MIND MAP FOR JEE ASPIRANTS

Physics

Calculus and motion in Straight line

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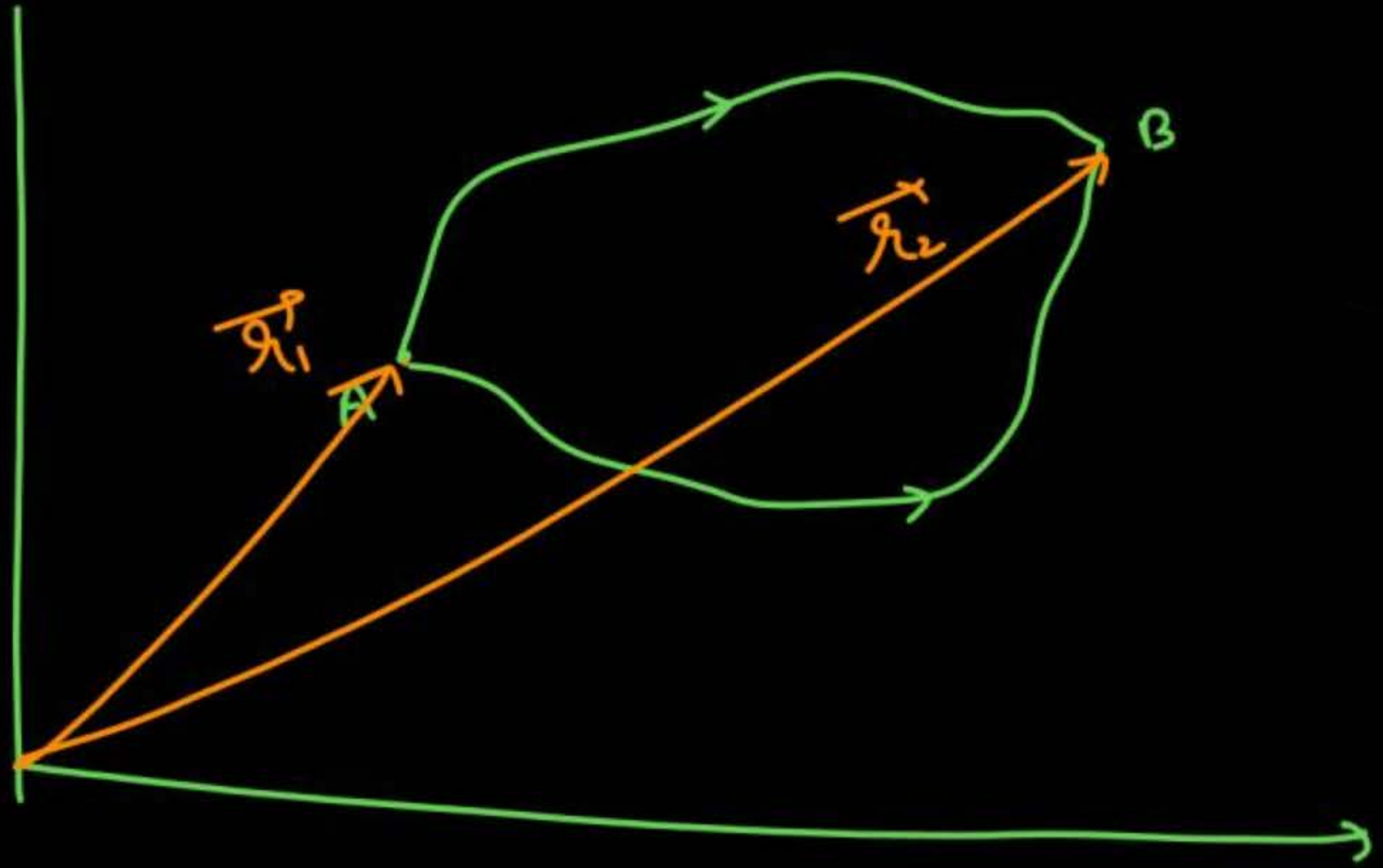
Today's Targets

1. Kinematics (motion in a st. line)
2. Calculus
3. [redacted]
4. [redacted]



$\vec{r_2} - \vec{r_1}$

Distance $\longrightarrow$ Total actual path travel by body.

Displacement $\longrightarrow$ vector, change in position vector, $\vec{r}_f - \vec{r}_i$, shortest distance b/w initial & final point

Velocity

$\longrightarrow$ Avrg velocity $= \dfrac{\text{total displacement}}{\text{total time}} = \dfrac{\Delta \vec{r}}{\Delta t}$

$\longrightarrow$ Rate of change of positive vector

$\longrightarrow$ Inst. Velocity $\vec{v} = \dfrac{d\vec{r}}{dt}$

If particle move on x-axis

$\vec{v}_x = \dfrac{dx}{dt}\hat{i}$ Similarly $v_y = \dfrac{dy}{dt}$

Acceleration $\longrightarrow$ Avrg acc $= \dfrac{\text{Change in velocity}}{\text{total time}} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\longrightarrow$ Inst-acc $= a = \dfrac{dv}{dt} =$ Rate of Change of Velocity.

Q $x = 3t^4$, $m = 2\,kg$

① find v and a at $t = 2$ sec

$v = \frac{dx}{dt} = 12t^3$ $\quad v = 12 \times 8 = 96$

$a = 12 \times 3t^2$

$a = 36 \times 4$

② Avrg velocity from $t=0$ to $t=2$ sec

$t=0,\ x=0$

$t=2,\ x=48$

$= \frac{48-0}{2} = 24$

③ Avrg acc from $t=0 \longrightarrow t=2$ sec.

$v = 12t^3$

$v_i = 0 \longrightarrow v_f = 96$

Avg acc $= \frac{96-0}{2} = 48$

④ Change in K.E.

from $t=0 \longrightarrow t=2$ sec

$t=0,\ (KE)_i = 0$

$t=2,\ (KE)_f = \frac{1}{2} \times 2 \times (96)^2$

⑤ $(WD)_{by}$ all the force from $t=0$ to $t=2$ sec.

$= (KE)_f - (KE)_i$

$$(\text{Slope})_{AB} = \tan\theta = \frac{y_2-y_1}{x_2-x_1} = \frac{\Delta y}{\Delta x}$$

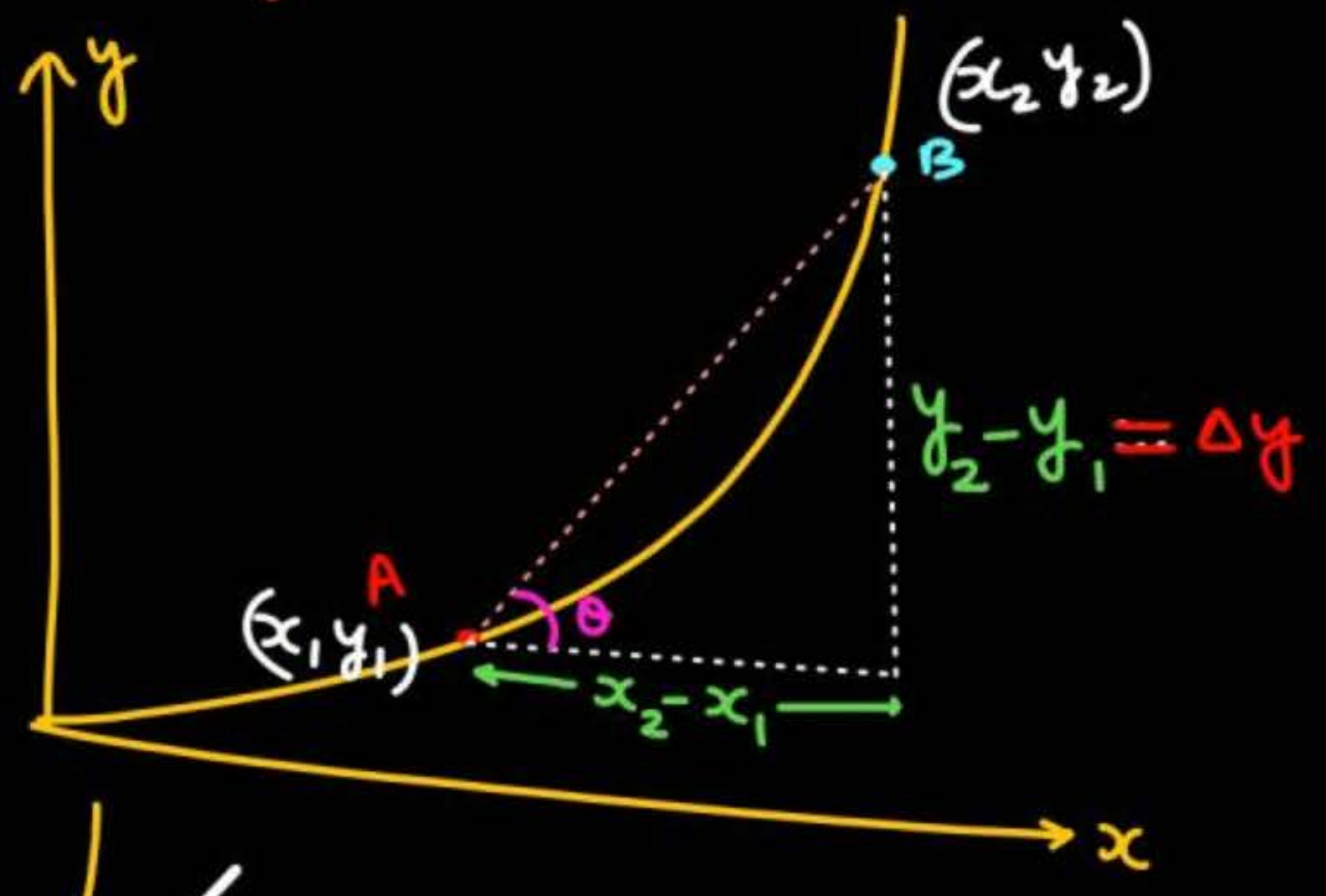
Slope of line AB

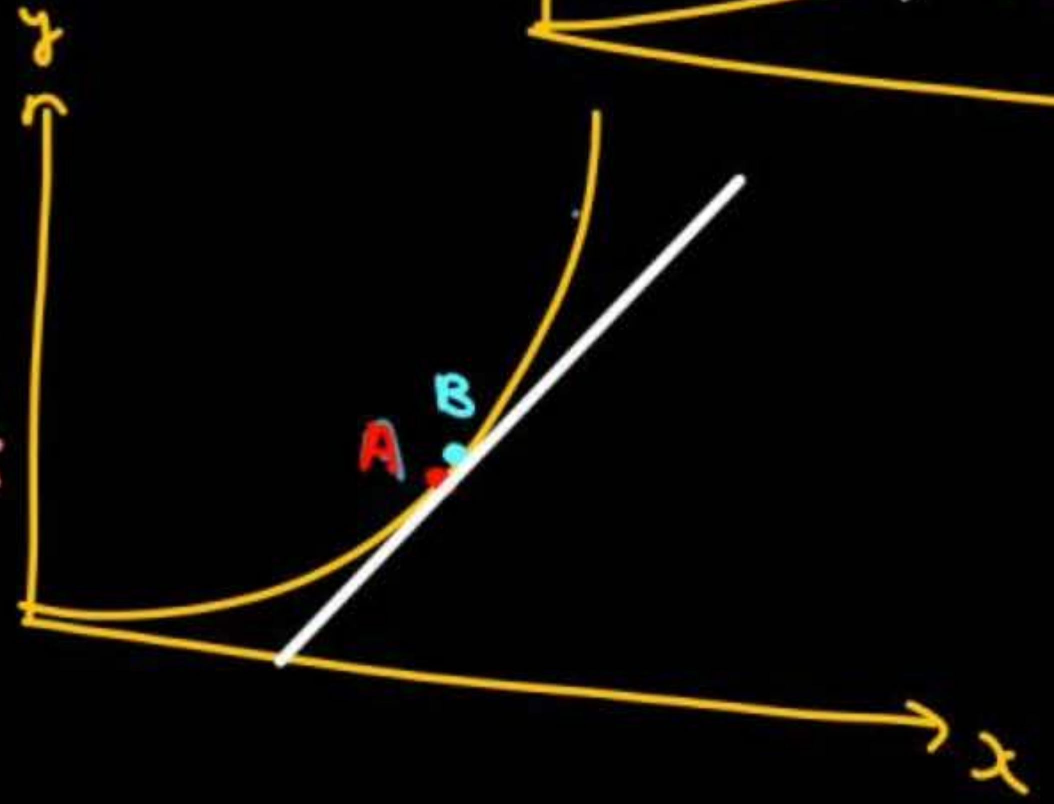
If Δy is very very small $\quad \Delta y \simeq dy$

 Δx $\quad$ " $\quad$ " $\quad \Delta x = dx$

$$\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Slope of tangent at A





$\frac{dy}{dx} \longrightarrow$ Slope of tangent of that point

$\longrightarrow$

$\longrightarrow$ Rate of change of y wrt x

$\longrightarrow$ diff. of y wrt x

$\longrightarrow$

$$\frac{d}{dx}x^n = n\,x^{n-1}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

Q $y = x^3 + \sin x - e^x + 10 + 5x^6$

$$\frac{dy}{dx} = 3x^2 + \cos x - e^x + 0 + 5(6x^5)$$

$y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$y' = uv' + vu'$

$y = \frac{u}{v}$

$$y' = \frac{vu' - uv'}{v^2}$$

Derivatives of Commonly Used Functions.

