$, $c_2 = \frac{7}{64}$

$\therefore y = \frac{e^{8x}}{64} - \frac{x}{8} + \frac{7}{64} \Rightarrow 64y = e^{8x} - 8x + 7$

59. (1)

Sol: $y_1 y_3 = 3y_2^2$

$\frac{y_3}{y_2} = 3 \frac{y_2}{y_1}$

$\Rightarrow \ ln\ y_2 = 3\ ln\ y_1 + ln\ c$

$y_2 = c{y_1}^3$

$\frac{y_2}{y_1^2} = cy_1$

$-\frac{1}{y_1} = cy + c_2$

$\frac{dx}{dy} = -cy - c_2$

$x = -\frac{cy^2}{2} - c_2 y + c_3$

$\therefore x = A_1 y^2 + A_2 y + A_3$

60. (1)

Sol: $\frac{xdx - ydy}{xdy - ydx} = \sqrt{\frac{1+x^2-y^2}{x^2-y^2}}$

$x = r\sec\theta$

$y = r\tan\theta$

$x^2 - y^2 = r^2$

$dx = r\sec\theta\tan\theta\, d\theta + \sec\theta\, dr$

$dy = r\sec^2\theta\, d\theta + \tan\theta\, dr$

$\Rightarrow \ \frac{r\ d\ r}{r^2 \sec\theta\ d\theta} = \sqrt{\frac{1+r^2}{r^2}}$

$\Rightarrow \ \int \frac{dr}{\sqrt{1+r^2}} = \int \sec\theta\ d\theta$

$\Rightarrow \ ln\ |r + \sqrt{1+r^2}\ | = ln\ |\sec\theta + \tan\theta| + ln\ c$

$\Rightarrow \ r + \sqrt{1+r^2} = c(\sec\theta + \tan\theta)$

on solving $\sqrt{x^2-y^2} + \sqrt{1+x^2-y^2}$

$= \frac{c\ (x+y)}{\sqrt{x^2-y^2}}$

61. (2)

Sol: $\frac{dy}{dx} = \frac{(x+y)+1}{2(x+y)+1}$

put $x + y = t$, $1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \ \frac{dt}{dx} = \frac{t+1}{2t+1} + 1 = \frac{3t+2}{2t+1}$

$\Rightarrow \ \int \frac{2t+1}{3t+2}\ \text{dt} = \int dx$

or $\frac{2t}{3} - \frac{1}{9}\ ln\ (3t+2) = x + c$

or $6(x + y) - ln\ (3x + 3y + 2) = 9x + c$

or $ln\ (3x + 3y + 2) = 6y - 3x + c$

since it passes throw (0, 0) hence equation of curve is $6y - 3x = ln \left|\frac{3x+3y+2}{2}\right|$

62. (4)

Sol: $e^{\left(d^3y/dx^3\right)^2} = 1 + \left(\frac{d^3y}{dx^3}\right)^2 + \frac{\left(\frac{d^3x}{dx^3}\right)^4}{2\ !} + ...$

here power of highest order is not defined, hence degree is not defined.

63. (1)

Sol: $y\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y\frac{dy}{dx} - x = 0$

$y\frac{dy}{dx}\left(\frac{dy}{dx} - 1\right) + x\left(\frac{dy}{dx} - 1\right) = 0$

$\left(y\frac{dy}{dx} + x\right)\left(\frac{dy}{dx} - 1\right) = 0$

$\therefore$ either $ydy + xdx = 0$ or $dy - dx = 0$

since the curves pass through the point (3, 4)

$\therefore x^2 + y^2 = 25$ or $x - y + 1 = 0$

64. **(3)**

Sol: $\frac{dy}{dx} = 2xe^{x^2-y} = 2xe^{x^2} \cdot e^{-y}$

$= \frac{2xe^{x^2}}{e^y}$

$\Rightarrow \quad e^y dy = 2xe^{x^2}$

$\Rightarrow \quad \int e^y dy = \int 2x \cdot e^{x^2} dx + c$

[Put $x^2 = z \;\Rightarrow 2x\, dx = dz$]

$\Rightarrow \quad e^y = e^z + c$

$\Rightarrow \quad e^y = e^{x^2} + c$

Integer Type Questions (65 to 73)

65. **(7)**

Sol: $\frac{dy}{dx} = y + 3$

$\frac{dy}{y+3} = dx$

$ln\,(y + 3) = x + c$

given at $x = 0$, $y = 2$

$x = 0 \;\; y = 2$

$ln\, 5 = c$

$\therefore\; ln\,(y + 3) = x + ln\, 5$

$ln\left(\frac{y+3}{5}\right) = x$

$y + 3 = 5e^x$

$y = 5e^x - 3$

$\therefore\; y(ln\, 2) = 5e^{\,ln2} - 3 = 7$

66. **(2)**

Sol: $\frac{dy}{dx} + \frac{y}{x\log x} = 2$

$I.F. = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

$y(\log x) = \int 2(\log x)dx$

$y(\log x) = 2[x \log x - x] + c$

at $x = 1$, $\quad c = 2$

$x = e$

$y = 2(e - e) + 2$

$y = 2$

67. **(3)**

Sol: $\frac{dy}{dx} + \frac{\cos x}{(2+\sin x)}\, y = \frac{-\cos x}{2+\sin x}$

on solving

$y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

68. **(2)**

Sol: $\frac{dy}{dt} - \left(\frac{t}{1+t}\right) y = \frac{1}{1+t}$

on solving

$y(t) = \frac{-1}{1+t}$

69. **(1)**

Sol: $\frac{dv}{dt} \propto -s$ and $s = \pi r^2$

$\because \frac{r}{h} = \tan\theta$

By solving

$T = \frac{H}{k}$

70. **(2)**

Sol: Let $x + y + 1 = t^2$

$1 + \frac{dy}{dx} = \frac{2tdt}{dx}$

$\left(\frac{2tdt}{dx} - 1\right) t = t^2 - 2$

$\frac{dt}{dx} = \frac{t^2 + t - 2}{2t^2}$ or $\int \frac{2t^2}{t^2 + t - 2} dt = \int dx$

or $\int \left[1 + \frac{1}{3(t-1)} - \frac{4}{3(t+2)}\right] dt = \int dx$

or $\quad 2\left[t + \frac{1}{3}\ell n\,|\,t-1\,| - \frac{4}{3}\ell n\,|\,t+2\,|\right] = x + c$

where $t = \sqrt{x+y+1}$

71. **(7)**

Sol: $(2x + y + 4)\, dy = (x - 2y + 3)\, dx$

$2(xdy + ydx) + ydy + 4dy = xdx + 3\ dx$

$2d(xy) + ydy + 4dy = xdx + 3dx$

on intergrating

$$2xy + \frac{y^2}{2} + 4y = \frac{x^2}{2} + 3x + c$$

72. **(4)**

Sol: $y\sin x = \int \sin^2 x dx$

$$\Rightarrow y\sin x = \frac{1}{4}(2x - \sin 2x) + c$$

73. **(9)**

Sol: $Y - y = m\ (X - x)$

Y-intercept (X = 0)

$Y = y - mx$

Given that $y - mx = x^3 \Rightarrow x\ \frac{dy}{dx} - y = -x^3$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = -x^2$$

Integrating factor ($I.F.$) = $e^{-\int \frac{1}{x} dx} = \frac{1}{x}$

solution $y.\ \frac{1}{x} = \int \frac{1}{x}\ .\ (-x^2)\ dx$

$$\Rightarrow f(x) = y = -\frac{x^3}{2} + cx$$

Given $f(1) = 1 \Rightarrow c = \frac{3}{2}$

$$\therefore f(x) = -\frac{x^3}{2} + \frac{3x}{2} \Rightarrow f(-3) = 9$$

PROBABILITY

Single Option Correct Type Questions (01 to 62)