- $y = \text{constant} \Rightarrow \frac{dy}{dx} = 0$
- $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$
- $y = e^x \Rightarrow \frac{dy}{dx} = e^x$
- $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$
- $y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x$
- $y = \tan x \Rightarrow \frac{dy}{dx} = \sec^2 x$
- $y = \cot x \Rightarrow \frac{dy}{dx} = -\operatorname{cosec}^2 x$
- $y = \operatorname{cosec} x \Rightarrow \frac{dy}{dx} = -\operatorname{cosec} x \cot x$
- $y = \sec x \Rightarrow \frac{dy}{dx} = \sec x \tan x$

Q $y = x^2 \sin x$

$$\frac{dy}{dx} = y' = x^2 \cos x + (\sin x) 2x$$

Q $y = e^x \cdot x^3$

$$y' = e^x 3x^2 + x^3 e^x$$

Q $y = x^4 \tan x$

$$\frac{dy}{dx} = x^4 \sec^2 x + (\tan x) 4x^3$$

Chain rule

$y = \quad x^3)$

Q $y = \sin x$

$\frac{dy}{dx} = \cos x$

$y = \sin x^3$

$\frac{dy}{dx} = (\cos x^3)3x^2$

$y = \sin(\cos(x^3))$

$\frac{dy}{dx} = \cos(\cos x^3)(-\sin x^3)$

$3x^2$

Q $y = \sin(4x+3)$

$\frac{dy}{dx} = \cos(4x+3) \times (4+0)$

Q $y = A\sin(\omega t + \phi)$ $\quad A, \omega, \phi \longrightarrow$ const

$\frac{dy}{dt} = A\cos(\omega t + \phi)(\omega + 0)$

$= A\omega \cos(\omega t + \phi)$

$$(\sin x)^3 \quad = \frac{d}{dx}(\text{कचरा})^3 = 3(\text{कचरा})^2 \cdot \frac{d}{dt}(\text{कचरा})$$

Q

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

$$y = \sin^3 x = (\sin x)^3$$

$$\frac{dy}{dx} = 3(\sin x)^2 \times \cos x$$

$$y = \sin^3 \cos(x^2)$$

$$y' = 3\sin^2(\cos x^2) \times \cos(\cos x^2) \times (-\sin x^2)\,2x$$

Q

$$x = t^3 - t^2 + 2t^5$$

$$v = \frac{dx}{dt} = 3t^2 - 2t + 10t^4$$

Q

$$y = \ln[\cos(e^x)]$$

$$y = \frac{1}{\cos(e^x)}\left[-\sin(e^x)\right] \times e^x$$

Q If radius of sphere changing at the rate of 10m/s find rate of change of its surface area & rate of change of its Vol^n

$$A = 4\pi r^2 \qquad \frac{dr}{dt} = 10 \qquad V = \frac{4}{3}\pi r^3$$

$$\frac{dA}{dt} = 4\pi \, 2r \frac{dr}{dt}$$

$$\frac{dv}{dt} = \frac{4}{3}\pi \, 3r^2 \cdot \frac{dr}{dt}$$

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$y = x^2$$

$$\frac{dy}{dt} = 2x \cdot \frac{dx}{dt}$$

Integration

$$\frac{d}{dx} f(x) = g(x)$$

$$\int g(x)\, dx = f(x) + c$$

$$\frac{d}{dx} x^3 = 3x^2$$

$$\frac{d}{dx}(x^3+5) = 3x^2$$

$$\int 3x^2\, dx = x^3 + c$$

Integrand $f(x) = \frac{dF(x)}{dx}$	Integral $\int f(x)dx = F(x) + C$
$k = \text{Constant}$	$kx+C$
x^n	$\frac{x^{n+1}}{n+1} + C \quad \text{If } n \neq -1$
x^{-1}	$\ln x + C$
e^x	$e^x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

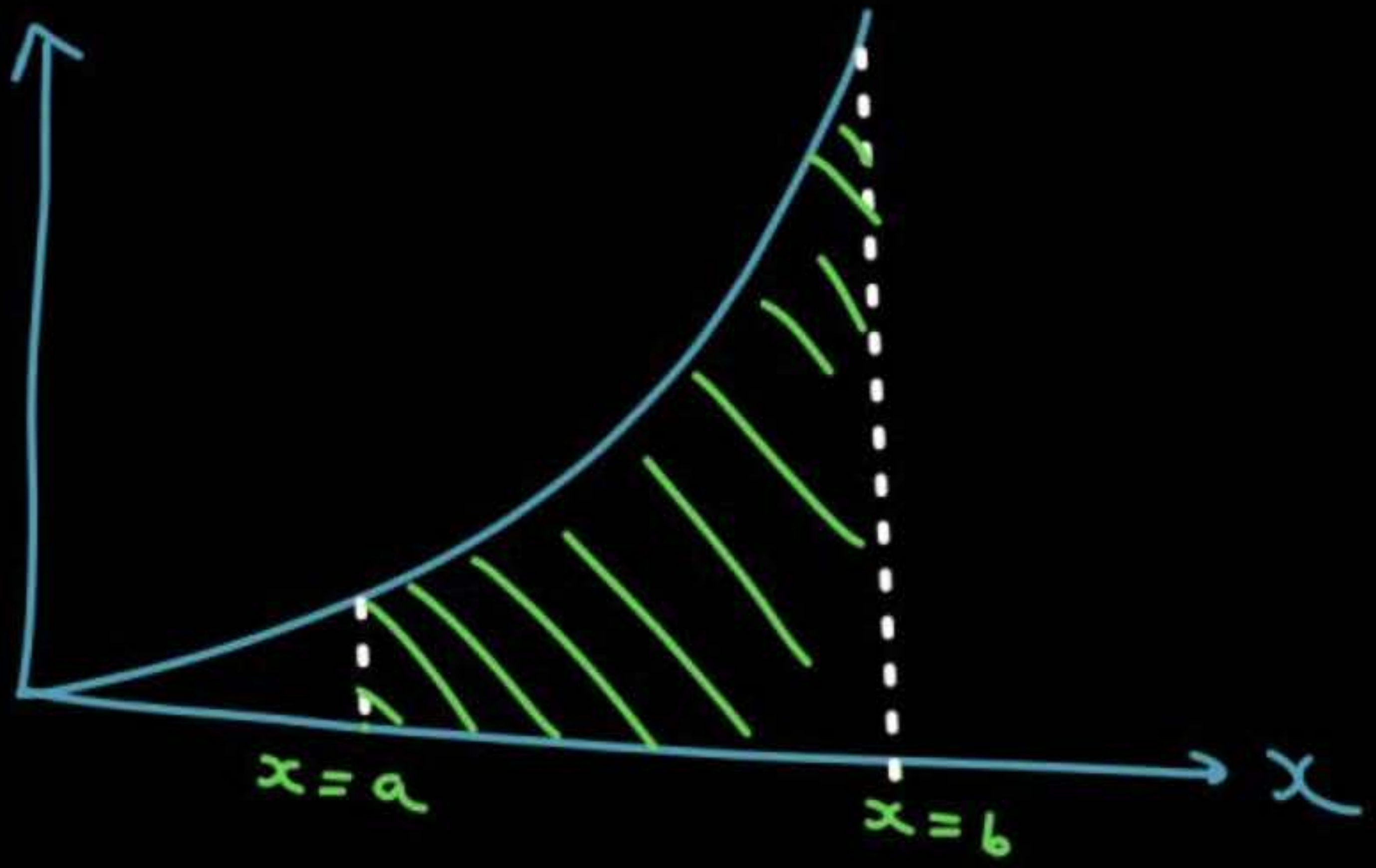
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int x^2 dx = \frac{x^3}{3} + c$$

$$\int_{x=a}^{x=b} y\,dx = \text{Area Under Curve}$$



$$F = \frac{dp}{dt} \Rightarrow \int dP = \int F\,dt$$

$$\Delta P = \int F \cdot dt$$

$$WD = \int F \cdot ds$$

$$(WD)_{gas} = \int P\,dV$$

$$\text{slope} = \frac{dy}{dx}$$

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

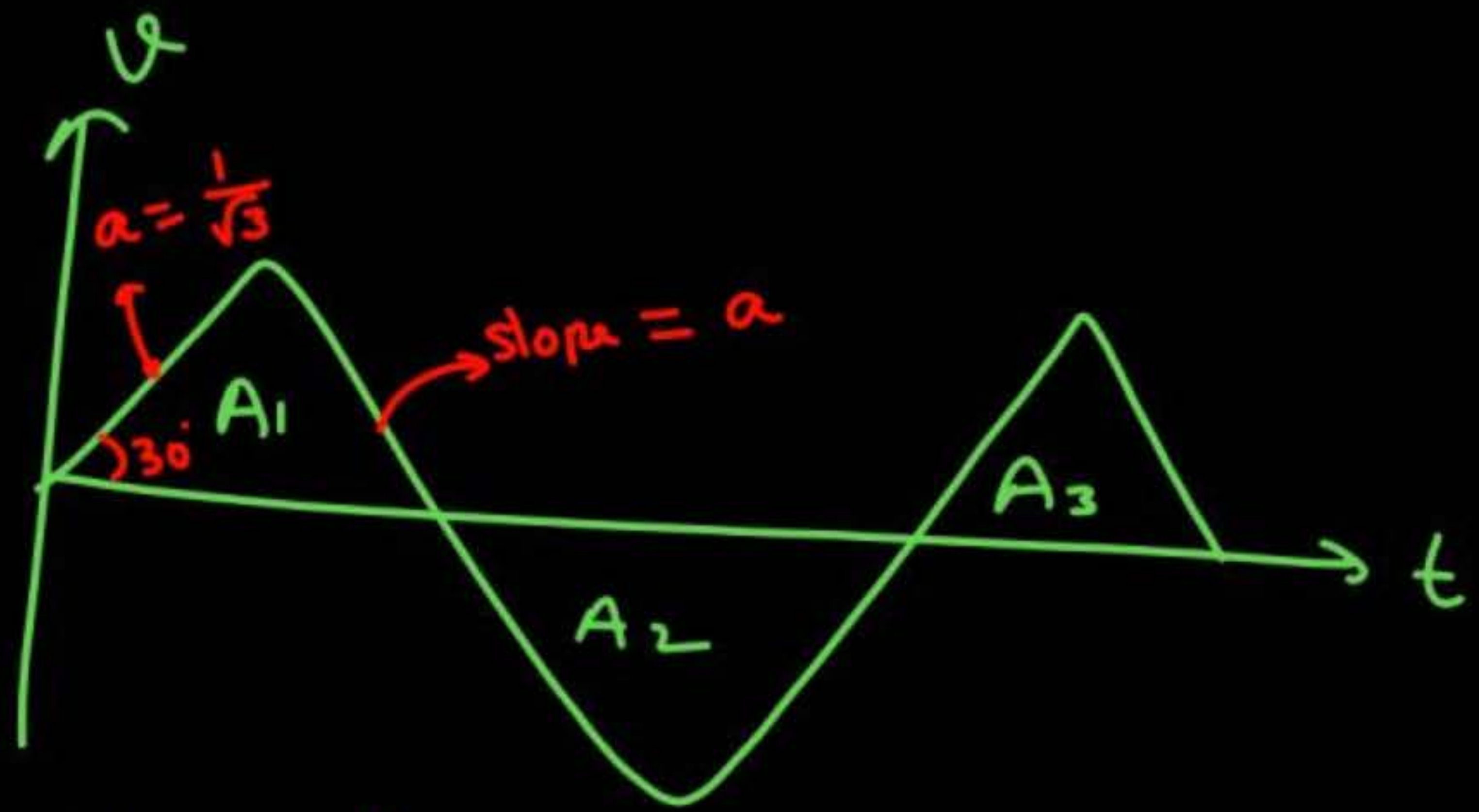
$$WD = \int F \cdot ds$$

$$(WD)_{gen} = \int P \cdot dV$$

$$\Delta V = \int a \cdot dt$$

$$\text{diplam} = \int v\,dl$$

$$\text{Area} = \int y\,dx$$



Displacement $= A_1 - A_2 + A_3$

Distance $= A_1 + A_2 + A_3$

$$\int_1^3 x^3\,dx = \left.\frac{x^4}{4}\right|_{x_i=1}^{x_f=3} = \frac{3^4}{4} - \frac{1^4}{4} = \checkmark$$

$(x-t)$ slope $\longrightarrow v$

$(v-t)_{\text{slope}} \longrightarrow a$

$(v-t)_{\text{area}} \longrightarrow$ Displacement

$(a-t)_{\text{area}} \longrightarrow$ change in velocity

If

(acc $\longrightarrow$ const)

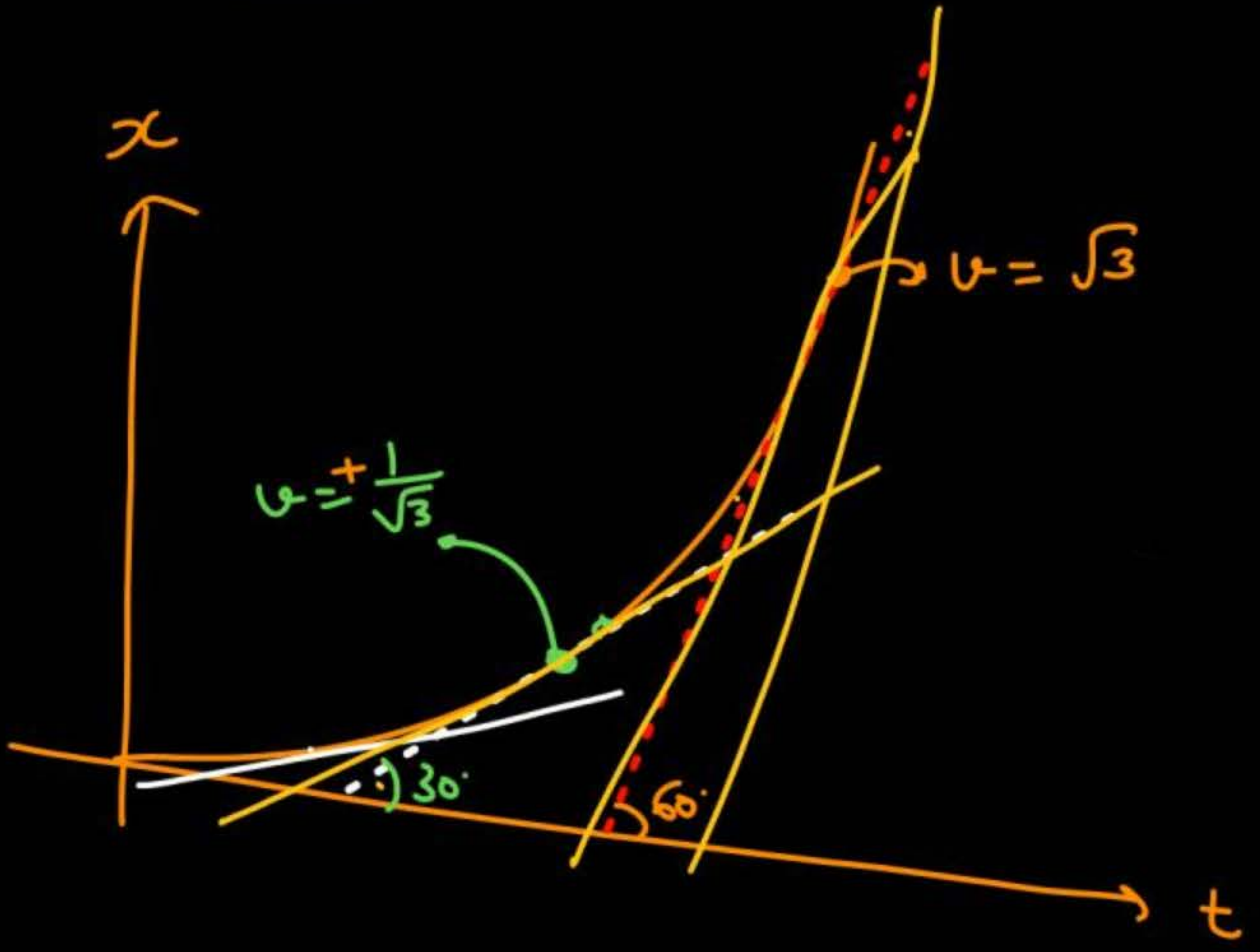
$(v-t) \longrightarrow$ st. line

$\Downarrow$

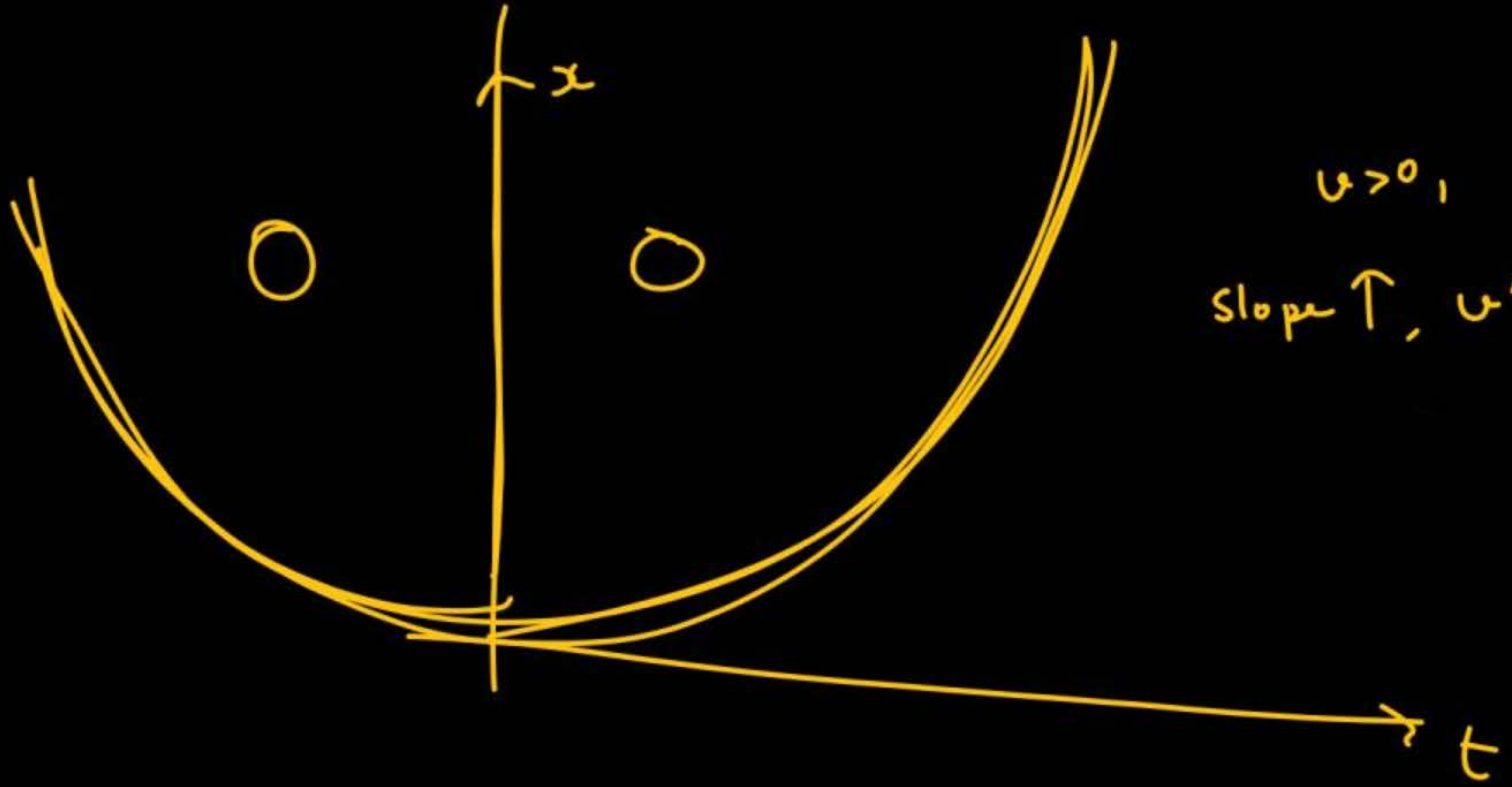
$(x-t)$ parabola

$\nearrow$ $a>0$ $(x-t)$ happy

$\searrow$ $a<0$ $(x-t)$ sad.



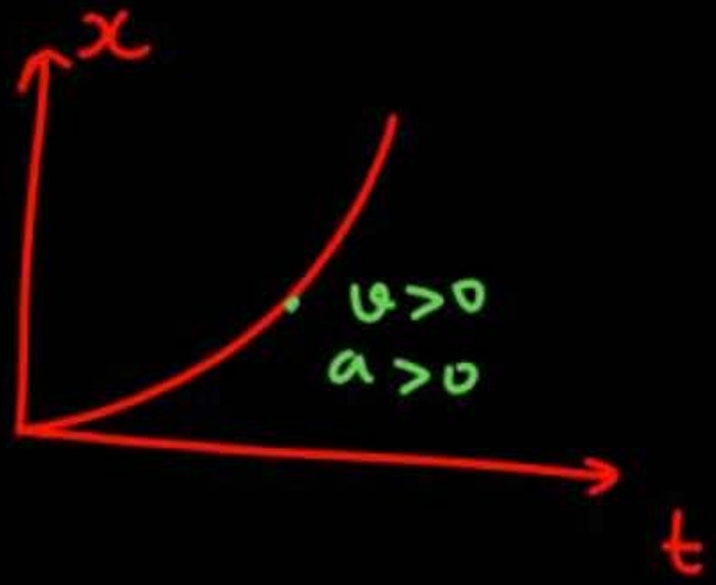
x
t
$v = \sqrt{3}$
$v = \frac{1}{\sqrt{3}}$
30°
60°