1. **(1)**

Sol. Max sum = 12

$$\left.\begin{aligned}6+6=12\\6+5=11\\6+4=10\\5+5=10\end{aligned}\right\} \text{6 cases}$$

$$P = \frac{6}{36} = \frac{1}{6} = \frac{1}{6}$$

2. **(3)**

Sol. $P(A) = \frac{1}{3}$ $P(\text{B}) = \frac{1}{4}$ and $P(C) = \frac{1}{5}$

Required probability

$= P(A\bar{B}\bar{C} \text{ or } \bar{A}B\bar{C} \text{ or } \bar{A}\bar{B}C)$

$= P(A).\ P(\bar{B}).\ P(\bar{C}) + P(\bar{A})\ P(B)\ P(\bar{C}) + P(\bar{A})\ P(\bar{B})\ P(C)$

$= \frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}$

$= \frac{12+8+6}{60} = \frac{13}{30}$

3. **(1)**

Sol. Given $f(x) = x^3 + 6ax^2 + 2bx + c$

$f'(x) = 3x^2 + 12ax + 2b$

for increasing function, $f'(x) \geq 0$

$\therefore\quad (12a)^2 - 4\cdot3\cdot2\ b \leq 0$

$6a^2 \leq b$

$\therefore\quad a = 1, b = 6$

where $c \in \{1, 2, ... 6\}$

$\therefore$ required probability $= \frac{6}{6^3} = \frac{1}{36}$

4. **(1)**

Sol. ${}^4C_1 \times {}^{13}C_9 \times {}^{39}C_4$ = Formula card

Suit any 9 cards any 4 cards from 39 cards

${}^{52}C_{13}$ = total case

5. **(4)**

Sol. 1st coupon can be selected in 9 ways

2nd coupon can be selected in 9 ways

3rd coupon can be selected in 9 ways

9^7 ways – when 9 is not take

$= 9^7 - 8^7$

Total $= 15^7$.

6. **(1)**

Sol. $$\frac{\dfrac{2n-2!}{n-1!\ n-1!2!}\times 2!}{\dfrac{2n!}{n!\ n!2!}} = P = \frac{n}{2n-1}$$

7. **(4)**

Sol. Favourable no. of ways = 12

total no. of ways = 220

$$P = \frac{12}{220} = \frac{3}{55}$$

8. **(4)**

Sol. Let A: card is spade

B: card is an ace.

$P(1) = \frac{13}{52}$ $P(2) = \frac{4}{52}$

$P(A \cap B) = \frac{1}{52}$

9. **(1)**

Sol. $P(\text{atleast one } W) = P(1W, 1M) + P(2W_1, 0M)$

$$= \frac{5\times8}{{}^{13}C_2}+\frac{{}^5C_2}{{}^{13}C_2}$$

10. **(2)**

Sol. $P(A) = \frac{3}{11}$

$P(B) = \frac{2}{7}$

$P(C) = P$

Now, $P(A) + P(B) + P(C) = 1$

$\frac{3}{11} + \frac{2}{7} + P = 1 \Rightarrow P = 1 - \frac{43}{77} = \frac{34}{77}$

odds against $C = 43:34$

11. **(3)**

Sol. Let x, y, z be probability of E_1, E_2, E_3 respectively

$\Rightarrow x(1-y)(1-z) = \alpha$

$\Rightarrow y(1-x)(1-z) = \beta$

$\Rightarrow z(1-x)(1-y) = \gamma$

$\Rightarrow (1-x)(1-y)(1-z) = P$

Putting in the given relation we get $x = 2y$ and

$y = 3z \Rightarrow x = 6z \Rightarrow \frac{x}{z} = 6$

12. **(1)**

Sol. odd —— 1, 3, 5.

$p\text{ (prime/ odd)} = \frac{2}{3}$

13. **(3)**

Sol. $2 + 6 = 8,\ p = \frac{1}{5}$

$3 + 5 = 8$

$4 + 4 = 8$

$5 + 3 = 8$

$6 + 2 = 8$

14. **(3)**

Sol. $P(S_1).\ P(\bar{S}_2)\ .\ P(\bar{S}_3) + P(\bar{S}_1)\,P(S_2)\ P(\bar{S}_3)$

$+ P(\bar{S}_1)\ P(\bar{S}_2)\ P(S_3)$

$= \frac{1}{3} \times \frac{5}{7} \times \frac{5}{8} + \frac{2}{3}.\frac{2}{7}.\frac{5}{8} + \frac{2}{3}.\frac{5}{7}.\frac{3}{8} = \frac{25}{56}$

15. **(2)**

Sol. 'R' be the event that selected marble is red

$P\left(\frac{R}{A}\right) = \frac{3}{5}, P\left(\frac{R}{B}\right) = \frac{5}{9}$

$P(A) = \frac{1}{2}, P(B) = \frac{1}{2}$

$$P\left(\frac{B}{R}\right) = \frac{P(B).P\left(\frac{R}{B}\right)}{P(B).P\left(\frac{R}{B}\right) + P(A).P\left(\frac{R}{A}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{5}{9}+\frac{1}{2}\times\frac{3}{5}} = \frac{\frac{5}{18}}{\frac{5}{18}+\frac{3}{10}}$$

$$= \frac{5\times10}{50+54} = \frac{50}{104} = \frac{25}{52}$$

16. **(1)**

Sol. $P(\bar{M} \cap \bar{N}\ ) = 1 - P(M \cup N)$

$= 1 - P(M) - P(N) + P(M)\ P(N)$

$= (1 - P(M))\ (1 - P(N)) = \overline{P(M)}\ \overline{P(N)}$

$P(M \cap\ ) = P(M) - P(M \cap N) = P(M) - P(M)$

$P(N) = P(M)\ P()$

and $P\left(\frac{M}{N}\right) + P\left(\frac{\bar{M}}{N}\right)$

$= \frac{P(M\cap N)}{P(N)} + \frac{P(N) - P(M\cap N)}{P(N)} = 1$

17. **(3)**

Sol. $p(\text{I}^{\text{st}}\text{ class}) = \frac{2}{3} \times 0.\ 28 + \frac{1}{3} \times 0.\ 25 = 0.\ 27$

18. **(4)**

Sol. Required probability

$$= \frac{^5C_1}{^{12}C_1} \times \frac{^4C_1}{^{12}C_1} + \frac{^7C_1}{^{12}C_1} \times \frac{^8C_1}{^{12}C_1} = \frac{76}{144}$$

19. **(2)**

Sol. $P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$

$\frac{1}{2}P(E_1) = P(E_1 \cap E_2)$ ... (i)

Similarly, $\frac{1}{4}P(E_2) = P(E_1 \cap E_2)$ ... (ii)

From (i) and (ii), $P(E_2) = \frac{1}{2}$

and $P(E_1 \cap E_2) = \frac{1}{8}$

$\therefore P(E_1 \cap E_2), P(E_1), P(E_2)$ are in G.P.

20. **(1)**

Sol. $U_1 - 1W + 1B$ $\quad U_2 \to 2W + 3B$

$U_3 \to 3W + 5B$ $\quad U_4 \to 4W + 7B$

$P(W) = \sum_{i=1}^{4}(u_1)\ P(w/u_i) = \sum_{i=1}^{4}\frac{i^2+1}{34}\ P(w/v_i)$

$= \frac{1^2+1}{34}\times\frac{1}{2}+\frac{2^2+1}{34}\times\frac{2}{5}+\frac{3^2+1}{34}\times\frac{3}{8}+\frac{4^2+1}{34}\times\frac{4}{11}$

$= \frac{569}{1496}$

21. **(3)**

Sol. LHS is an integer.