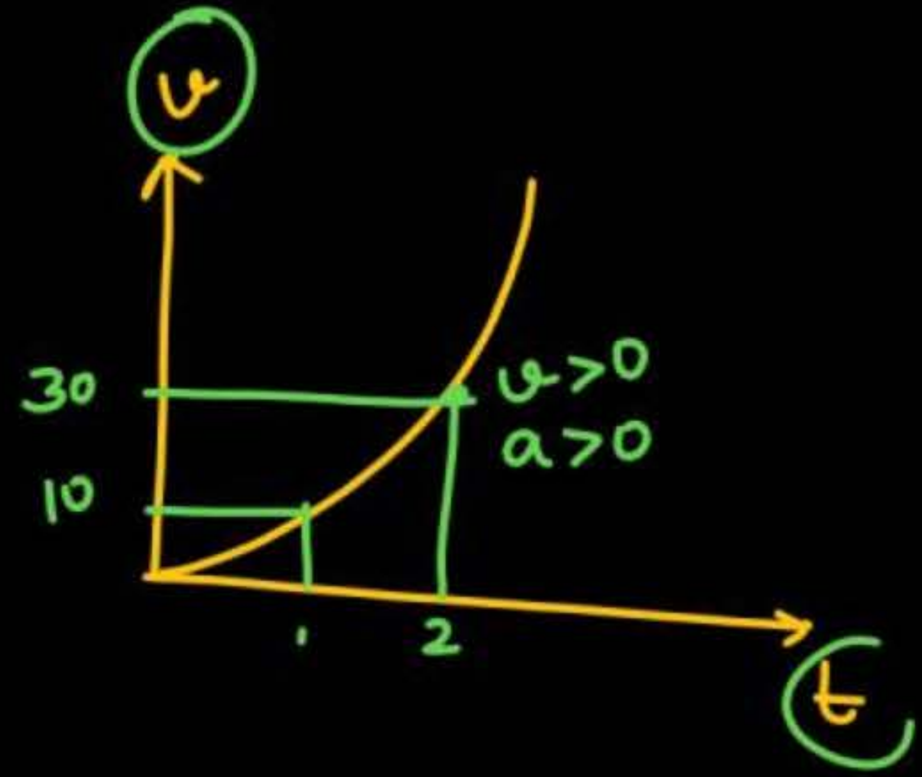


$v > 0,$

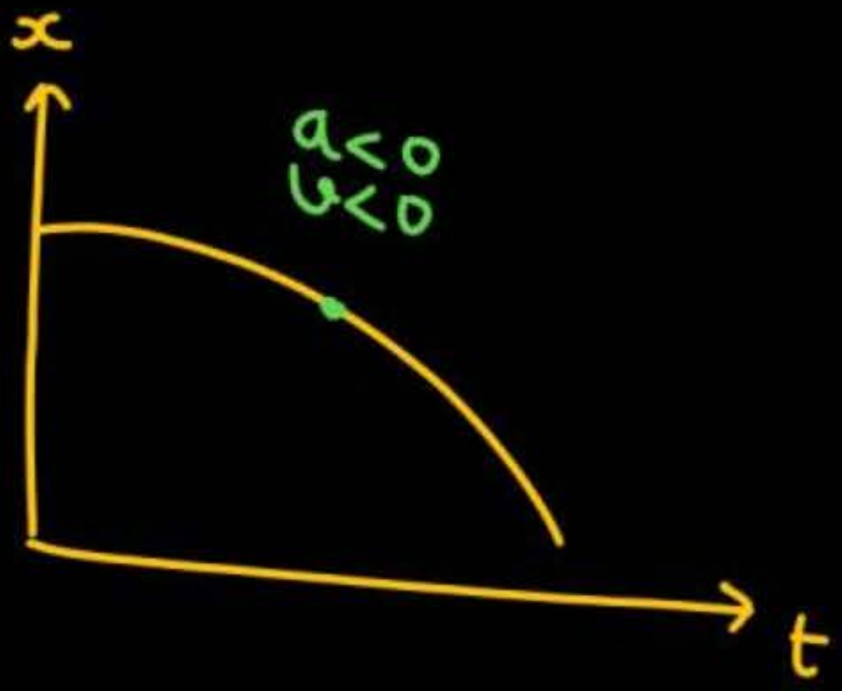
slope $\uparrow$, $v \uparrow$, $a > 0$

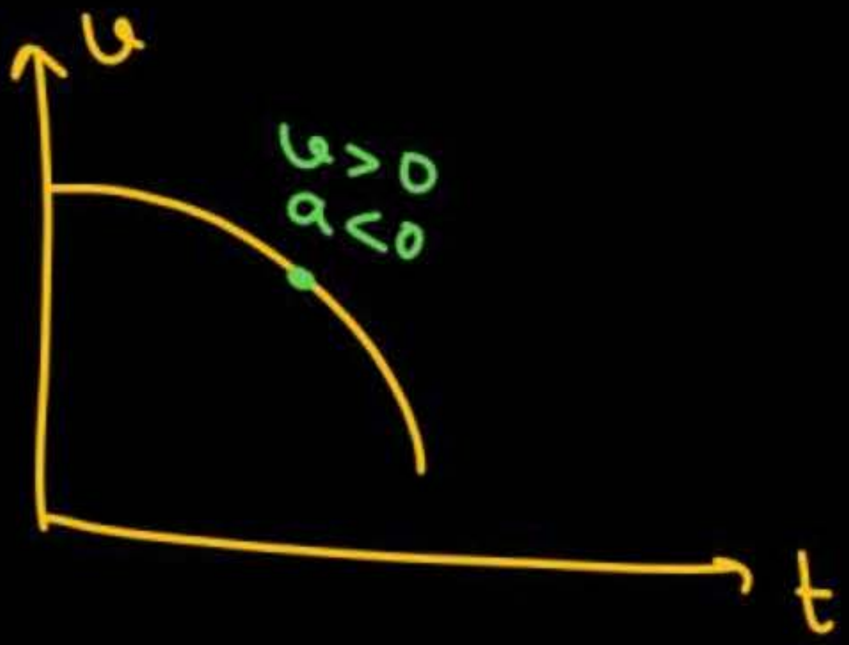
Q



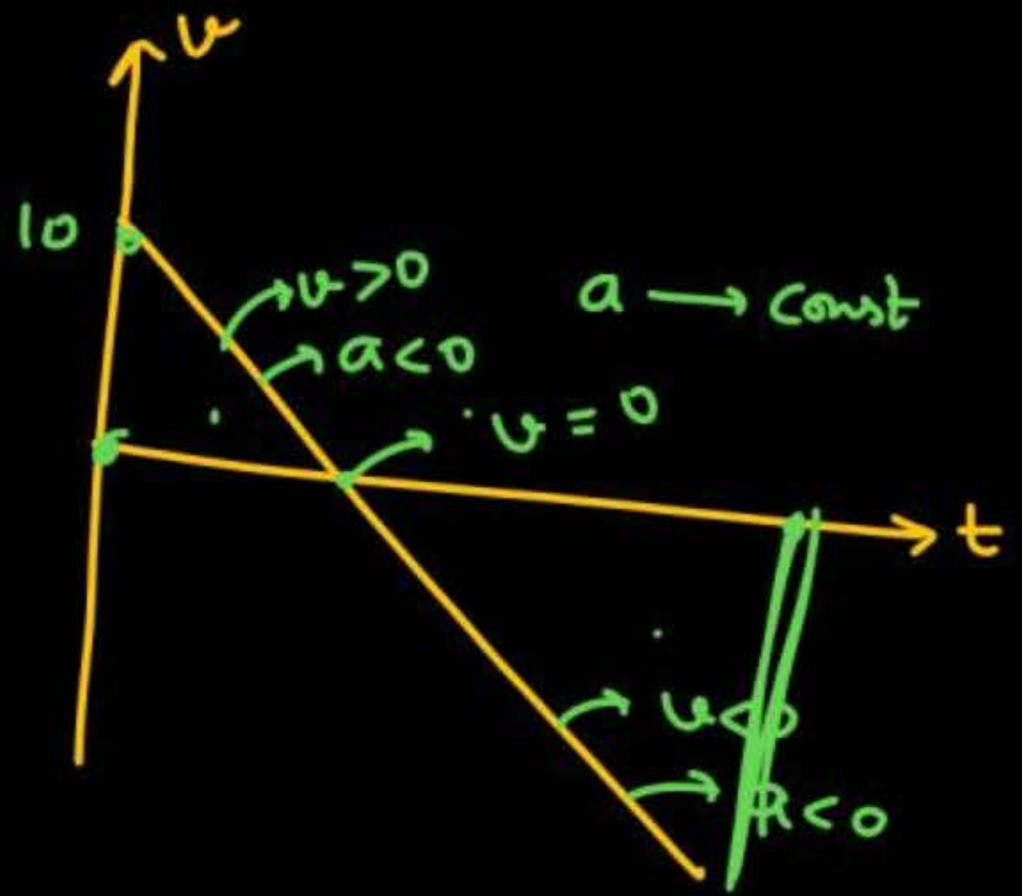


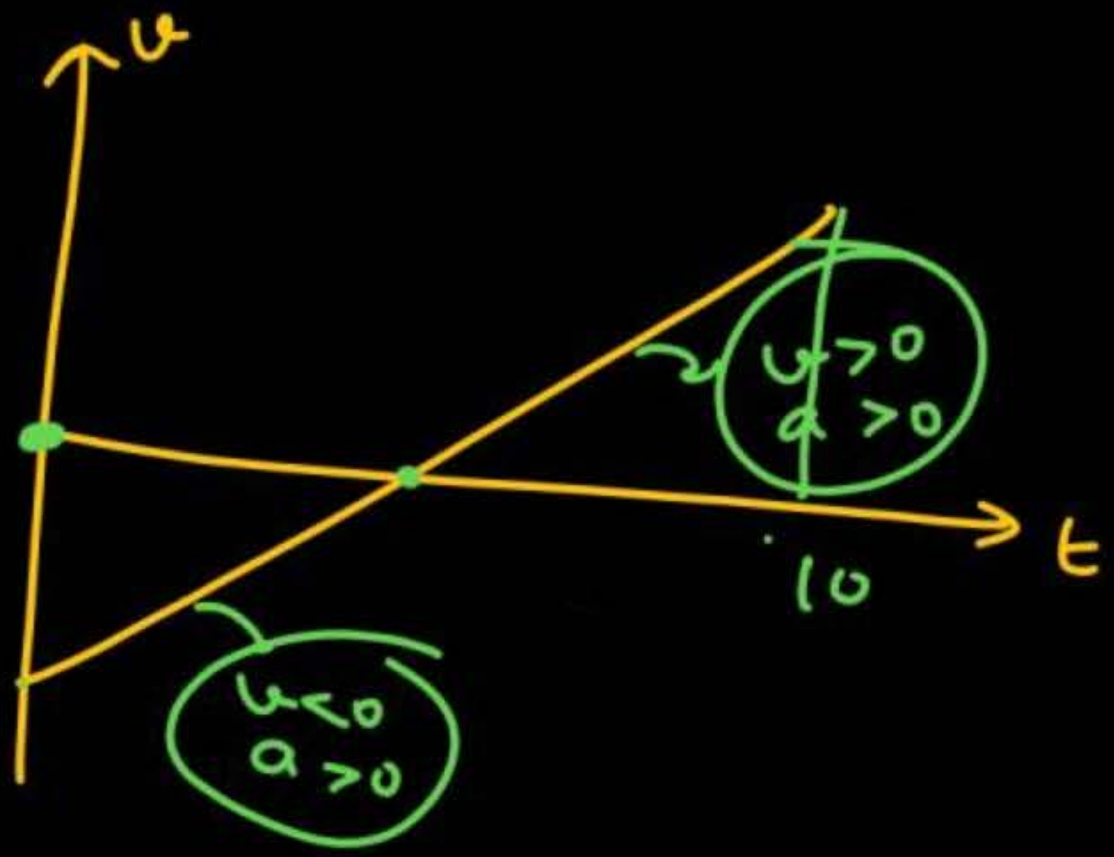
q



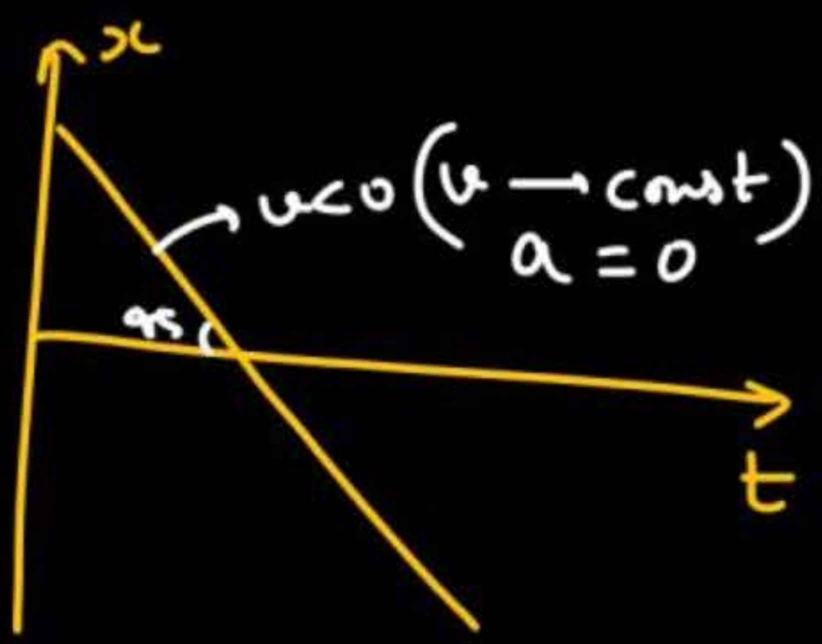


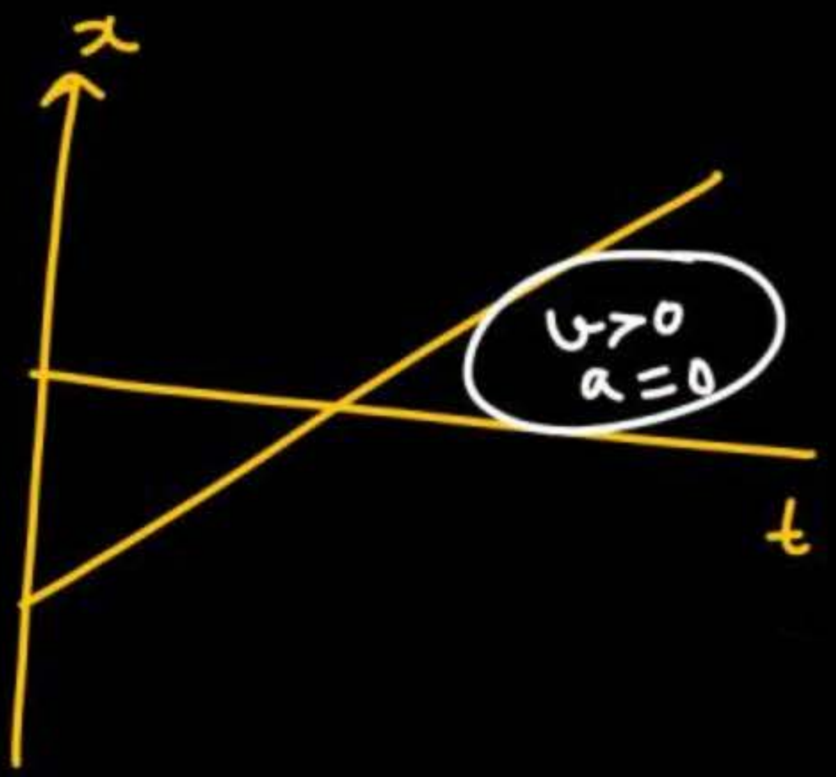
Q

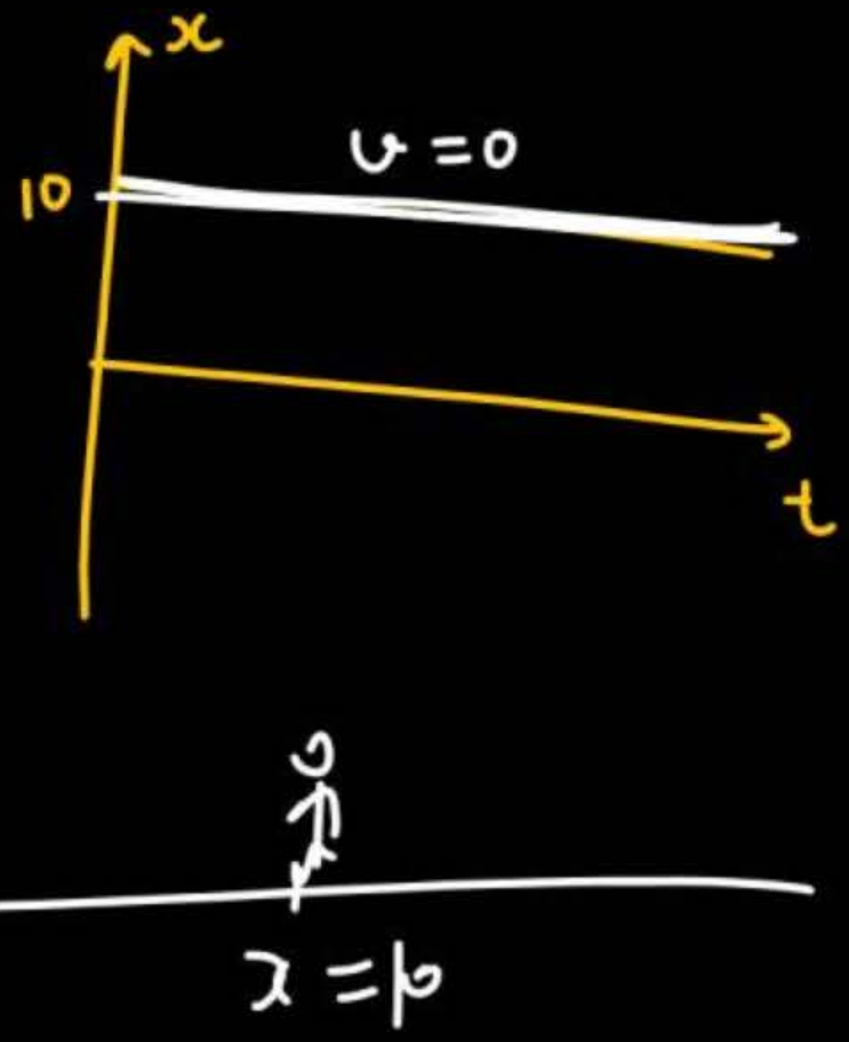




Q







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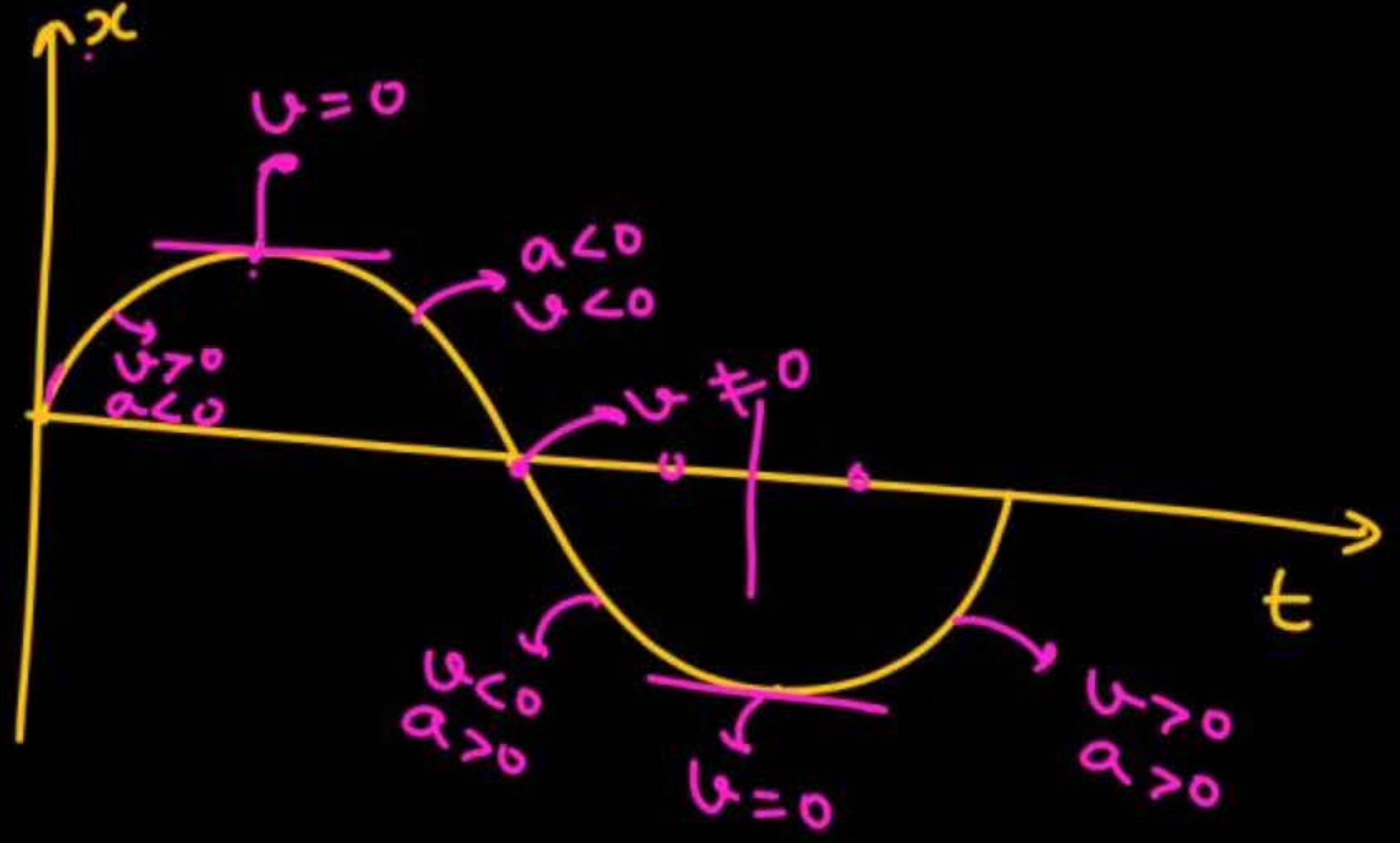

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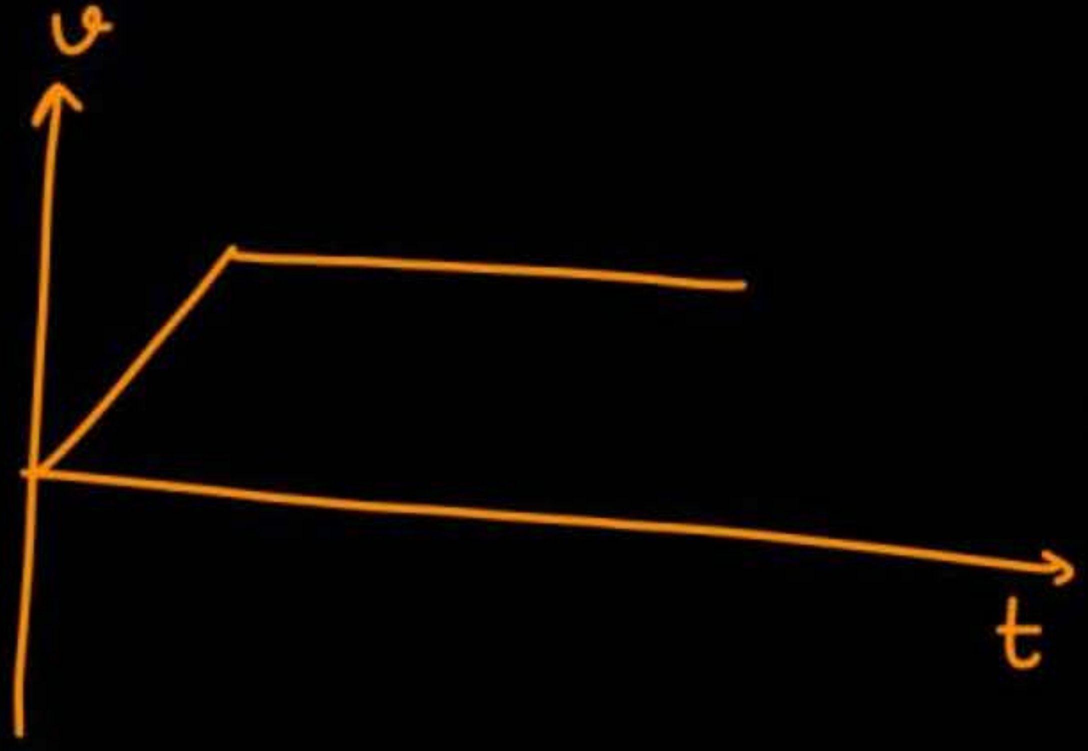
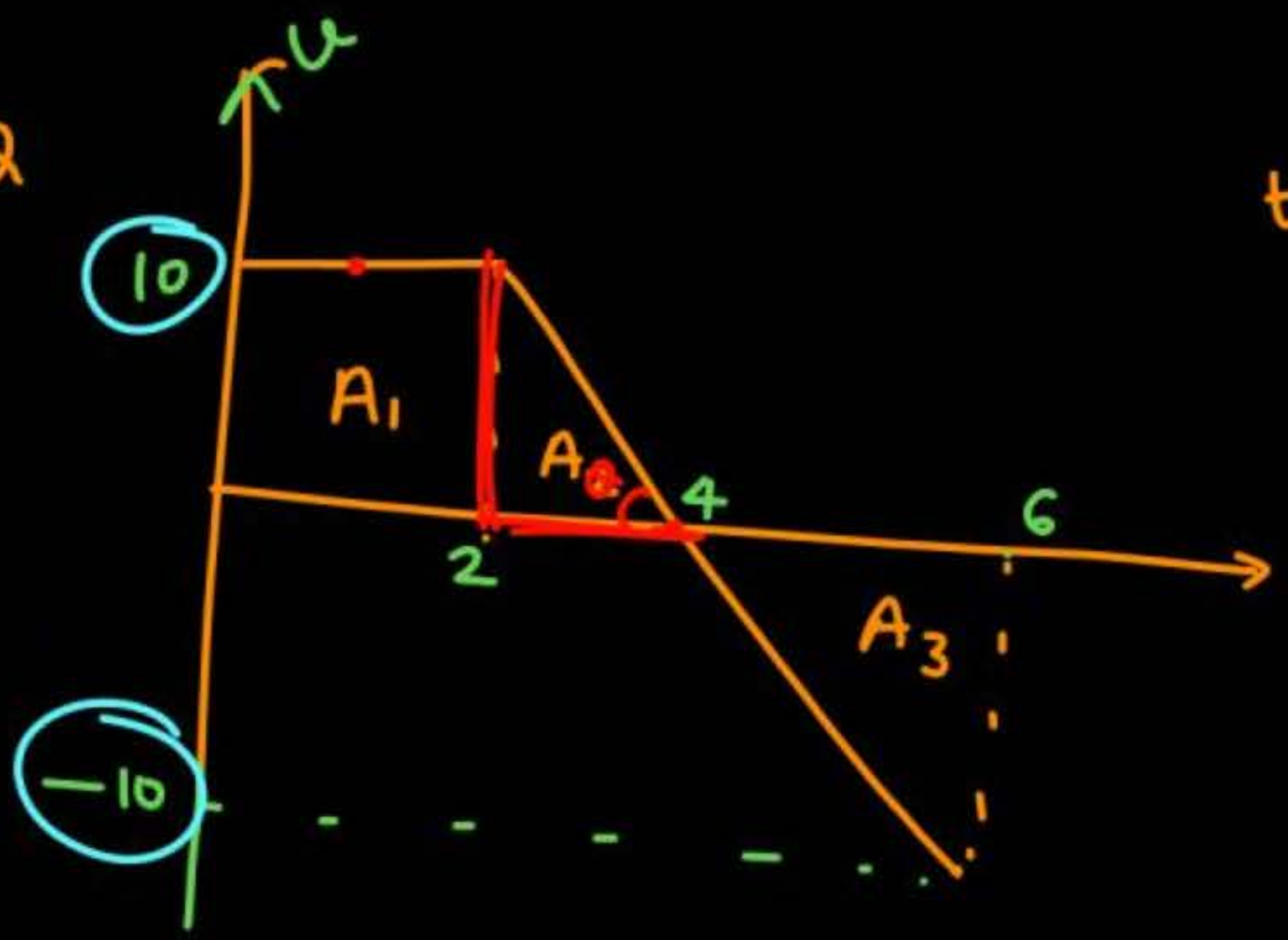


Q



$A_1 = 20$

$A_2 = \frac{1}{2} \times 2 \times 10 = 10$

$A_3 = 10$

$t=0 \longrightarrow t=6$ distance $= 20 + 10 + 10$

$\langle speed \rangle = \frac{40}{6}$

Displacement $= (20 + 10) - (10) = 20$

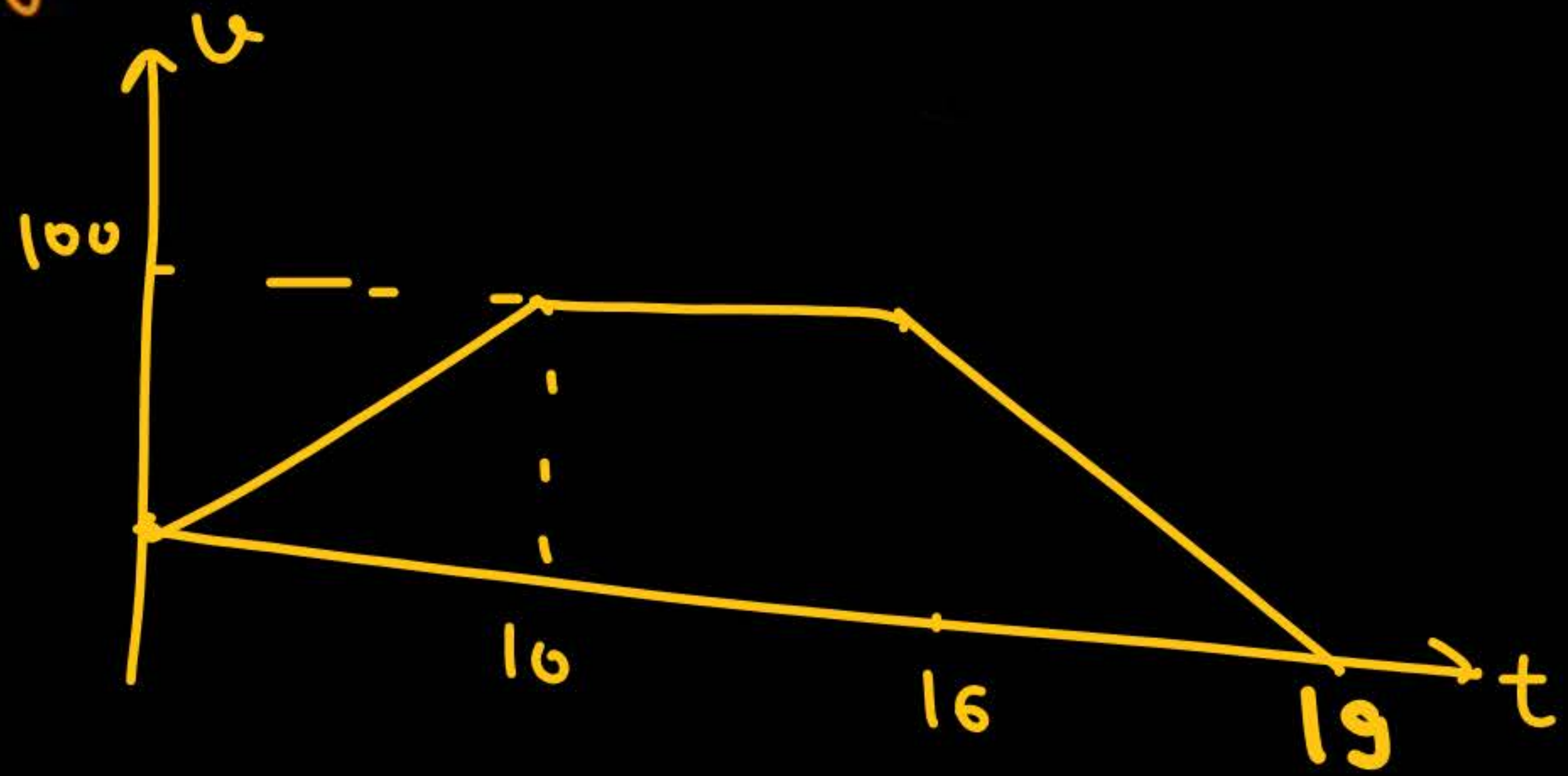
acc $t = 1, \Rightarrow a = 0$

$t = 4 \quad a = -\frac{10}{2}$

$t=2 \longrightarrow t=6$ Avg acc $= \frac{(-10) - (10)}{6 - 2}$

Q A particle start motion from rest having acc $+10\,m/s^2$ for 10 sec after that it move with const velocity for next 6 sec and in third part of journey it comes to rest and took 3 more sec.
3

find Avg. velocity.