$\Rightarrow$ RHS must be an integer for P is a multiple of 30.

i.e., $P = 30, 60, 90, \ldots\ldots., 990$.

22. **(3)**

Sol. Since sum of 1 + 2 + 3 +......9 = $\frac{9\times10}{2}$ = 45 is divisible by 9, hance all no. will be divisible by 9.

23. **(3)**

Sol. $\frac{6.7.5}{^{18}C_3} = \frac{35}{136}$

24. **(1)**

Sol. Favourable case: (3,3,3,3) or (3,3,3,5)

$= 1+\frac{4!}{3!} = 5$

total number of way $\to 2^4$

$p = \frac{5}{2^4}$

25. **(3)**

Sol. Required probability = 1 – both numbers are not divisible by 5 = $1 - \frac{8}{10}\times\frac{8}{10} = \frac{9}{25}$

26. **(3)**

Sol. KRISHNAGIRI or DHARMAPURI

$A = RI$ is visible

B_1 = its from KRISHNAGIRI

B_2 = its from DHARMAPURI

$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1)+P(A/B_2)P(B_2)}$

$= \frac{\frac{1}{2}\times\frac{2}{10}}{\frac{1}{2}\times\frac{2}{10}+\frac{1}{2}\times\frac{1}{9}} = \frac{9}{14}$

27. **(1)**

Sol. $\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\left(\frac{13}{52}\right)^2\times\frac{6!}{2!2!2!} = P$

28. **(4)**

Sol. E_1 = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

E_2 = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

E_3 = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

clearly (1), (2) and (3) are correct.

29. **(3)**

Sol. $P(C/D) = \frac{P(C\cap D)}{P(D)}$

$P(C/D) = \frac{P(C)\cdot P(D/C)}{P(D)}$ .... (i)

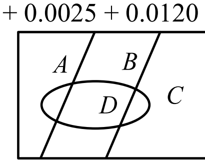
$P(D) = P(A\cap D)+P(B\cap D)+P(C\cap D)$

$= P(A)\cdot P(D/A)+P(B)\cdot P(D/B)$

$+P(C)\cdot P(D/C)$

= (0.35) (0.02) + (0.25) (0.01) + (0.40) (0.03)

= 0.0070 + 0.0025 + 0.0120 = 0.0215



$P(C/D) = \frac{0.0120}{0.0215} = \frac{120}{215} = \frac{24}{43}$

30. **(2)**

Sol. 10 coins 95 paisa 10 coins 5 paisa 11 Rs.

$p = p$ (1 Rs. transferred + Back transferred) + p (1 Rs. not transferred) $\frac{{}^{9}c_8}{{}^{10}c_9} \times \frac{{}^{18}c_8}{{}^{19}c_9} + \frac{{}^{9}c_9}{{}^{10}c_9}$

$= \frac{10}{19}$

31. **(1)**

Sol. $p(A) = \frac{13}{52} + \left(\frac{39}{52}\right)^3\left(\frac{13}{52}\right) + \left(\frac{39}{52}\right)^6\left(\frac{13}{52}\right) + ...$

$p(B) = \left(\frac{39}{52}\right)\frac{13}{52} + \left(\frac{39}{52}\right)^4 \frac{13}{52} + \left(\frac{39}{52}\right)^7\left(\frac{13}{52}\right) + ...$

$p(C) = \left(\frac{39}{52}\right)^2\left(\frac{13}{52}\right) + \left(\frac{39}{52}\right)^5\left(\frac{13}{52}\right) + ...$

32. **(3)**

Sol.

$\frac{(11-4)(11-5)}{(11-1)(11-2)} = \frac{7}{15}$

33. **(4)**

Sol. $\frac{2}{5} = (1-P)P + (1-P)^3 P + (1-P)^5 p + ...$

$\frac{2}{5} = P(1-P)\{1 + (1-P)^2 + (1-P)^4 + ...\}$

$\frac{2}{5} = P(1-P)\left[\frac{1}{1-(1-P)^2}\right]$

$\frac{2}{5} = P(1-P)\left[\frac{1}{P(2-P)}\right]$

$3P = 1 \Rightarrow P = 1/3$

34. **(3)**

Sol. **Statement - 2:** True (By definition)

Statement -1: False because the sample points are not equally likely.

35. **(1)**

Sol. Statement-2 $P(A/B) = P(A) \Leftrightarrow P(A)$

$= \frac{P(A \cap B)}{P(B)} \Leftrightarrow P(A)\,P(B) = P(A \cap B)$

A and B are mutually exclusive

$\Rightarrow P(A \cap B) = 0$

$\Rightarrow P(B)\,P(A/B) = 0 \Rightarrow \quad P(A/B) = 0$

$\Rightarrow P(A/B) \neq P(A)$

$\therefore$ statement-2 is true

Statement-1 Suppose A and B are mutually exclusive, then by statement-2 $P(A/B) \neq P(A)$ which is a contradiction.

$\therefore$ statement-1 is true.

36. **(2)**

Sol. Required probability $= \frac{5}{25}\cdot\frac{4}{24} + \frac{20}{25}\cdot\frac{5}{24} = \frac{1}{5}$

37. **(2)**

Sol. The total number of ways in which numbers can be chosen = 25 × 25 = 625

The number of ways in which either players can choose same numbers = 25

$\therefore$ Probability that they with a prize

$= \frac{25}{625} = \frac{1}{25}$

Thus, the probability that they will not win a prize in a single trial $= 1 - \frac{1}{25} = \frac{24}{25}$

38. **(2)**

Sol. Let A_1, A_2 and A_3 be the events of match winning in first, second and third match respectively. And whose probabilities are

$P(A_1) = P(A_2) = P(A_3) = \frac{1}{2}$

$\therefore$ Required probability

$= P(A_1 A'_2, A_3) + P(A'_1, A_2 A_3)$

$= P(A_1)\, P(A'_2)\, P(A_3) + P(A'_1)\, P(A_2)\, P(A_3)$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}.$

39. (3)

Sol. Here,

$P(A)$ = probability that A will hit $B = \frac{2}{3}$

$P(B)$ = probability that B will hit $A = \frac{1}{2}$

$P(C)$ = probability that C will hit $A = \frac{1}{3}$

$P(E)$ = probability that A will be hit

$P(E) = 1 - P(\bar{B}).\, P(\bar{C}) = 1 - \frac{1}{2}.\frac{2}{3} = \frac{2}{3}$

Probability if A is hit by B and not by C.

$\Rightarrow P(B \cap \bar{C} / E)$

$$\Rightarrow \frac{P(B).P(\bar{C})}{P(E)} = \frac{\frac{1}{2}.\frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$$

40. (4)

Sol. The probability that Mr. A selected the loosing horse $= \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$

The probability that Mr. A selected the winning horse $= 1 - \frac{3}{5} = \frac{2}{5}$

41. (1)

Sol. Since, $0 \le P(\text{A}) \le 1$, $0 \le P(\text{B}) \le 1$, $0 \le P(\text{C}) \le 1$ and $0 \le P(\text{A}) + P(\text{B}) + P(\text{C}) \le 1$

$\therefore\ 0 \le \frac{3x+1}{3} \le 1$

$\Rightarrow\ -\frac{1}{3} \le x \le \frac{2}{3}$ ...(i)

$0 \le \frac{1-x}{4} \le 1$

$\Rightarrow\ -3 \le x \le 1$ ...(ii)

$0 \le \frac{1-2x}{2} \le 1$

$\Rightarrow\ -\frac{1}{2} \le x \le \frac{1}{2}$ ...(iii)

and $0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$\Rightarrow\ 0 \le 13 - 3x \le 12$

$\Rightarrow\ \frac{1}{3} \le x \le \frac{13}{3}$ ………...(iv)

From Eqs. (i), (ii), (iii) and (iv), we get

$\frac{1}{3} \le x \le \frac{1}{2}$

42. (3)

Sol. Given probabilities of speaking truth are

$P(A) = \frac{4}{5}$ and $P(B) = \frac{3}{4}$

And their corresponding probabilities of not speaking truth are

$P(\bar{A}) = \frac{1}{5}$ and $P(\bar{B}) = \frac{1}{4}$

The probability that they contradict each other

$= P(\text{A}) \times P(\bar{B}) + P(\bar{A}) \times P(\text{B})$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4}$

$= \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$

43. (2)

Sol. Given that,

$P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{1}{4}$ and $P(\overline{A \cup B}) = \frac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = \frac{1}{6} \Rightarrow 1 - P(\text{A}) - P(\text{B}) + P(A \cup B) = \frac{1}{6} \Rightarrow P(\bar{A}) - P(\text{B}) + \frac{1}{4} = \frac{1}{6}$

$\Rightarrow P(\text{B}) = \frac{1}{4} + \frac{1}{4} - \frac{1}{6}$

$\Rightarrow P(\text{B}) = \frac{1}{3}$ and $P(\text{A}) = \frac{3}{4}$

Now, $P(A \cap B) = \frac{1}{4} = \frac{3}{4} \times \frac{1}{3} = P(A)\, P(B)$.

Hence, the events A and B are independent events but not equally likely.

44. **(3)**

Sol. All the three persons has three options to apply a house.

$\therefore$ Total number of cases = 3^3

Now, favourable cases = 3 (An either all has applied for house 1 or 2 or 3)

$\therefore$ Required probability = $\frac{3}{3^3} = \frac{1}{9}$.

45. **(4)**

Sol. $P\left(\frac{A \cap B}{\bar{A} \cup \bar{B}}\right) = P\left(\frac{A \cap B}{\overline{A \cap B}}\right) = 0$

46. **(2)**

Sol. Given that, $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$

and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

we know, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ ...(i)

and $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$ ...(ii)

$\therefore P(B) = \frac{P\left(\frac{B}{A}\right).P(A)}{P\left(\frac{A}{B}\right)} = \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$

47. **(3)**

Sol. $\because A = \{4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$

$\therefore A \cap B = \{4\}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1.$

48. **(4)**

Sol. Case in which sum of digits in 8 are 08, 17, 26, 35, 44

Total cases: 00, 01, 02, ..., 09, 10, 20, 30, 40

Required probability = $\frac{1}{14}$

49. **(2)**

Sol. Statement-1 Total ways = ${}^{20}C_4$

number of $AP's$ of common difference 1 is = 17

number of $AP's$ of common difference 2 is = 14

number of $AP's$ of common difference 3 is = 11

number of $AP's$ of common difference 4 is = 8

number of $AP's$ of common difference 5 is = 5

number of $AP's$ of common difference 6 is = 2

total = 57

probability = $\frac{57}{{}^{20}C_4} = \frac{1}{85}$

Statement-2 common difference can be ± 6, so statement -2 is false

50. **(1)**

Sol. $= \frac{{}^3C_1\, {}^4C_1\, {}^2C_1}{{}^9C_3} = \frac{3\,.\,4\,.\,2}{\frac{9\,.\,8\,.\,7}{3\,.\,2\,.\,1}} = \frac{2}{7}$

51. **(2)**

Sol. Let Event (Given: {1, 2, 3,.........8})

A: Maximum of three numbers is 6.

B: Minimum of three numbers is 3

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$

52. **(2)**

Sol. Let E_1 = the event of getting 5 in a roll of two dice

= {(1, 4), (2, 3), (3, 2), (4, 1)}

$\therefore P(E_1) = \frac{4}{36} = \frac{1}{9}$

Let E_2 = the event of getting either 5 or 7

= {(1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$\therefore \; P(E_2) = \frac{10}{36} = \frac{5}{18}$

$\therefore \; P(\bar{E}_2) = \frac{13}{18}$

Now, the probability of getting 5 before 7

$= P(E_1) + P(\bar{E}_2 E_1) + P(\bar{E}_2 \bar{E}_2 E_1) + \ldots$

$= \frac{1}{9} + \frac{13}{18}.\frac{1}{9} + \frac{13}{18}.\frac{13}{18}.\frac{1}{9} + \ldots$

$= \frac{1}{9}\left[1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + \ldots \text{to } \infty\right]$

$= \frac{1}{9}\left[\frac{1}{1 - \frac{13}{18}}\right] = \frac{1}{9}.\frac{18}{5} = \frac{2}{5}$

53. (3)

Sol. E_1: {(4, 1) ,............... (4,6,)}

E_2: {(1,2), ... (6,2)}

E_3: 18 cases (sum of both are odd)}

$P(E_1) = \frac{6}{36} = \frac{1}{6} = P(E_2)$

$P(E_3) = \frac{18}{36} = \frac{1}{2}$

$P(E \cap E_2) = \frac{1}{36}$

$P(E_2 \cap E_3) = \frac{1}{12}$

$P(E_3 \cap E_1) = \frac{1}{12}$

$P(E_1 \cap E_2 \cap E_3) = 0$

$\therefore$ E_1, E_2, E_3 are not independent

54. (2)

Sol. $P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

$P(A) + P(C) - 2P(A \cap C) = \frac{1}{4}$

$P(A \cap B \cap C) = \frac{1}{16}$

$\therefore\; P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$

55. (1)

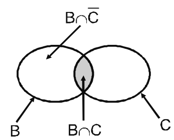
Sol. $P = \frac{6}{{}^{11}C_2} = \frac{6}{55}$

56. (1)

Sol. We have $P(B \cap \bar{C}) = P[(A \cup \bar{A}) \cap (B \cap \bar{C})]$

$= P(A \cap B \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C})$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$



$P(B \cap C) = P(B) - P(B \cap \bar{C}) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

57. (4)

Sol. Such numbers are 6, 12, 18, ... 96 i.e. 16 such numbers. Hence required probability $= \frac{{}^{16}C_3}{{}^{100}C_3}$

$= \frac{16 \times 15 \times 14}{100 \times 99 \times 98} = \frac{4}{1155}$

58. (3)

Sol. If A: Indian Men sit with their wife

B: American men sit with their wife

$P\left(\frac{A}{B}\right) = \frac{P\ \ A \cap B}{P(B)} = \frac{n\ \ A \cap B}{n(B)}$

$= \frac{4!\ \ 2!^{\ 5}}{5!(2!)^4} = \frac{2}{5}$

59. (3)

Sol. $x + y + z = 10$

Total number of non-negative solutions

$= {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Now Let $z = 2n$.