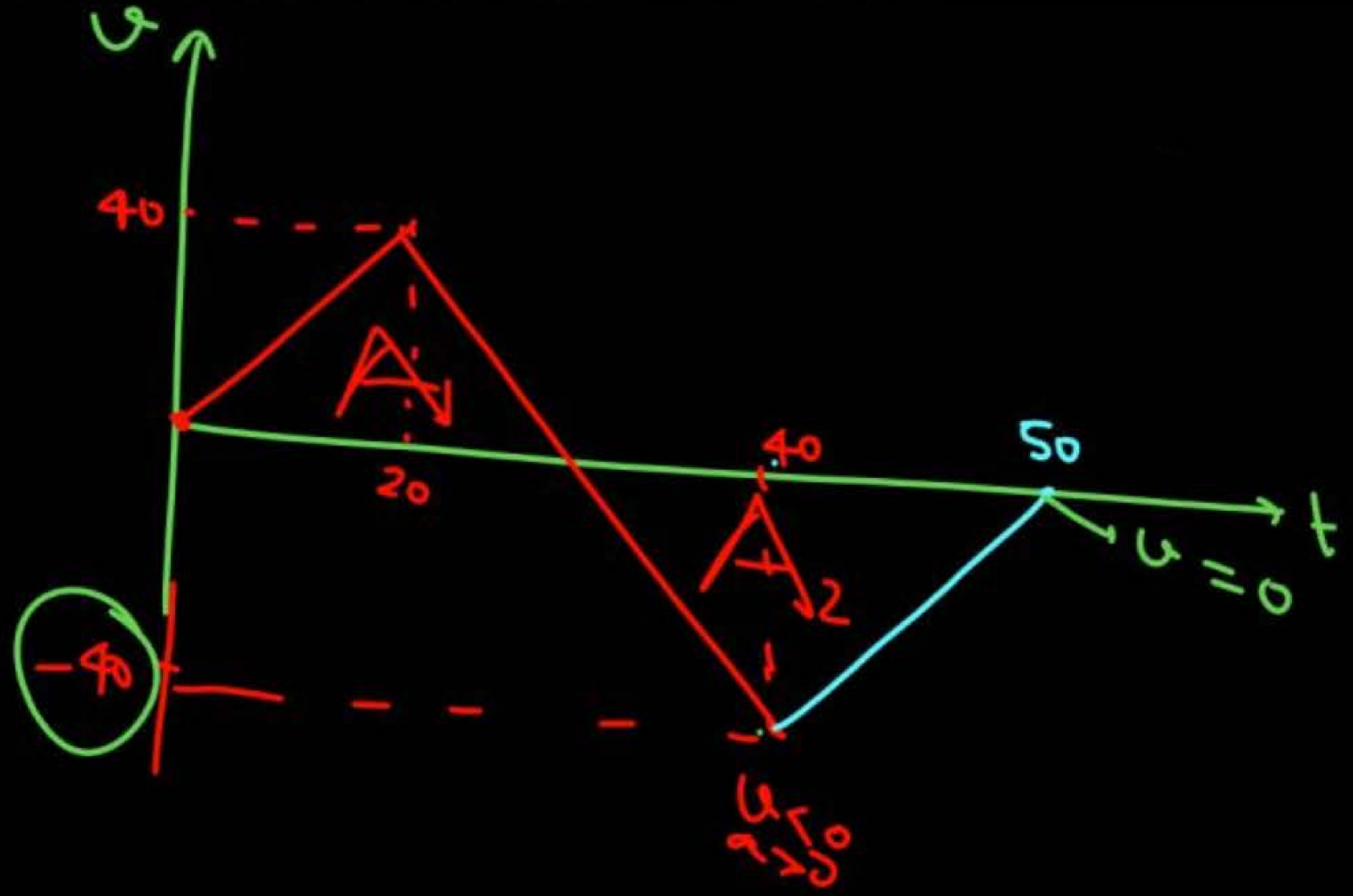
A particle starts from rest at t = 0 and x = 0 to move with a constant acceleration = +2 m/s^2, for 20 seconds. After that, it moves with –4 m/s^2 for the next 20 seconds. Finally, it moves with positive acceleration for 10 seconds until its velocity becomes zero. $a = +4$

Const

(a) What is the value of the acceleration in the last phase of motion?

(b) What is the final x-coordinate of the particle?

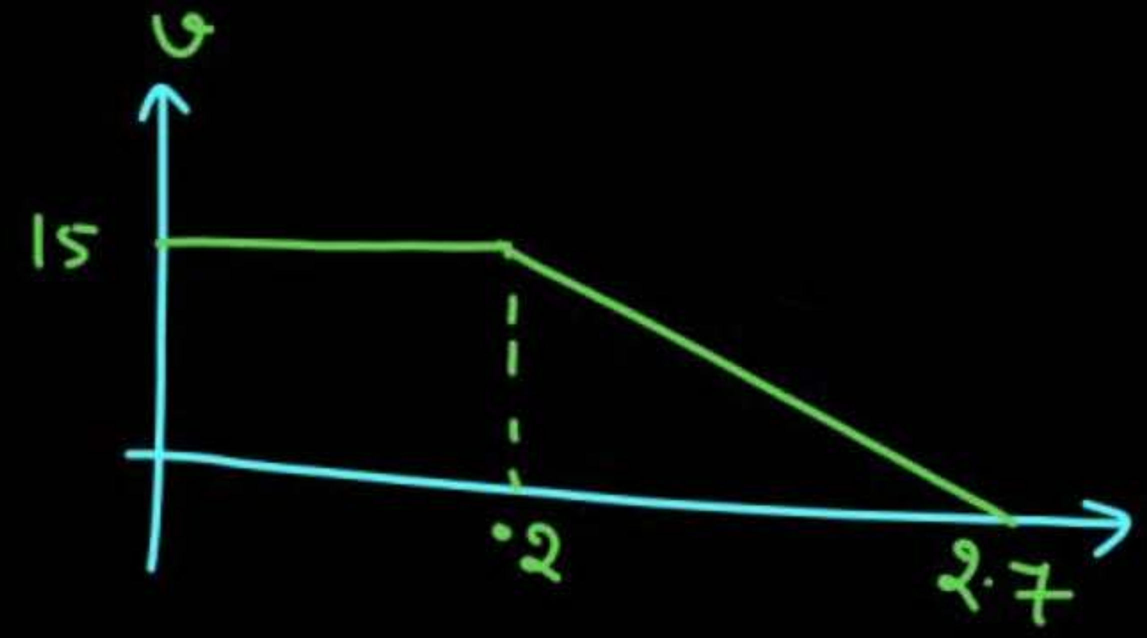
(c) Find the total distance covered by the particle during the whole motion.



Ex. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of $6.0 m/s^2$, find the distance travelled by the car after he sees the need to put the brakes on

$$u = 54 \times \frac{5}{18} = 15 \, m/s$$

$$a = -6$$



$v = u + at$

$S = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2aS$

$S_{n^{th}} = u + \frac{1}{2}(2n-1)a$

$\rightarrow a \rightarrow \text{const}$

A particle moving in one-dimension with constant acceleration of 10 m/s^2 is observed to cover a distance of 100 m during a 4s interval. How far will the particle move in the next 4s?