$x + y + 2n = 10; n \geq 0$

Total number of non-negative solutions

$= 11 + 9 + 7 + 5 + 3 + 1 = 36$

Required probability $= \frac{36}{66} = \frac{6}{11}$

60. **(3)**

Sol. $P(B / A \cup B)^c)) = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$

$= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$

$= \frac{P(A) - P(A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)}$

$= \frac{0.7 - 0.5}{0.8} = \frac{1}{4}$

61. **(2)**

Sol. Total number of ways in which 8 person can speak is ${}^8P_8 = 8!$

The number of ways which can be arranged in the specified speaking order is ${}^8C_3$. There are 5! ways in which the other five can speak. So, favourable number of ways is ${}^8C_3 \times 5!$

Required probability $= \frac{{}^8C_3 \times 5!}{8!}$

$= \frac{8! \times 5!}{3! \times 5! \times 8!} = \frac{1}{6}$

62. **(2)**

Sol. Unit digit of $3^a = 3, 9, 7, 1$

Unit digit of $2^b = 2, 4, 8, 6$

3^a	2^b
3	8

3^a	2^b
9	2

3^a	2^b
7	4

$\Rightarrow \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100} + \frac{25}{100} \times \frac{25}{100}$

$= \frac{3}{16}$

Integer Type Questions (63 to 72)

63. **(1)**

Sol. Required probability $= \frac{m}{n} = \frac{24}{120} = \frac{1}{5}$

64. **(5)**

Sol. $P(A) = \frac{1}{3},\ P(B) = \frac{2}{3}$

$\Rightarrow P(\bar{A} \cap B) = \frac{5}{12}$

65. **(4)**

Sol. P(hitting the target at least once) > 0.99

$\Rightarrow 1 - \underbrace{\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \dots \frac{1}{4}}_{n \text{ times}} > 0.99$

$\Rightarrow 1 - \left(\frac{1}{4}\right)^n > 0.99$

$\Rightarrow 4^n > \frac{1}{0.01}$

$\Rightarrow 4^n > 100$

So minimum value of n to satisfy the inequality is 4.

66. **(49)**

Sol. $b^2 < 4c$

$$\Rightarrow \begin{cases} c = 1, b = 1 \\ c = 2, b = 1, 2 \\ c = 3, b = 1, 2, 3 \\ c = 4, b = 1, 2, 3 \\ c = 5, 6, b = 1, 2, 3, 4 \\ c = 7, 8, 9, b = 1, 2, 3, 4, 5 \end{cases}$$

Total cases = 32

$\Rightarrow$ Required probability $= \frac{32}{81}$

67. **(5)**

Sol. $\frac{7}{12} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$ solve for n of get

$\Rightarrow n = 5$

68. **(2)**

Sol. $p = \frac{4}{12} \cdot \frac{3}{11} \cdot 2$

69. **(3)**

Sol.

Unit digit in number	**Unit digit in number**	**Unit digit in product**
Odd	Odd	Odd
Odd	Even	Even
Even	Odd	Even
Even	Even	Even

$p = \frac{3}{4} \Rightarrow q = \frac{1}{4} \Rightarrow \frac{p}{q} = 3$

70. **(8)**

Sol. $\frac{6}{10} \times \frac{5}{9} + \frac{4}{10} \times \frac{6}{9} = \frac{54}{90} = \frac{3}{5}$

71. **(2)**

Sol. Even integers ends in 0, 2, 4, 6, 8. Square of an even integer ends in 4 only when the integer ends either in 2 or 8.

$\therefore$ probability $= \frac{2}{5}$

72. **(27)**

Sol. If 6 boxes are selected from 8 boxes then there are only two ways in which one row remains empty.

So, required probability $= \frac{{}^{8}C_{6} - 2}{{}^{8}C_{6}} = \frac{13}{14}$

STATISTICS

Single Option Correct Type Questions (01 to 66)

1. **(1)**

Sol. $\dfrac{1+2+4.....2^n}{n+1}$

$= \dfrac{(2^{n+1}-1)}{2-1} \cdot \dfrac{1}{n+1}$

2. **(1)**

Sol. Mean $= \dfrac{\sum_{i=1}^{n} A+iB}{n} = \dfrac{A.n+B\dfrac{n\ \ n+1}{2}}{n}$

Mean $= A + B\ \dfrac{n+1}{2}$

3. **(3)**

Sol. Geometric mean

$\dfrac{M\ \ x_1, x_2, x_3.....x_n}{M\ \ y_1, y_2, y_3.....y_n}$

$= \dfrac{x_1.x_2.x_3......x_n{}^{1}{}_{n}}{y_1.y_2.y_3......y_n{}^{1}{}_{n}} = \left(\dfrac{x_1}{y_1}.\dfrac{x_2}{y_2}.....\dfrac{x_n}{y_n}\right)^{\frac{1}{n}}$

$= \text{M}\left(\dfrac{x_1}{y_1}, \dfrac{x_2}{y_2}, \dfrac{x_3}{y_3}, ..., \dfrac{x_n}{y_n}\right)$

$\Rightarrow$ While in case of 1 & 2 it is not true always

4. **(3)**

Sol. Frequency of $f = {}^{10}C_5$ which has maximum value

5. **(1)**

Sol. Mean of 21 observation $\bar{x} = 40$, so

Sum of numbers = 21 × 40 = 840

$\Rightarrow$ As numbers greater then median increased by 21, so 10 observations will increase by 21.

Now sum of all observations

= 840 + 10×(21) = 1050

$\Rightarrow$ So now new mean is $= \dfrac{1050}{21} = 50$

6. **(1)**

Sol. 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

median $= \dfrac{46+48}{2} = 47$

$\sum | x_i - M | = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 16 + 23 = 86$

so mean deviation about median $= \dfrac{86}{10} = 8.6$

7. **(4)**

Sol. $n = 88$

Median $= \dfrac{44^{th}\text{ value} + 45^{th}\text{ value}}{2}$

$= \dfrac{56+57}{2} = 56.5$

M.D.(median) $= \dfrac{\sum_{i=1}^{88} |x_i - 56.5|}{88}$

$= \dfrac{43.5+42.5+...+0.5+0.5+...+43.5}{88}$

$= \dfrac{1+3+5+....\ \ +85+87}{88} = 22$

8. **(2)**

Sol. If x_1 ..... x_n are the observations then the new observations are (1.02) $x_1,\ldots,(1.02)x_n$

Therefore the new mean = (1.02) $\bar{x}$

New mean deviation

$$= \frac{1}{n}\sum|(1.02)x_i-(1.02)\bar{x}|$$

$$= (1.02)\ \frac{1}{n}\sum|x_i-\bar{x}|$$

$$= (1.02) \times 50 = 51$$

9. **(1)**

Sol. If x_1 ..... x_n are the observations then the new observations are (0.95) $x_1,\ldots,$ (0.95) x_n

Therefore the new mean = (0.95) $\bar{x}$

New mean deviation

$$= \frac{1}{n}\sum|(0.95)x_i-(0.95)\bar{x}|$$

$$= (0.95)\ \frac{1}{n}\sum|x_i-\bar{x}|$$

$$= (0.95) \times 80 = 76$$

10. **(4)**

Sol. $\bar{x} = \frac{1}{2n+1}\ [a+(a+d)+.....+(a+2nd)]$

$$= \frac{1}{2n+1}\ [(2n+1)\,a + d\,(1+2+.....+2n)]$$

$$= a + d\ \frac{2n}{2}\ \frac{(1+2n)}{2n+1} = a + nd$$

M.D. from mean = $\frac{1}{2n+1}\ 2|d|\ (1+2+....+n)$

$$= \frac{n(n+1)\,|d|}{(2n+1)}$$

11. **(1)**

Sol. See properties of AM

12. **(4)**

Sol. $\sum_{i=1}^{20}(x_i-30)=20 \Rightarrow \sum_{i=1}^{20}x_i-20\times30=20$

$$\sum_{i=1}^{20}x_i = 600+20=620$$

Mean = $\frac{620}{20} = 31$ Ans.

13. **(1)**

Sol. $\bar{x} = \frac{\sum_{i=1}^{20} n}{20} = \frac{21}{2}$, $\frac{\sum_{i=1}^{20} n^2}{20} = \frac{287}{2}$

$$\Rightarrow\ \sigma^2 = \frac{\sum_{i=1}^{20} n^2}{20} - \left(\frac{\sum_{i=1}^{20} n}{20}\right)^2$$

$$= \frac{287}{2} - \left(\frac{21}{2}\right)^2 = \frac{133}{4}$$

14. **(2)**

Sol. Let two observations are x and y

then $\frac{x+y+2+4+10+12+14}{7} = 8$

$x + y + 42 = 56 \Rightarrow x + y = 14$ ...(A)

and $\frac{x^2+y^2+4+16+100+144+196}{7}$

$$-\frac{(x+y+42)^2}{49} = 16$$

$$\Rightarrow\ \frac{x^2+y^2+460}{7} = 16 + 64 = 80$$

$\Rightarrow$ $x^2 + y^2 = 560 - 460 = 100$ ...(B)

$\therefore$ on solving (A) & (B) we get $x = 6$, $y = 8$

15. **(1)**

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \bar{x}^{\,2}$

$$\sigma^2 = \frac{1560}{10} - \sqrt{12}^{\,2} = 144$$

$$\Rightarrow\ \text{S.D.} = \sqrt{\sigma^2} = 12$$

16. **(3)**

Sol. Coefficient of variation = 0.58 = $\frac{\sigma}{\bar{x}}$

σ(S.D.) = .58 × 4 = 2.32

17. **(2)**

Sol. $\sum_{i=1}^{10}(x_i - 50)^2 = 250$

$\sigma = \sqrt{\frac{\sum_{i=1}^{10}(x_i-50)^2}{10}} = 5$

coeff. of variation = $\frac{\sigma}{\bar{x}} \times 100 = 10\%$

18. **(3)**

Sol. We known that if $d_1 = \frac{x_i + 3/2}{-1/2}$ so $h = -\frac{1}{2}$.

Thus $\sigma_d = \frac{1}{|h|}\ \sigma_x = 2 \times 3.5 = 7$

19. **(2)**

Sol. We known that $y_i = \frac{3}{2}x_i$ so $h = \frac{3}{2}$

Thus $\sigma_y = |h|\ \sigma_x = 1.5 \times 9 = 13.5$

20. **(2)**

Sol. $\frac{^{2n+1}C_0 + \ldots\ldots\ ^{2n+1}C_n}{n+1} = \frac{2^{2n}}{n+1}$

21. **(2)**

Sol. $\bar{x} = \frac{0.^nC_0 + 1\ .^nC_1 + 2^nC_2 + \ldots\ldots + n^nC_n}{^nC_0 + ^nC_1 + ^nC_2 + \ldots\ldots + ^nC_n}$

$= \frac{n.2^{n-1}}{2^n} = \frac{n}{2}$

22. **(1)**

Sol.

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	1	0	0
1	9	9	9
2	7	14	28
3	5	15	45
4	3	12	48
	$\sum f_i = 25$	$\sum f_i x_i = 50$	$\sum f_i x_i^2 = 130$

$$\sigma^2 = \left[\frac{\sum f_i\ x_i^2}{\sum f_i} - \left(\frac{\sum f_i\ x_i}{\sum f_i}\right)^2\right]$$

$$= \left[\frac{130}{25} - \left(\frac{50}{25}\right)^2\right] = 1.2$$

So, variance of $A = 1.2 < 1.25$ = variance of B

so more consistent team = A

23. **(3)**

Sol. $\frac{\Sigma x_i}{200} = 25,\ \frac{\Sigma y_i}{300} = 10$

$\Rightarrow \Sigma x_i = 5000,\ \Sigma y_i = 3000$

$\sigma_x = 3$ and $\sigma_y = 4$

$\Rightarrow \frac{\Sigma x_i^2}{200} - (25)^2 = 9$

and $\frac{\Sigma y_i^2}{300} - (10)^2 = 16$

$\Rightarrow \Sigma x_i^2 = 126800$ and $\Sigma y_i^2 = 34800$

$\therefore\ \sigma = \frac{\Sigma z_i^2}{n} - \left(\frac{\Sigma z_i}{n}\right)^2$

$= \frac{\Sigma(x_i^2 + y_i^2)}{500} - \left(\frac{\Sigma x_i + \Sigma y_i}{500}\right)^2$

$= \frac{161600}{500} - \left(\frac{8000}{500}\right)^2 = 67.2$

24. **(2)**

Sol. $\sigma_x = 3$

$\Rightarrow \frac{\sum x_i^2}{100} - \bar{x}^2 = 9$

$\Rightarrow \sum x_i^2 = 23400$

$\sum z_i = 250 \times 15.6 = 3900$

$\therefore \quad \sum y_i = \sum z_i - \sum x_i = 3900 - 1500 = 2400$

$\sigma_z^2 = 13.44$

$\Rightarrow \quad \frac{\sum x_i^2 + \sum y_i^2}{250} - (15.6)^2$

$= 13.44$

$\Rightarrow \quad \sum y_i^2 = 40800$

$\Rightarrow \quad \sigma_y$

$= \sqrt{\frac{\sum y_i^2}{150} - \left(\frac{\sum y_i}{150}\right)^2} = \sqrt{\frac{40800}{150} - \left(\frac{2400}{150}\right)^2} = 4$

25. **(4)**

Sol. $\bar{x} = 60, \quad \bar{y} = 40$

$(\sigma_x^2) = 16, (\sigma_y^2) = 36$

$\sigma_x^2 = \frac{\Sigma x_1^2}{10} -, \quad \bar{x}^2 \quad \sigma_y^2 = \frac{\Sigma y^2}{10} - \bar{y}^2$

$\Sigma x_1^2 = 160 + (60)^2 - 10$

$\Sigma y_1^2 = 360 + (40)^2.10$

$\sigma^2(\text{overall}) = \frac{\Sigma x_1^2 + \Sigma y_1^2}{20} - \left(\frac{10\bar{x} + 10\bar{y}}{20}\right)^2$

$= \frac{520 + 52000}{20} - (50)^2$

$\sigma^2 = 2626 - 2500 = 126$

$\text{S.D.} = +\sqrt{\sigma^2} = 11.2$

26. **(1)**

Sol. Mean $\bar{x} = 4$, variance = 5.2

$a_1, a_2, a_3 = 1, 2, 3.$

Let x_1, x_2 are remaining values

Mean $\bar{x} = \frac{a_1 + a_2 + a_3 + x_1 + x_2}{5}$

$\Rightarrow \quad x_1 + x_2 = 11 \qquad ....(1)$

variance $\sigma^2 = 5.2 = \frac{a_1^2 + a_2^2 + a_3^2 + x_1^2 + x_2^2}{5}$

$- \bar{x}^2 \Rightarrow x_1^2 + x_2^2 = 65 \qquad ....(2)$

$\Rightarrow \quad |x_1 - x_2| = 3$

$\Rightarrow$ So $\lambda = 11 \Rightarrow 10 - x^2 - 2x = \lambda$

$\Rightarrow \quad (x + 1)^2 = 0$ one solution

27. **(1)**

Sol. Let x_n misread value $(x_n) = 10 \; (x_n)_{\text{actual}} = 12$

$\sigma^2 = 3.3 \; \bar{x} = 11.3$

$\Rightarrow \quad \sum_{i=1}^{n-1} x_i = 113 - 10 = 103 = 10. \quad \bar{x} - 10$

$\sigma^2 = \frac{\sum_{i=1}^{n-1} x_i^2 + x_n^2}{10} - \bar{x}^2$

$\sum_{i=1}^{n-1} x_i^2 = -67 + 10 \; \bar{x}^2 \qquad ...(1)$

$\Rightarrow \quad (\sigma^2)_{\text{actual}} = \frac{\sum_{i=1}^{n} x_1^2 + x_n^2{}_{\text{actual}}}{10} - \bar{x}_{\text{actual}}$