$u = ?$

$a = 10$

$t = 4$

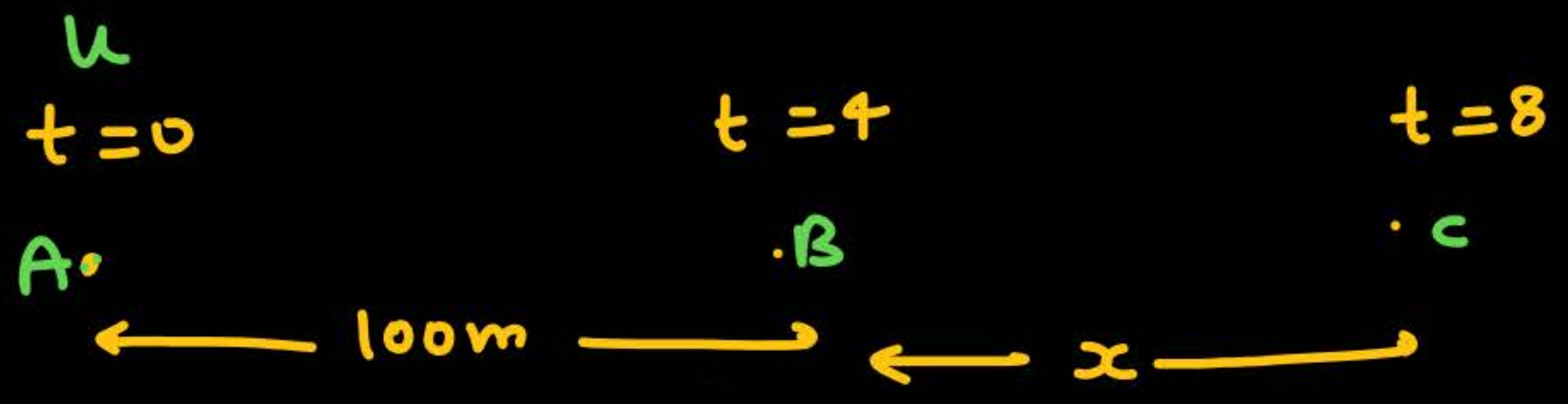
$s = 100$

$s = ut + \frac{1}{2}at^2$

$100 = u \times 4 + \frac{1}{2} \times 10 \times 4^2$

$100 = 4u + 80$

$\boxed{u = 5}$



$(A \longrightarrow C)$

$100 + x = 5 \times 8 + \frac{1}{2} \times 10 \times 8^2$

Motion Under Gravity

① $T = 16\ \text{sec}$

② $t = 12\ \text{sec}$ $\quad v = u + at = 80 - 10 \times 12$

$v = -40$

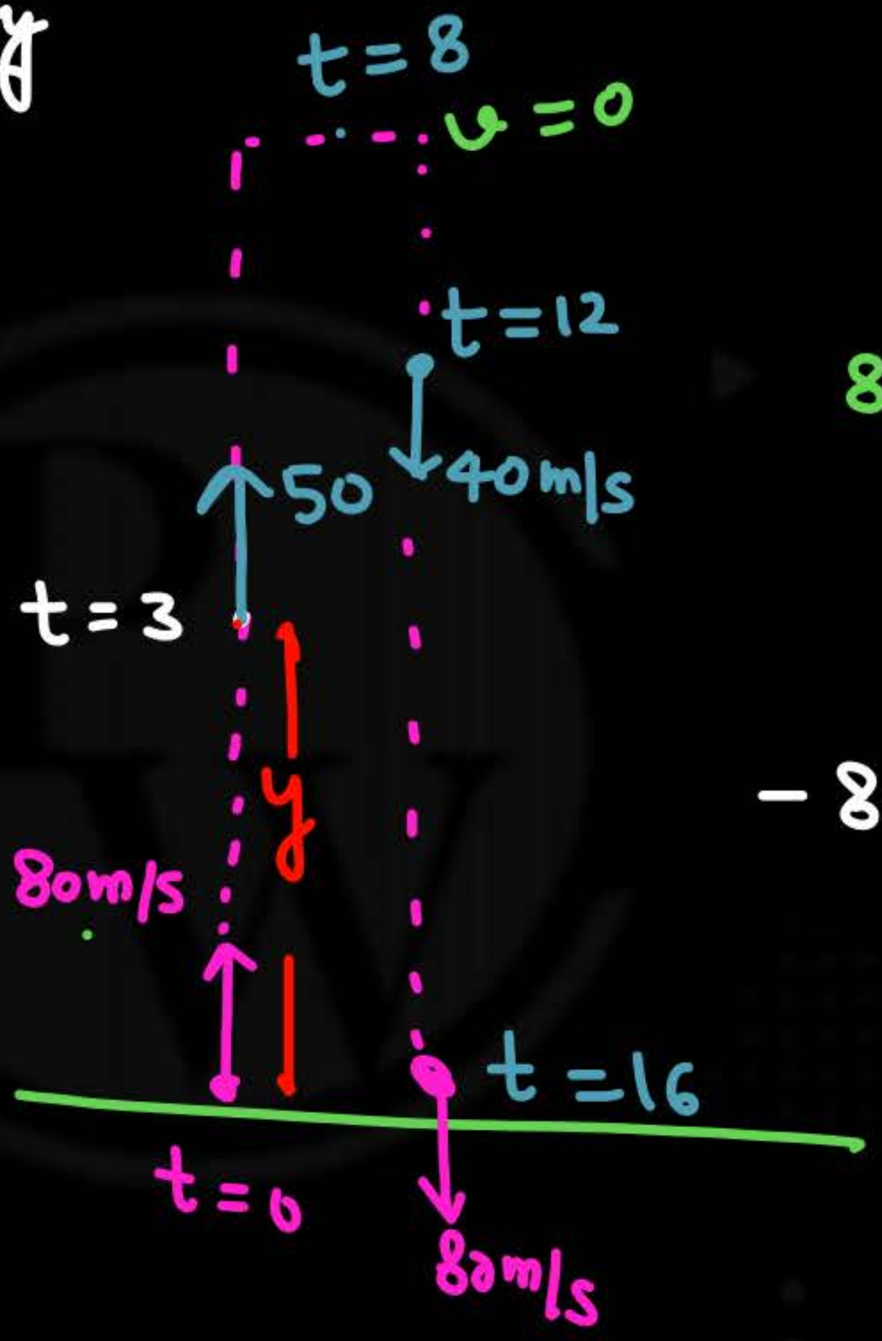
③ $v^2 = u^2 + 2as$

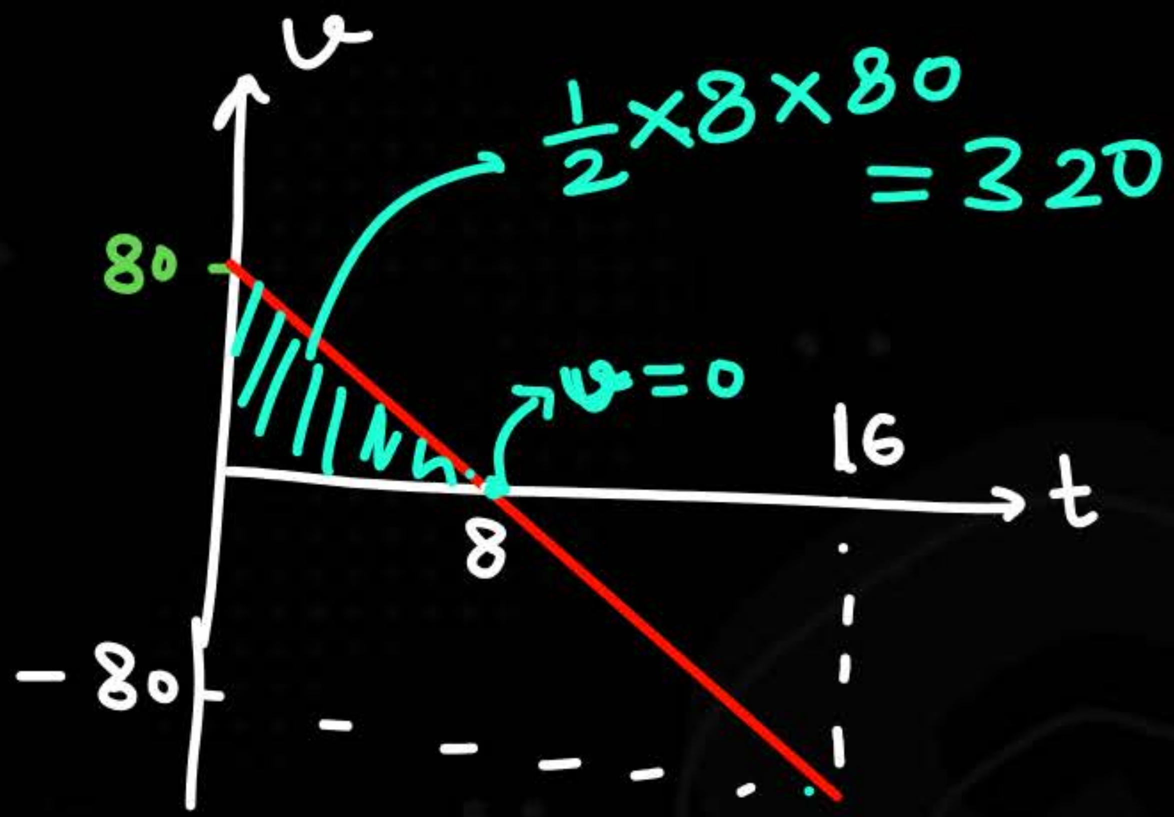
$0 = 80^2 + 2(-10)(h_{max})$

$h_{max} = \dfrac{6400}{20} = 320$

④ location at $t = 3\ \text{sec}$

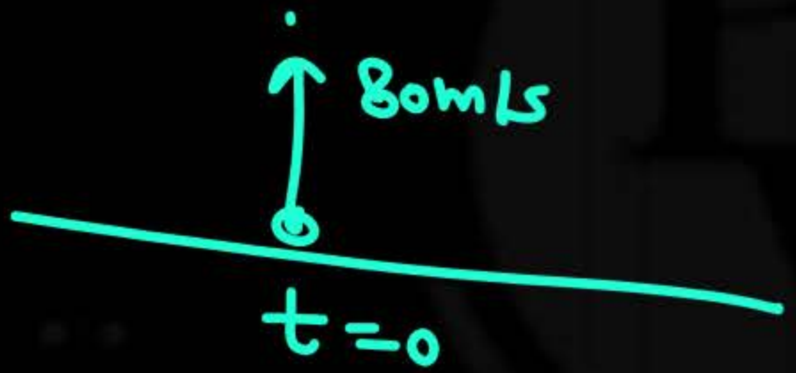
$y = 80 \times 3 - \frac{1}{2} \times 10 \times 3^2$

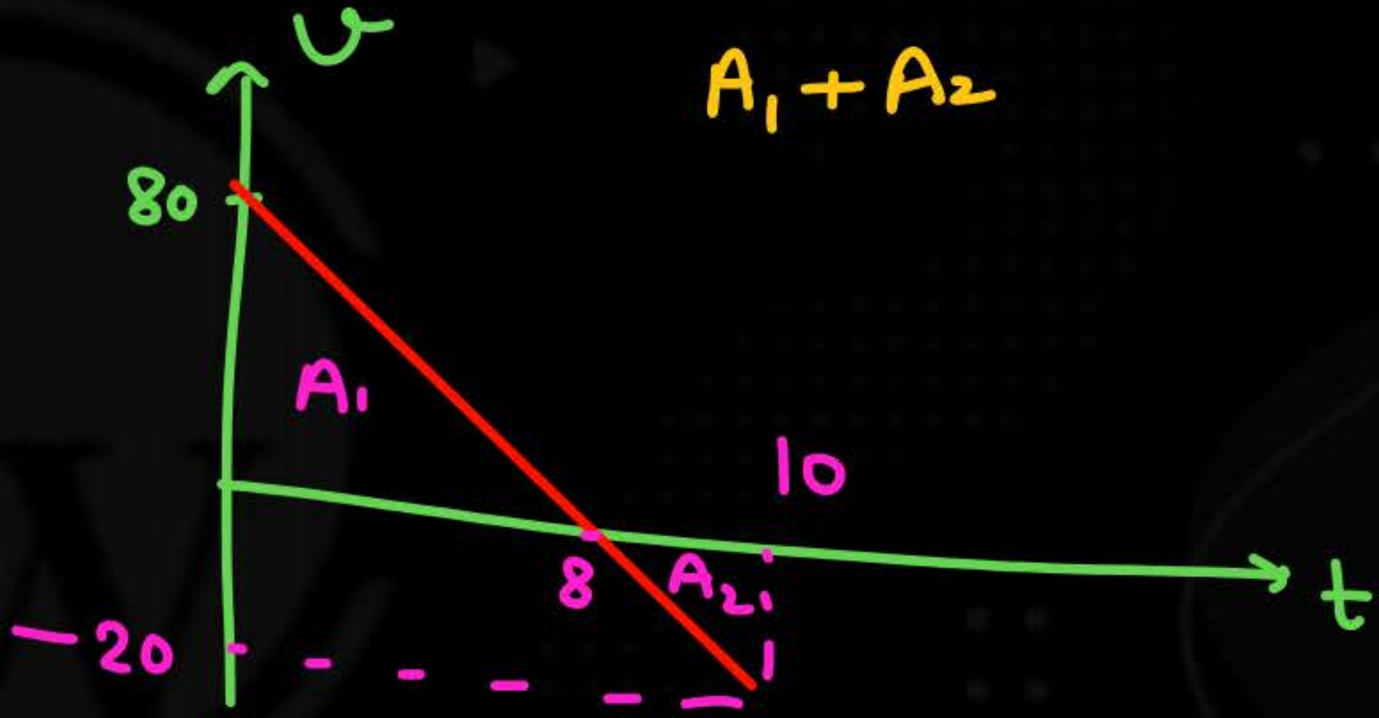




Q :

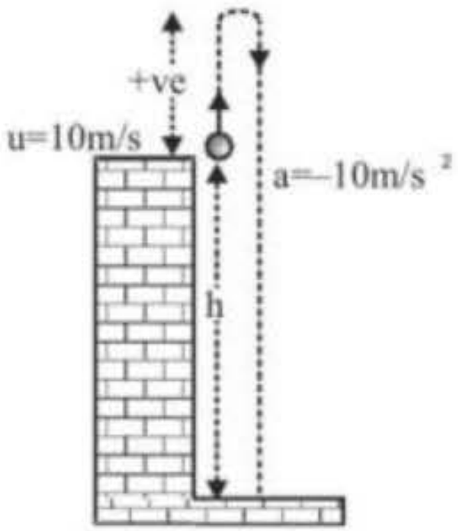
<speed>, distan $t=0 \longrightarrow t=10$

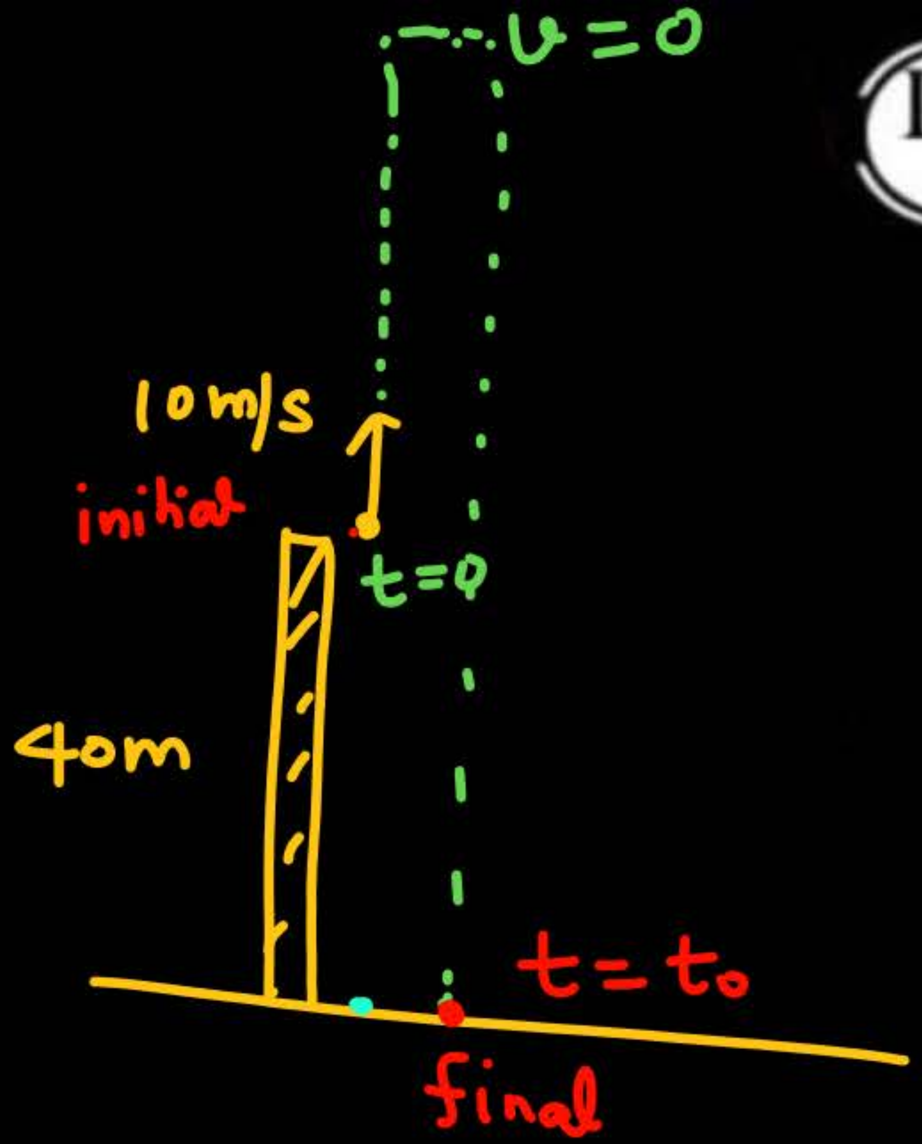




$A_1 + A_2$

Ex. A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10\ m/s^2$)



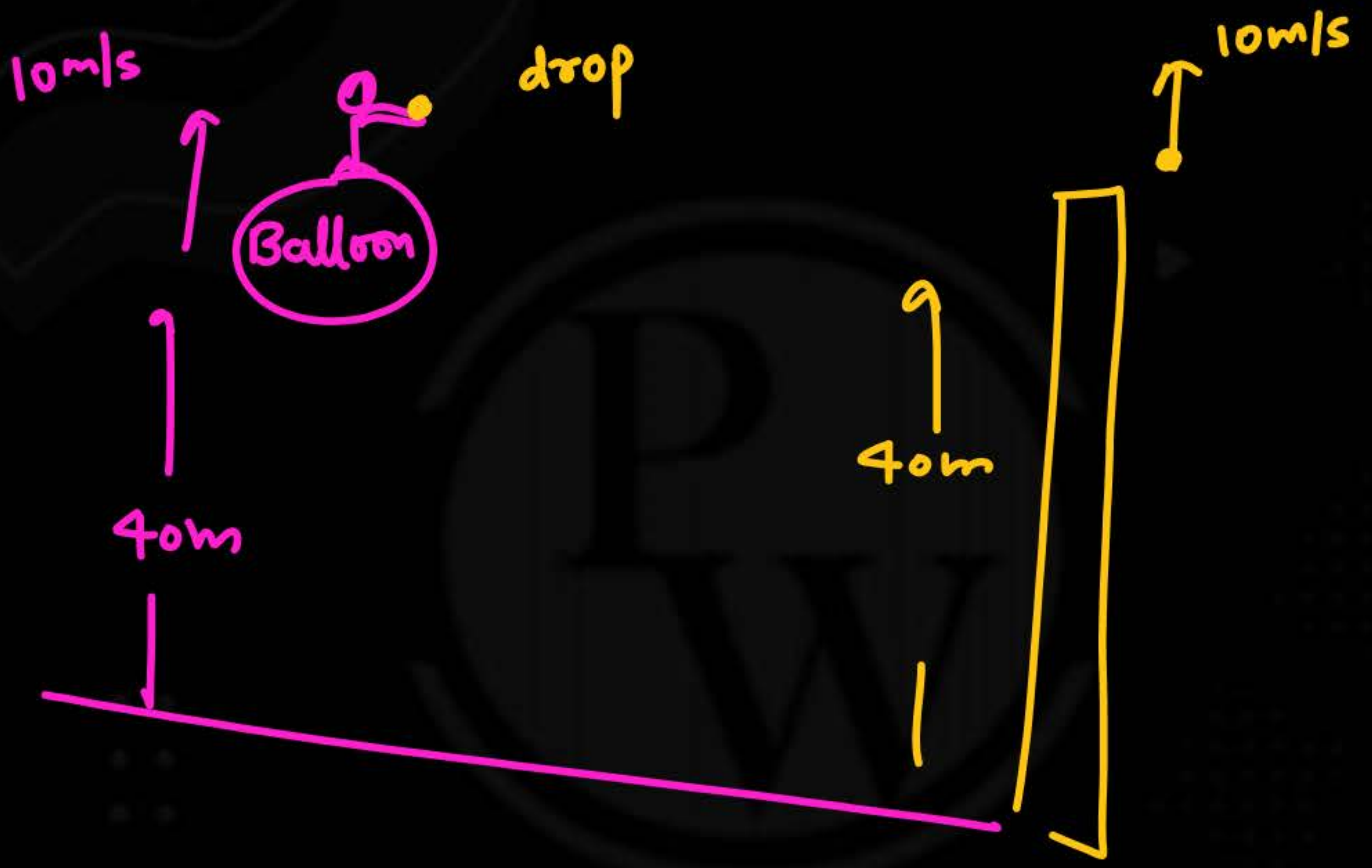


$u = +10$ (upward +ve)