$\Rightarrow \quad (\sigma^2)_{\text{actual}}$

$= \frac{\sum_{i=1}^{n} x_1^2 + x_n^2{}_{\text{actual}}}{10} - \bar{x}_{\text{actual}}$

$= \frac{-67 + 10 \; \bar{x}^2 + 144}{10} - \left(\frac{10 \; \bar{x} - 10 + 12}{10}\right)$

$= (\sigma^2_{\text{actual}}) = 3.14$

28. **(1)**

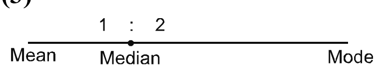
Sol. $\text{S.D.}(x_i) = \text{S.D.}(x_i - 8)$

$= \sqrt{\frac{\sum (x_i - 8)^2}{n} - \left(\frac{\sum (x_i - 8)}{n}\right)^2}$

$= \sqrt{\frac{45}{9} - 1} = 2$

29. **(3)**

Sol.



so median = 22

$= \frac{1 \times \text{mode} + 2 \times 21}{3}$, mode = 24

30. (1)

Sol. Variances remain uneffected by adding some constant to all observations

so $V_A = V_B$

so $V_A/V_B = 1$

31. (3)

Sol. Let no. of student = 100 number of boys = n,

$$\frac{n\times 52+(100-n)\times 42}{100} = 50 \Rightarrow n = 80$$

so 80%

32. (1)

Sol. $$\frac{a+b+8+5+10}{5} = 6 \Rightarrow a + b = 7 \quad ...(1)$$

$$\frac{(a-6)^2+(b-6)^2+2^2+1^2+4^2}{5} = 6.80$$

$\Rightarrow$ $(a - 6)^2 + (b - 6)^2 = 13$

solve $a = 3, b = 4$

33. (4)

Sol. **Statement-1:** $\dfrac{\sum n^2}{n} - \left(\dfrac{\sum n}{n}\right)^2$

$$= \frac{(n+1)\ \ (2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}$$

Statement-2: Obvious

34. (3)

Sol. $$\bar{x} = \frac{1+(1+d)+(1+2d)+(1+100d)}{101}$$

$= 1 + 50d$

Mean deviation = $\dfrac{\sum_{i=0}^{100}|x_i-\bar{x}|}{101}$

$$= \sum\frac{|(1+id)-(1+50d|}{101}$$

$$= \sum_{i=0}^{100}\frac{|(i-50)\ \ d|}{101}$$

$$=225\Rightarrow \frac{(50+49+1+0+1+50)|d|}{101}$$

$$= 225 \Rightarrow \frac{50\times 51}{101}.|d| = 225 \Rightarrow |d| = 10.1$$

35. (1)

Sol. $$\sigma_x^2 = 4 \Rightarrow \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = 4$$

$$\Rightarrow \frac{\sum x_i^2}{5} - (2)^2 = 4 \Rightarrow \sum x_i^2 = 40$$

similarly, $\sum y_i^2 = 105$

$$\therefore\ \sigma^2 = \frac{\sum x_i^2+\sum y_i^2}{10} - \left(\frac{\sum x_i + \sum y_i}{10}\right)^2$$

$$= \frac{145}{10} - \left(\frac{10+20}{10}\right)^2 = 5.5$$

36. (1)

Sol. Correct mean = observed mean + 2

30 + 2 = 32

Correct S.D. = observed S.D. = 2

37. (4)

Sol. A.M. of $2x_1, 2x_2 ..... 2x_n$ is $\dfrac{2x_1+2x_2+....+2x_n}{n}$

$$= 2\left(\frac{x_1+x_2+.....+x_n}{n}\right) = 2\bar{x}$$

So statement-2 is false

variance $(2x_i) = 2^2$ variance $(x_i) = 4\sigma^2$

so statement-1 is true.

38. (4)

Sol. If initially all marks were x_i then σ_1^2

$$= \frac{\sum(x_i-\bar{x})^2}{N}$$

Now each is increased by 10

$$\sigma_2^2 = \frac{\sum\ \left[(x_i+10)\ -(\bar{x}+10)\right]^2}{N} = \sigma_1^2$$

So variance will not change whereas mean, median and mode will increase by 10.

39. **(2)**

Sol. First 50 even natural numbers

Mean = $\dfrac{\sum_{i=1}^{50} 2n}{50} = \dfrac{2+4+6+.....+100}{50}$

$= 2\left(\dfrac{50\times 51}{2\times 50}\right) = 51$

variance = $\dfrac{\sum_{i=1}^{50} 2n^{\ 2}}{50} - 51^2$

$= \dfrac{2^2+4^2+......+100^2}{50} - 2601 = 833$

40. **(4)**

Sol. $\dfrac{x_1+x_2........x_{16}}{16} = 16$

If $x_1 = 16$

$= \dfrac{x_1+x_2........x_{10}-16+3+4+5}{18}$

$= \dfrac{16\times 16-16+12}{18} = \dfrac{240+12}{18} = \dfrac{252}{18} = 14$

41. **(1)**

Sol. Standard deviation of numbers 2, 3, a and 11 is 3.5

$\therefore \quad (3.5)^2 = \dfrac{\sum x^2_i}{4} - (\overline{x})^2$

$\Rightarrow \quad (3.5)^2 = \dfrac{4+9+a^2+121}{4} - \left(\dfrac{2+3+a+11}{4}\right)^2$

on solving, we get

$3a^2 - 32a + 84 = 0$

42. **(1)**

Sol. Let the number of men be x

$\dfrac{70x+55(150-x)}{150} = 60 \qquad x = 50$

43. **(3)**

Sol. $\overline{x}_{new} = \overline{x}_{old} - 5$

44. **(3)**

Sol. We known that $y_i = \dfrac{100}{60} x_i = \dfrac{5}{3} x_i$

so $h = \dfrac{5}{3}$ Thus $\sigma_y = |h|\ \sigma_x = \dfrac{5}{3} \times 5 = \dfrac{25}{3}$

so new variance = $\left(\dfrac{25}{3}\right)^2 = \dfrac{625}{9}$

45. **(2)**

Sol. variance $(x_i) = \dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2$

$1^2 + 2^2 + 3^2$ ....10^2

$= \dfrac{n(n+1)(2n+1)}{6} = \dfrac{10\times 11\times 21}{6} = 385$

1 + 2 + 3 10

$= \dfrac{n(n+1)}{2} = \dfrac{10 \quad 11}{2} = 55$

var(xi) = $\dfrac{385}{10} - \left(\dfrac{55}{10}\right)^2 = \dfrac{825}{100} = \dfrac{33}{4}$

46. **(3)**

Sol. 4, 8, 12, 17, 19, 23, 27, $\Rightarrow$ Median = 17

47. **(4)**

Sol. Most frequent data = 3

48. **(2)**

Sol. Mode + 2 mean = 3 median

(mean –3) + 2 mean = 3 median

3(mean – median) = 3

49. **(4)**

Sol. Most frequent data

50. **(3)**

Sol. $S_n = \dfrac{n}{2}$ [2 + (n–1)3]

$\overline{x} = \dfrac{S_n}{n} = \dfrac{1}{2}$ [3n–1]

51. **(3)**

Sol. variance $(ax_i + b) = a^2$ var(x_i)

52. **(4)**

Sol. Variance $(x_i) = \dfrac{\sum_{i=1}^{n} x_i - \overline{x}^{\ 2}}{n}$

53. (2)

Sol. $\frac{\Sigma x_i}{7} = 7$

$2 + 4 + 7 + 11 + 10 + a + b = 49$

$\Rightarrow a + b = 15$

$\frac{\Sigma(x_i - \bar{x})^2}{7} = \frac{100}{7}$

$\Rightarrow 25 + 9 + 0 + 16 + 9 + (7 - a)^2 + (7 - b)^2 = 100$

$\Rightarrow (7 - a)^2 + (7 - b)^2 = 41$

$\Rightarrow a = 3$

$b = 12$

54. (3)

Sol. Standard deviation is independent of change of origin bu not scale.

55. (3)

Sol. Actual data is more close to mean, therefore less variance

56. (3)

Sol. $\text{Var}(3x_i + 4) = 9\sigma^2$

57. (3)

Sol. Arranging the data in ascending order 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

median = $\frac{46+48}{2} = 47$

$\Sigma|x_i - 47|$

$= 9 + 23 + 1 + 13 + 5 + 8 + 16 + 1 + 7 + 3$

$= 86.$

Mean deviation = $\frac{\Sigma|x_i - 47|}{10} = 8.6$

58. (2)

Sol. $\bar{x} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$

$$\Rightarrow \sigma^2 = \left(\frac{\sum_{i=1}^{n} x_i^2}{n}\right) - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2 - 1}{12}$$

59. (3)

Sol. Arrange marks in ascending order

(................), 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29

8 boys failed

median = $\frac{18+19}{2} = 18.5$

60. (4)

Sol. $\frac{\sigma_1}{\bar{x}_1} = 0.6$; $\frac{\sigma_2}{\bar{x}_2} = 0.7$

$\frac{\sigma_1}{\sigma_2} = \frac{0.6 \times \bar{x}_1}{0.7 \times \bar{x}_2}$

$\Rightarrow \frac{21}{14} \times \frac{0.7}{0.6} = \frac{\bar{x}_1}{\bar{x}_2} \Rightarrow \frac{7}{4} = \frac{\bar{x}_1}{\bar{x}_2}$

61. (2)

Sol. Mean and median both will be increased by 2

62. (4)

Sol. $\frac{2 \times 1 + 14 \times 2 + 8 \times 5 + 32 \times 7}{2 + 14 + 8 + 32}$

$= \frac{294}{56} = 5.25$

63. (1)

Sol. We have: $\Sigma X = a\,\Sigma U + b\Sigma V$. Therefore,

$\bar{X} = \frac{1}{n}\sum X = a.\frac{1}{n}\sum U + \text{b}.\frac{1}{n}\sum V$

$= \text{a}\bar{U} + \text{b}\bar{V}$

64. (2)

Sol. Let the n-numbers be $x_1, x_2, \ldots, x_n$ then,

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$\Rightarrow \bar{X} = \frac{x_1 + x_2 + \ldots x_{n-1} + x_n}{n}$

$\Rightarrow \bar{X} = \frac{k + x_n}{n}$ $\quad [\because x_1 + x_2 + \ldots\ldots x_{n-1} = k]$

$\Rightarrow x_n = n - \bar{X}\text{k}$

65. **(4)**

Sol. Let n_1 and n_2 be the number of observations in two groups having means $\overline{X}_1$ and $\overline{X}_2$ respectively. Then

$$\overline{X} = \frac{n_1\overline{X} + n_2\overline{X}_2}{n_1 + n_2}$$

Now $\overline{X} - \overline{X}_1$

$$= \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2} - \overline{X}$$

$$= \frac{n_2(\overline{X}_2 - \overline{X}_1)}{n_1 + n_2} > 0 \qquad [\because \overline{X}_2 - \overline{X}_1\ ]$$

$\Rightarrow \overline{X}_1 - \overline{X}_2$ .... (i)

And $\overline{X} - \overline{X}_2$

$$= \frac{n(\overline{X}_1 - \overline{X}_2)}{n_1 + n_2} < 0 \qquad [\because \overline{X}_2 > \overline{X}_1\ ]$$

$\Rightarrow \overline{X} < \overline{X}_2$ .... (ii)

From (i) and (ii) $\overline{X}_1 < \overline{X} < \overline{X}_2$.

66. **(1)**

Sol. Let σ_1 and σ_2 be the standard deviations of the two data, then

$$\frac{50}{100} = \frac{\sigma_1}{30} \text{ and } \frac{60}{100} = \frac{\sigma_2}{25}$$

$\Rightarrow \sigma_1 = 15$ and $\sigma_2 = 15$

Integer Type Questions (67 to 75)

67. **(2)**

Sol. $\overline{x} = 0$

$$\frac{|x_i - \overline{x}|^2}{2n} = a^2 \Rightarrow \text{S.D.} = |a| = 2$$

68. **(4)**

Sol. Median = 25.5 a

Mean deviation about median = 50

$$\Rightarrow \frac{\Sigma\ |x_i - 25.5a|}{50} = 50$$

$\Rightarrow$ 24.5 a + 23.5a + ... + 0.5a + 0.5a + ... + 24.5a = 2500

$\Rightarrow$ a + 3a + 5a + ... + 49a = 2500

$$\Rightarrow \frac{25}{2}(50a) = 2500$$

$\Rightarrow a = 4$

69. **(3)**

Sol. $$\text{variance } (x_i - 4) = \text{var}(x_i) = \frac{44}{11} - \left(\frac{11}{11}\right)^2 = 3$$

$$(x_i - 4) = \text{var}(x_i) = \frac{44}{11} - \left(\frac{11}{11}\right)^2 = 3$$

70. **(12)**

Sol. variance $(ax_i) = a^2 var(x_i)$

S.D. $(ax_i) = |a|\sqrt{\text{var}(x_i)}$

71. **(3)**

Sol. $$\frac{\sigma}{\overline{x}} \times 100 = \text{coefficient of variation}$$

$\sigma = 3$

72. **(12)**

Sol. Median

$$e = \frac{24^{th}\,term + 25^{th}\,term}{2} = \frac{76+77}{2}$$

= 76.5

mean deviation

$$= \frac{23.5 + 22.5......0.5 + 0.5 + ...23.5}{48} = 12$$

73. **(16)**

Sol. $\sum x_i = 20 \times 11 = 220$

$\Rightarrow \sum y_i = 10 \times 8 = 80$

variance (x_i)

$$= \frac{\sum x_i^{\,2}}{20} - (121)$$

$\Rightarrow \quad 2500 = \sum x_i^{\ 2}$

variance $(y_i) = \dfrac{\sum y_i^{\ 2}}{10} - 64$

$\Rightarrow \quad 980 = \Sigma\, y_i^2$

$\Sigma x_i^2 + \Sigma y_i^2 = \quad x_i^2 + y_i^2 \quad = 3480$

$\Rightarrow \quad \Sigma x_i + \Sigma y_i = \dfrac{\Sigma x_i^2 + y_i^2}{30} - \left(\dfrac{\Sigma x_i + y_i}{30}\right)^2$

$= 116 - 100 = 16$

74. **(8)**

Sol. Variations

$= \dfrac{2^2 + 4^2 + 6^2 + 8^2 + 10^2}{5} - \left(\dfrac{2+4+6+8+10}{5}\right)^2$

$= 44 - 36 = 8$

75. **(81)**

Sol. $\sum x_i = 63\times 50 = 3150$;

$\sum y_i = 40\times 54 = 2160$

$\text{var}(x_i) = 81 = \dfrac{\sum x_i^{\ 2}}{50} - (63)^2$, $\text{var}(y_i)$

$= 36 = \dfrac{\sum y_i^{\ 2}}{40} - (54)^2$

$\sum x_i^{\ 2} = 202500$

$\sum y_i^{\ 2} = 118080$

combined variance

$= \dfrac{\sum x_i^{\ 2} + \sum y_i^{\ 2}}{90} - \left(\dfrac{\sum x_i + \sum y_i}{90}\right)^2$

$= \dfrac{320580}{90} - (59)^2$

$= 3562 - 3481 = 81$