$a = -10$

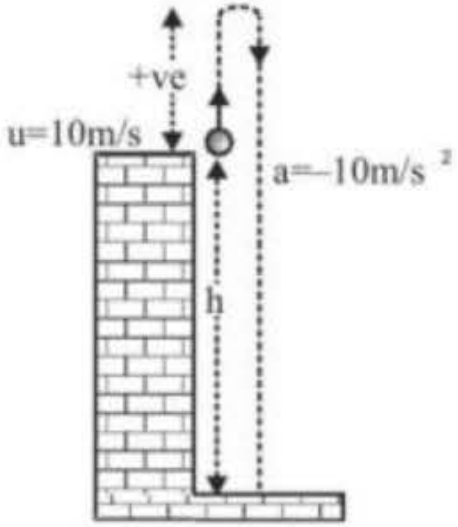
$s = -40$

$$-40 = 10 \times t - \frac{1}{2} \times 10 \times t^2$$


10m/s
drop
Balloon
40m
40m
10m/s

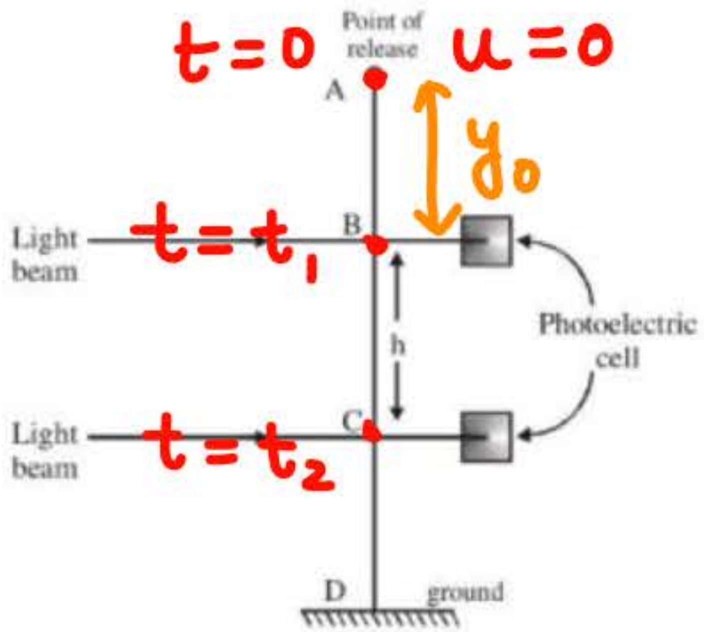

A
B
10m

Ex. A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10\ m/s^2$)



The acceleration of free fall at a planet is determined by timing the fall of a steel ball photo –electrically. The ball passes B and C at times t_1 and t_2 after release from A. The acceleration of free fall is given by

स्टील गेंद को प्रकाश वैद्युत रूप से किसी ग्रह पर मुक्त रूप से गिराकर, गिरने के समय द्वारा उस ग्रह के गुरूत्वीय त्वरण को ज्ञात किया जाता है। A से छोड़ने के बाद गेंद t_1 तथा t_2 समय पर B तथा C से गुजरती है। मुक्त रूप से गिरने के त्वरण को निम्न व्यंजक द्वारा दिया जाता है :-

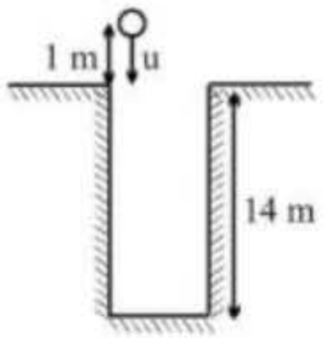


$$y_0 = 0 + \frac{1}{2}at_1^2$$

$$y_0 + h = 0 + \frac{1}{2} \times at_2^2$$

A boy throws a ball with speed u in a well of depth 14 m as shown. On bounce with bottom of the well the speed of the ball gets halved. What should be the minimum value of u (in m/s) such that the ball may be able to reach his hand again? It is given that his hands are at 1 m height from top of the well while throwing and catching.

एक लड़का किसी गेंद को u चाल से चित्रानुसार 14 m गहरे कुँए में फेंकता है। कुँए के तल से टकराने पर गेंद की चाल आधी हो जाती है। u (m/s में) का न्यूनतम मान क्या होना चाहिये ताकि गेंद पुनः उसके हाथों तक पहुँच सके? गेंद को फेंकते तथा पकड़ते समय लड़के के हाथ कुँए के शीर्ष से 1 m की ऊँचाई पर होते है।



A balloon rises from rest on the ground with constant acceleration $\frac{g}{3}$. A stone is dropped when the balloon has rises to a height 60 metre. The time taken by the stone to reach the ground is.

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

$$a = v\frac{dv}{dx}$$

Q $v = x^3 + 2x^2$

find acc at $x = 1$

$$a = v\frac{dv}{dx}$$

$$= (x^3 + 2x^2)(3x^2 + 4x)$$

put

. A particle is projected with velocity v_0 along x–axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e., $a = -\alpha x^2$. The distance at which the particle stops is:-

एक कण x अक्ष के अनुदिश v_0 वेग से प्रक्षेपित किया जाता है। कण का मंदन, मूल बिन्दु से इसकी दूरी के वर्ग के समानुपाती है अर्थात् $a = -\alpha x^2$ है। किस दूरी पर कण रूक जायेगा ?

(A) $\sqrt{\frac{3v_0}{2\alpha}}$ (B) $\left(\frac{3v_0}{2\alpha}\right)^{\frac{1}{3}}$ (C) $\sqrt{\frac{3v_0^2}{2\alpha}}$ (D) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

distance $\rightarrow x$

stops $\Rightarrow v = 0$

$a = -\alpha x^2$

a, x

v_0 ⟶ - - - - $v_f = 0$

$0 \qquad x_0$

$a = -\alpha x^2$

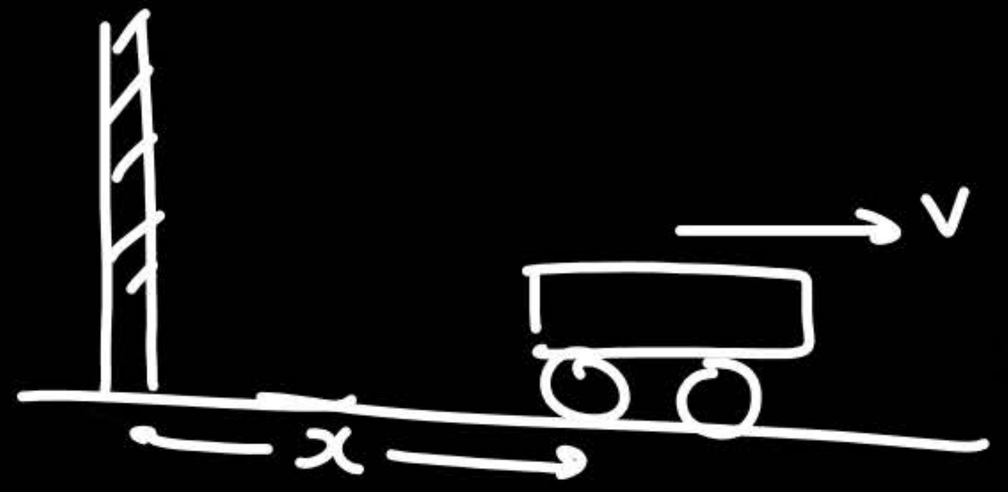
$v\frac{dv}{dx} = -\alpha x^2$

$$\int_{v_0}^{0} v\,dv = -\alpha \int_{0}^{x_0} x^2\,dx$$

Velocity of a car depends on its distance ℓ from a fixed pole on a straight road as $v = 2\sqrt{\ell}$, where ℓ is in meters and v in m/s. Find acceleration (in m/s^2) when $\ell = 8$m.

$$\frac{dv}{dx} = 2\,\frac{1}{2}\,x^{-\frac{1}{2}}$$

$$v = 2\sqrt{x} = 2x^{\frac{1}{2}}$$



$$v^2 = 4x$$

$$v^2 = 0^2 + 2 \times 2 \times x$$

$$v^2 = u^2 + 2as$$

$$a = v\frac{dv}{dx}$$

$$= 2\sqrt{x} \times x^{-\frac{1}{2}}$$

$$= 2x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}}$$

$$= 2$$

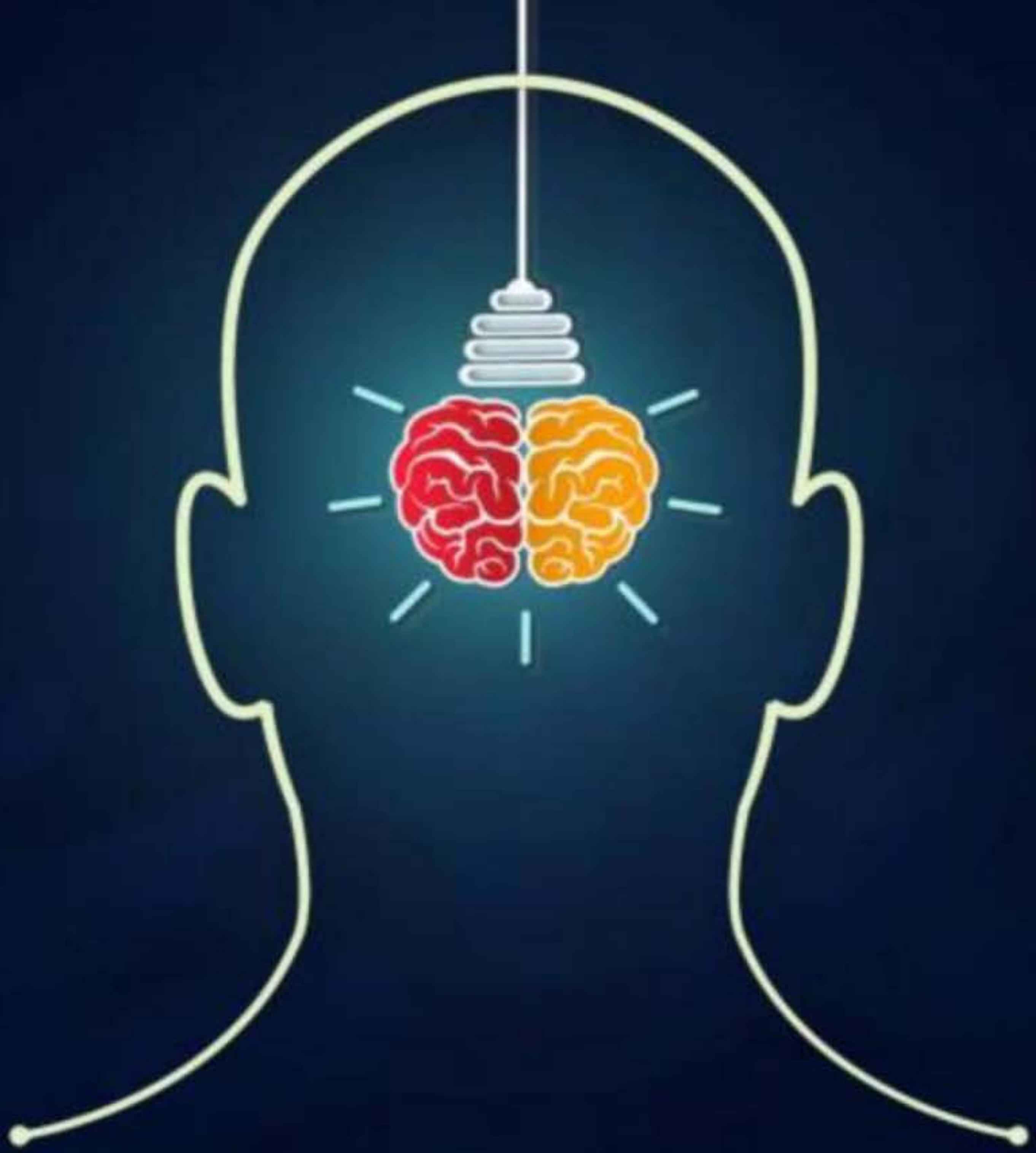
$x = 8$ पर acc

x, a

Ex. A rocket is fired vertically up from the ground with a resultant vertical acceleration of $10 m/s^2$. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After finishing fuel, calculate the time for which it continues its upwards motion. (Take $g = 10 m/s^2$)



THANK YOU